Ficha nº 8 - Intervalos de confiança — Estatística Aplicada

1. a) $P = 95\%$ $\quad \alpha = 1 - 0.95 = 0.05$

$m = 20$

$\sigma^2 = 225$ $\quad \mu = \bar{x} \pm z_{(1-\alpha/2)} \frac{\sigma}{\sqrt{m}} = 64.3 \pm 1.96 \times \frac{\sqrt{225}}{\sqrt{20}}$

$\bar{x} = 64.3$ $\quad = 64.3 \pm 6.57$

$z_{1-0.05/2} = z_{0.975} = 1.96$

b) $\alpha = 1 - 0.90 = 0.10$ $\quad \mu = \bar{x} \pm z_{1-\alpha/2} = 64.3 \pm 1.65 \times \frac{\sqrt{225}}{\sqrt{20}}$

$z_{1-\alpha/2} = z_{1-0.10/2} = z_{0.95} = 1.65$ $\quad = 64.3 \pm 5.53$

2. 2.3 1.9 2.1 2.8 2.3 3.6 1.4 1.8 2.1 3.2 2.0 1.9

a) $\bar{x} = \frac{2.3 + 1.9 + 2.1 + 2.8 + 2.3 + 3.6 + 1.4 + 1.8 + 2.1 + 3.2 + 2.0 + 1.9}{12} = 2.28$

$$s = \sqrt{\frac{(2.3-2.28)^2 + (1.9-2.28)^2 + (2.1-2.28)^2 + (2.8-2.28)^2 + (2.3-2.28)^2 + (3.6-2.28)^2 + (1.4-2.28)^2 + *}{12-1}} =$$

$* \Rightarrow + (1.8-2.28)^2 + (2.1-2.28)^2 + (3.2-2.28)^2 + (2.0-2.28)^2 + (1.9-2.28)^2 \quad = 0.625$

$P = 99\%$ $\quad \alpha = 1 - 0.99 = 0.01$

$t_{\alpha/2,\, m-1} = t_{0.005,\, 11} = 3.106$

$\bar{x} \pm t_{\alpha/2,\, m-1} \times \frac{s}{\sqrt{m}} = 2.28 \pm 3.106 \times \frac{0.625}{\sqrt{12}} = 2.28 \pm 0.56$

b)

90%	95%
$\alpha = 1 - 0.90 = 0.10$	$\alpha = 1 - 0.95 = 0.05$
$t_{0.05,\, 11} = 1.796$	$t_{0.025,\, 11} = 2.201$
$1.796 \times \frac{0.625}{\sqrt{12}} = 0.324$	$2.201 \times \frac{0.625}{\sqrt{12}} = 0.4$
2.28 ± 0.32	2.28 ± 0.4

3. $m = 100$ $\quad \bar{x} = 177500$ $\quad s = 9000$ $\quad P = 95\%$

$\alpha = 1 - 0.95 = 0.05$

$z_{1-\alpha/2} \frac{s}{\sqrt{m}} = z_{0.975} \times \frac{9000}{\sqrt{100}} = 1.96 \times 900 = 1764$

177500 ± 1764

$175736 < \mu < 179264$

4. $\bar{x}=45 \quad \sigma=5.8 \quad p=95\% \quad \alpha=0.05$

a) $m=30$

$z_{1-\alpha/2} = z_{1-0.05/2} = z_{0.975} = 1.96$

$$\bar{x} \pm z_{1-\alpha/2} \frac{\sigma}{\sqrt{m}} = 45 \pm 1.96 \times \frac{5.8}{\sqrt{30}} = 45 \pm 2.08$$

$$42.92 < \mu < 47.08$$

b) $m=60$

$$\bar{x} \pm z_{1-\alpha/2} \frac{\sigma}{\sqrt{m}} = 45 \pm 1.96 \times \frac{5.8}{\sqrt{60}} = 45 \pm 1.47$$

$$43.53 < \mu < 46.47$$

c) $m=90$

$$\bar{x} \pm z_{1-\alpha/2} \frac{\sigma}{\sqrt{m}} = 45 \pm 1.96 \times \frac{5.8}{\sqrt{90}} = 45 \pm 1.20$$

$$43.8 < \mu < 46.2$$

5. $P\left[\bar{X}_1 - \bar{X}_2 - \frac{\sigma}{5} < \mu_1 - \mu_2 < \bar{X}_1 - \bar{X}_2 + \frac{\sigma}{5}\right] = 0.90$

$\alpha = 1 - 0.90 = 0.10$

$$z_{1-\alpha/2} \sqrt{\frac{\sigma^2+\sigma^2}{m}} = \frac{\sigma}{5} \Leftrightarrow z_{1-0.10/2} \sqrt{\frac{2\sigma^2}{m}} \Leftrightarrow z_{0.950}\, \sigma \sqrt{\frac{2}{m}} = \frac{\sigma}{5}$$

$$\Leftrightarrow \frac{1.645\sqrt{2}}{\sqrt{m}} = \frac{1}{5} \Leftrightarrow m = (5 \times 1.645 \times \sqrt{2})^2 \Leftrightarrow m \simeq 136$$

6. $Y-X \quad 100 \quad 20 \quad -20 \quad 30 \quad 70 \qquad m=5$

$$\bar{D}_f = \frac{100+20-20+30+70}{5} = 40 \qquad \alpha = 1-0.90 = 0.10$$

$$\sigma_m = \sqrt{\frac{(100-40)^2+(20-40)^2+(-20-40)^2+(30-40)^2+(70-40)^2}{4}} = 46.37$$

$$t_{\alpha/2,\, m-1} = t_{0.05,\,4} = 2.132$$

$$\bar{D}_f \pm t_{\alpha/2,\, m-1} \cdot \frac{\sigma_m}{\sqrt{m}} = 40 \pm 2.132 \times 46.37/\sqrt{5}$$

$$= 40 \pm 44.2119$$

$$-4.2119 < \mu_y - \mu_x < 84.2119$$

7. a) $\bar{x}_A = 8260$ $s_A = 251.89$ $P = 99\%$ $n = 5$

$\bar{x}_B = 7930$ $s_B = 206.52$ $\alpha = 1 - 0.99 = 0.01$

$GL = 5 + 5 - 2 = 8$

$t_{\alpha/2, GL} = t_{0.005, 8} = 3.355$ $s_p^2 = \frac{(5-1)\,251.89 + (5-1)\,206.52}{5+5-2} = 229.205$

$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2, GL} \cdot s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 8260 - 7930 \pm 3.355 \cdot 229.205 \sqrt{\frac{1}{5} + \frac{1}{5}}$

$= 330 \pm 486.35$

b) $P = 90\%$ $\alpha = 1 - 0.90 = 0.10$

$t_{0.05, 8} = 1.860$ $330 \pm 1.860 \cdot 229.205 \sqrt{\frac{1}{5} + \frac{1}{5}} = 330 \pm 269.63$

8. a) $n_1 = 90$ $\bar{x}_1 = 91.1$ $s_1 = 5.4$ $P = 95\%$

$n_2 = 50$ $\bar{x}_2 = 92.3$ $s_2 = 7.6$ $\alpha = 1 - 0.95 = 0.05$

$(\bar{x}_1 - \bar{x}_2) \pm z_{1-\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 91.1 - 92.3 \pm z_{0.975} \sqrt{\frac{5.4^2}{50} + \frac{7.6^2}{50}}$

$= -1.2 \pm 1.96 \sqrt{\frac{5.4^2}{50} + \frac{7.6^2}{50}}$

$= -1.2 \pm 2.58$

b) $z_{1-\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 2.58$

9. a) $p = \frac{70}{250} = 0.28$

b) $P = 95.4\%$ $\alpha = 1 - 0.954 = 0.046$

$z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{n}} = z_{1-0.046/2} \sqrt{\frac{0.28(1-0.28)}{250}} = 0.0568$

$z_{0.977} = 2.00$

10. $n = 500$ $p = \frac{41}{500} = 0.082$ $P = 95\%$

$\alpha = 1 - 0.95 = 0.05$

$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{n}} = 0.082 \pm 1.96 \sqrt{\frac{0.082(1-0.082)}{500}} = 0.082 \pm 0.024$

$z_{1-0.05/2} = z_{1.975} = 1.96$

11. $m = 150$ $\quad p = \frac{30}{150} = 0.2$ $\quad P = 95\%$

$\alpha = 1 - 0.95 = 0.05$

$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{m}} = 0.2 \pm 1.96 \times \sqrt{\frac{0.2 \times 0.8}{150}} = 0.2 \pm 0.064$

$z_{1-0.05/2} = z_{0.975} = 1.96$

12. a) $m = 60$ $\quad x = 35$ $\quad p = \frac{35}{60} = 0.58$

b) $P = 95\%$ $\quad \alpha = 1 - 0.95 = 0.05$

$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{m}} = 0.58 \pm 1.96 \sqrt{\frac{0.58(1-0.58)}{60}} = 0.58 \pm 0.125$

$z_{1-\alpha/2} = z_{1-0.05/2} = z_{0.975} = 1.96$

13. $m = 400$ $\quad x = 140$

a) $P = 90\%$ $\quad \alpha = 1 - 0.90 = 0.10$

$p = \frac{140}{400} = 0.35$ $\quad z_{1-\alpha/2} = z_{1-0.10/2} = z_{0.950} = 1.65$

$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{m}} = 0.35 \pm 1.65 \times \sqrt{\frac{0.35 \times (1-0.35)}{400}} = 0.35 \pm 0.039$

b)

95%	98%
$\alpha = 1 - 0.95 = 0.05$	$\alpha = 1 - 0.98 = 0.02$
$z_{1-\alpha/2} = z_{1-0.05/2} = z_{0.975} = 1.96$	$z_{1-\alpha/2} = z_{1-0.02/2} = z_{0.990} = 2.33$
$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{m}} = 0.35 \pm 0.047$	$p \pm z_{1-\alpha/2} \sqrt{\frac{p(1-p)}{m}} = 0.35 \pm 0.056$

14. $x_A = 132$ $\quad x_B = 90$ $\quad P = 99\%$

$m_A = 400$ $\quad m_B = 150$ $\quad \alpha = 1 - 0.99 = 0.01$

$p_A = \frac{132}{400} = 0.33$ $\quad p_B = \frac{90}{150} = 0.6$ $\quad z_{1-\alpha/2} = z_{0.995} = 2.58$

$(p_A - p_B) \pm z_{1-\alpha/2} \sqrt{\frac{p_A(1-p_A)}{m_A} + \frac{p_B(1-p_B)}{m_B}} = -0.27 \pm 0.120$

15. $m_1 = m_2 = 1000$ $x_1 = 825$ $x_2 = 760$ $\alpha = 0.05$

$p_1 = \frac{825}{1000} = 0.825$ $p_2 = \frac{760}{1000} = 0.760$ $z_{1-\alpha/2} = z_{0.975} = 1.96$

$$(p_1 - p_2) \pm z_{1-\alpha/2} \sqrt{\frac{p_1(1-p_1)}{m_1} + \frac{p_2(1-p_2)}{m_2}} = 0.065 \pm 0.0354$$

16. $m = 10$ $\bar{x} = 7\,\Omega$ $\Omega = 4\,\Omega$ 90%

$\chi^2_{\alpha/2, m-1} = \chi^2_{0.05,9} = 16.91898$ $\alpha = 1 - 0.90 = 0.10$

$\chi^2_{1-\alpha/2, m-1} = \chi^2_{0.950,9} = 3.32511$

$$\sqrt{\frac{(m-1)S^2}{\chi^2_{\alpha/2,m-1}}} < \sigma < \sqrt{\frac{(m-1)S^2}{\chi^2_{1-\alpha/2,m-1}}} \Leftrightarrow \sqrt{\frac{9 \times 4^2}{16.91898}} < \sigma < \sqrt{\frac{9 \times 4^2}{3.32511}}$$

$$\Leftrightarrow \sigma \in]2.92, 6.58[$$

17. $m = 10$ $\bar{x} = 32.98\,\Omega$ $\Omega = 0.04\,\Omega$ 90%

$\chi^2_{\alpha/2, m-1} = \chi^2_{0.05,9} = 16.91898$ $\alpha = 1 - 0.90 = 0.10$

$\chi^2_{1-\alpha/2, m-1} = \chi^2_{0.950,9} = 3.32511$

$$\frac{(m-1)\Omega^2}{\chi^2_{\alpha/2,m-1}} < \sigma^2 < \frac{(m-1)\Omega^2}{\chi^2_{1-\alpha/2,m-1}} \Leftrightarrow \frac{9 \times 0.04^2}{16.91898} < \sigma^2 < \frac{9 \times 0.04^2}{3.32511}$$

$$\Leftrightarrow \sigma \in]0.000851, 0.0043[$$

18. $m_1 = 21$ $S_1^2 = 1432$ 95%

$m_2 = 25$ $S_2^2 = 3761$ $\alpha = 1 - 0.95 = 0.05$

$\nu_1 = 20$ $\nu_2 = 24$

$F_{\alpha/2, \nu_1, \nu_2} = F_{0.025, 20, 24} = 2.33$

$$F_{1-\alpha/2, \nu_1, \nu_2} = \frac{1}{F_{\alpha/2, \nu_2, \nu_1}} = \frac{1}{F_{0.025, 24, 20}} = \frac{1}{2.40} = 0.42$$

$$\frac{S_1^2}{S_2^2} \frac{1}{F_{\alpha/2, \nu_1, \nu_2}} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{S_1^2}{S_2^2} \frac{1}{F_{1-\alpha/2, \nu_1, \nu_2}}$$

$$\Leftrightarrow \frac{1432}{3761} \times \frac{1}{2.33} < \frac{\sigma_1^2}{\sigma_2^2} < \frac{1432}{3761} \times 2.40$$

$$\Leftrightarrow \frac{\sigma_1^2}{\sigma_2^2} \in]0.163, 0.9138[$$