Formulário

Fórmula fundamental dos erros $\boxed{\delta_f \leq M_{x_1}\delta_{x_1} + M_{x_2}\delta_{x_2} + ... + M_{x_n}\delta_{x_n}}$

em que $|\frac{\partial f}{\partial x_i}(\xi)| \leq M_{x_i}$ com $\xi \in [x_1 - \delta_{x_1}, x_1 + \delta_{x_1}] \times \cdots \times [x_n - \delta_{x_n}, x_n + \delta_{x_n}]$.

Uma equação não linear	
Método da Secante	**Método de Newton**
$x_{k+1} = x_k - \dfrac{(x_k - x_{k-1})f(x_k)}{f(x_k) - f(x_{k-1})}$, $k = 2, 3, ...$	$x_{k+1} = x_k - \dfrac{f(x_k)}{f'(x_k)}$, $k = 1, 2, ...$

Critério de Paragem:
$\dfrac{\lvert x_{k+1} - x_k\rvert}{\lvert x_{k+1}\rvert} \leq \epsilon_1$ e $\lvert f(x_{k+1})\rvert \leq \epsilon_2$

Método de Newton para sistemas de equações não lineares

Equação iterativa	**Jacobiano**	**Critério de Paragem**
$J(x^{(k)})\Delta_x = -f(x^{(k)})$ $x^{(k+1)} = x^{(k)} + \Delta_x$	$J = \begin{pmatrix} \frac{\partial f_1(x_1,x_2,\ldots,x_n)}{\partial x_1} & \cdots & \frac{\partial f_1(x_1,x_2,\ldots,x_n)}{\partial x_n} \\ \cdots & & \cdots \\ \frac{\partial f_n(x_1,x_2,\ldots,x_n)}{\partial x_1} & \cdots & \frac{\partial f_n(x_1,x_2,\ldots,x_n)}{\partial x_n} \end{pmatrix}$	$\dfrac{\lVert\Delta_x\rVert}{\lVert x^{(k+1)}\rVert} \leq \epsilon_1$ e $\lVert f(x^{(k+1)})\rVert \leq \epsilon_2$

Tabela das diferenças divididas

$$[x_j, x_{j+1}] = \frac{f_j - f_{j+1}}{x_j - x_{j+1}}, \qquad j = 0, \ldots, n-1 \qquad \text{(diferença dividida de ordem 1)}\ (dd1)$$

$$[x_j, x_{j+1}, x_{j+2}] = \frac{[x_j, x_{j+1}] - [x_{j+1}, x_{j+2}]}{x_j - x_{j+2}}, \qquad j = 0, \ldots, n-2 \qquad (dd2)$$

$$[x_0, x_1, \ldots, x_{n-1}, x_n] = \frac{[x_0, x_1, \ldots, x_{n-2}, x_{n-1}] - [x_1, x_2, \ldots, x_{n-1}, x_n]}{x_0 - x_n} \qquad (ddn)$$

$$[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

Polinómio interpolador de Newton

$$\boxed{p_n(x) = f_0 + (x - x_0)\,[x_0, x_1] + (x - x_0)(x - x_1)\,[x_0, x_1, x_2] + \cdots + (x - x_0)\cdots(x - x_{n-1})\,[x_0, \ldots, x_n]}$$

Erro de truncatura $\boxed{R_n(x) \leq |(x - x_0)(x - x_1)\cdots(x - x_n)\, dd_{n+1}|}$

Expressão do segmento i da *spline* cúbica

$$\boxed{\begin{aligned} s_3^i(x) = &\ \tfrac{M_{i-1}}{6(x_i - x_{i-1})}(x_i - x)^3 + \tfrac{M_i}{6(x_i - x_{i-1})}(x - x_{i-1})^3 + \left[\tfrac{f(x_{i-1})}{(x_i - x_{i-1})} - \tfrac{M_{i-1}(x_i - x_{i-1})}{6}\right](x_i - x) \\ &+ \left[\tfrac{f(x_i)}{(x_i - x_{i-1})} - \tfrac{M_i(x_i - x_{i-1})}{6}\right](x - x_{i-1}) \text{ para } i = 1, 2, \ldots, n \end{aligned}}$$

Expressão para os pontos interiores (nó i)

$$\boxed{\begin{aligned} &(x_i - x_{i-1})M_{i-1} + 2(x_{i+1} - x_{i-1})M_i + (x_{i+1} - x_i)M_{i+1} = \\ &\tfrac{6}{(x_{i+1} - x_i)}(f(x_{i+1}) - f(x_i)) - \tfrac{6}{(x_i - x_{i-1})}(f(x_i) - f(x_{i-1})) \end{aligned}}$$

***Spline* Natural**	***Spline* Completa**
$M_0 = 0$	$2(x_1 - x_0)M_0 + (x_1 - x_0)M_1 = \frac{6}{(x_1 - x_0)}(f(x_1) - f(x_0)) - 6f'(x_0)$
$M_n = 0$	$2(x_n - x_{n-1})M_n + (x_n - x_{n-1})M_{n-1} = 6f'(x_n) - \frac{6}{(x_n - x_{n-1})}(f(x_n) - f(x_{n-1}))$

Erro de truncatura *spline* cúbica $\boxed{|f(x) - s_3(x)| \leq \frac{5}{384}h^4 M_4}$ $\boxed{\left|f'(x) - s_3'(x)\right| \leq \frac{1}{24}h^3 M_4}$ com

$\max_{\xi \in [x_0, x_n]} |f^{(iv)}(\xi)| \leq M_4 \qquad h = \max_{0 \leq i \leq n-1}(x_{i+1} - x_i)$

Fórmulas simples Newton-Cotes		
Trapézio	$\int_a^b f(x)dx \approx \frac{(b-a)}{2}[f(a)+f(b)]$	$ET = \lvert -\frac{(b-a)^3}{12} f''(\xi)\rvert,\ \xi \in [a,b]$
Simpson	$\int_a^b f(x)dx \approx \frac{(b-a)}{6}\left[f(a)+4f(\frac{a+b}{2})+f(b)\right]$	$ET = \lvert -\frac{(b-a)^5}{2880} f^{(iv)}(\xi)\rvert,\ \xi \in [a,b]$
$\frac{3}{8}$	$\int_a^b f(x)dx \approx \frac{(b-a)}{8}\left[f(a)+3f(\frac{2a+b}{3})+3f(\frac{a+2b}{3})+f(b)\right]$	$ET = \lvert -\frac{(b-a)^5}{6480} f^{(iv)}(\xi)\rvert,\ \xi \in [a,b]$

Fórmulas compostas Newton-Cotes	
Trapézio	$\int_a^b f(x)dx \approx \frac{h}{2}[f_0+2f_1+2f_2+...+2f_{n-2}+2f_{n-1}+f_n]$ $ET = \lvert -\frac{h^2}{12}(b-a)f''(\eta)\rvert,\ \eta \in [a,b]$
Simpson	$\int_a^b f(x)dx \approx \frac{h}{3}[f_0+4f_1+2f_2+4f_3+...+4f_{n-3}+2f_{n-2}+4f_{n-1}+f_n]$ $ET = \lvert -\frac{h^4}{180}(b-a)f^{(iv)}(\eta)\rvert,\ \eta \in [a,b]$
$\frac{3}{8}$	$\int_a^b f(x)dx \approx \frac{3h}{8}[f_0+3f_1+3f_2+2f_3+...+2f_{n-3}+3f_{n-2}+3f_{n-1}+f_n]$ $ET = \lvert -\frac{h^4}{80}(b-a)f^{(iv)}(\eta)\rvert,\ \eta \in [a,b]$

Mínimos Quadrados (amostra m)

Polinómios ortogonais

$$p_n(x) = c_0P_0(x) + c_1P_1(x) + \ldots + c_nP_n(x)$$

$$\boxed{P_{i+1} = A_i(x-B_i)P_i(x) - C_iP_{i-1}(x)},\ \boxed{P_0(x)=1},\ \boxed{P_{-1}(x)=0}$$

$$\boxed{A_i = 1}\ \boxed{B_i = \frac{\sum_{j=1}^m x_jP_i(x_j)\,P_i(x_j)}{\sum_{j=1}^m P_i(x_j)\,P_i(x_j)}}\ \boxed{C_0 = 0 \text{ e } C_i = \frac{\sum_{j=1}^m P_i(x_j)\,P_i(x_j)}{\sum_{j=1}^m P_{i-1}(x_j)\,P_{i-1}(x_j)}}$$

Coeficientes do modelo polinomial

$$\boxed{c_i = \frac{\sum_{j=1}^m f_jP_i(x_j)}{\sum_{j=1}^m P_i(x_j)^2},\ \ i=0,\ldots,n}$$

Modelo não polinomial linear

$$M(x;c_1,c_2,\ldots,c_n) = c_1\phi_1(x) + c_2\phi_2(x) + \ldots + c_n\phi_n(x)$$

$$\begin{pmatrix} \sum_{j=1}^m \phi_1^2(x_j) & \ldots & \sum_{j=1}^m \phi_1(x_j)\phi_n(x_j) \\ \ldots & \ldots & \ldots \\ \sum_{j=1}^m \phi_n(x_j)\phi_1(x_j) & \ldots & \sum_{j=1}^m \phi_n^2(x_j) \end{pmatrix} \begin{pmatrix} c_1 \\ \ldots \\ c_n \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^m f_j\phi_1(x_j) \\ \ldots \\ \sum_{j=1}^m f_j\phi_n(x_j) \end{pmatrix}$$

Resíduo $\boxed{\sum_{j=1}^m (f_j - M(x_j))^2}$