

## How to apply advantage to a PDF?

In the old ProbDist implementation, this was done with the following code:

```
fn apply_positive_advantage(&mut self, adv: usize) {
    for _ in 0..adv {
        let mut total_above = 0.0;
        let mut next = BTreeMap::new();
        let cumulative = self.get_cumulative_prob();
        let mut rev_iter = self.keys().rev();
        let first = rev_iter.next().unwrap();
        // The cumulative probabilities, in descending order, with the first one
        // skipped and a zero appended to the end
        // Because we always need the cumulative probability of the  $X \leq x - 1$ 
        let mut rev_cumul =
cumulative.values().rev().skip(1).chain([0.0]).iter();
        //  $P_a(X = \max) = 1 - P(X \leq \max - 1)^2$ 
        total_above += 1.0 - rev_cumul.next().unwrap().powi(2);
        next.insert(*first, total_above);

        // Now for the rest of them
        for (outcome, cumul_below) in rev_iter.zip(rev_cumul) {
            //  $P_a(X=x<\max) = 1 - P(X \leq x - 1)^2 - P_a(X > x)$ 
            let new_prob = 1.0 - cumul_below.powi(2) - total_above;
            // Add new probability to total
            total_above += new_prob;
            // Insert new entry
            next.insert(*outcome, new_prob);
        }

        // Next should now contain the probability weight function with advantage
        // applied
        *self = ProbDist(next);
    }
}
```

First of all, it is good to take note of the function signature. The keys don't have to change, and the ProbDist remains valid, so it can be 'mut &self'. The process the function follows to apply advantage is as follows:

1. Create new BTreeMap ( $P_a$ )
2. Get the cumulative PDF ( $P(X \leq x)$ )
3. Create a reverse iterator over the keys and cumulative pdf, then take out the first element of both
4. Add a zero to the end of the reverse cumulative pdf
5. Then calculate  $P_a(X = \max) = 1 - P(X \leq \max - 1)^2$  and insert in into the new BTreeMap
6. Then for the rest of the elements, calculate  $P_a(X = x < \max) = 1 - P(X \leq x - 1)^2 - P_a(X > x)$  and insert into the new BTreeMap.

From this it follows that:

$$P_a(X = x) = 1 - P(X \leq x - 1)^2 - P_a(X > x) \quad 1.$$

The following results from facts and logic (and Demorgan:  $A \cup B = \overline{\overline{A} \cap \overline{B}}$ ):

$$P(X \leq x - 1) = P(X < x)$$

$$P_a(X > x) = \text{Chance that one die rolls higher} \stackrel{\text{Demorgan}}{=} \text{Chance that both don't roll higher} = 1 - P(X \leq x)^2 \quad 2.$$

Now substituting these relations in  $P_a(X = x)$ :

$$\begin{aligned} P_a(X = x) &= 1 - P(X \leq x)^2 - 1 + P(X \leq x)^2 = P(X \leq x)^2 - P(X < x)^2 \\ &= \text{Chance that both roll } x \text{ or lower} - \text{Chance that both roll lower than } x \end{aligned} \quad 3.$$

This suggests that this is generalizable to higher order, and possibly negative, advantages. So where  $n$  is the level of advantage, the following may hold:

$$P_a(X = x) = P(X \leq x)^{n+1} - P(X < x)^{n+1} \quad 4.$$

The most efficient way of calculating this is probably this:

1. Let  $a = P(X < x)^{n+1}$  with a trailing 1
2. Then  $P_a(X = x) = a[x + 1] - a[x]$

The logic behind Equation 3 suggests that it is generalizable to the following:

$$P(X = \max(x_1, \dots, x_n)) = \prod_{i=1}^n P(X \leq x_i) - \prod_{i=1}^n P(X < x_i) \quad 5.$$

This is more difficult to implement for arbitrary random variables than the simple case of  $x_1 = x_2$  (AKA advantage).

## Disadvantage

From Equation 5, it follows that disadvantage can be based on the following

$$P(X = \min(x_1, \dots, x_n)) = \prod_{i=1}^n P(X \geq x_i) - \prod_{i=1}^n P(X > x_i) \quad 6.$$

, and the algorithm for  $x_1 = x_2$  might be (confirm with testing):

1. Let  $a = P(X > x)^{n+1}$  with a leading 1
2. Then  $P_a(X = x) = a[x] - a[x + 1]$

### Small note on composable operations

It feels like it should be possible to just use a lazy-iterator-like system to perform all these computations with great efficiency. This would be tricky to implement, because some algorithms require access to more than pointwise operations.