

Exercise 1

What are the values of weights w_0 , w_1 and w_2 for the perceptron with decision boundary illustrated in figure below? Assume the surface crosses the x_1 axis at -1 and the x_2 axis at 2.

$$w_0 = 2$$

$$w_1 = 1$$

$$w_2 = 2$$

Exercise 2

(a) Design a two-input perceptron that implements the Boolean function $A \text{ AND } (\text{NOT } B)$.

This is logically displayed as follows:

$$A \cap \neg B$$

x_1	x_2	$A \wedge \neg B$
0	0	0
1	0	1
0	1	0
1	1	0

With $\theta_0 + \theta_1 x_1 + \theta_2 x_2$ and picking $\theta_0 = -20$, $\theta_1 = 30$, $\theta_2 = -20$, the hypothesis will be:

$$h_{\theta}(x) = \begin{cases} 1 & \text{if } \theta_0 + \theta_1 x_1 + \theta_2 x_2 > 0 \\ -1 & \text{otherwise} \end{cases}$$

(b) Design a two-layer network of perceptrons that implements $A \text{ XOR } B$.

This is logically displayed as follows:

$$A \oplus B = (A \cap \neg B) \cup (\neg A \cap B)$$

Or, alternatively:

$$A \oplus B = (A \cup B) \cap \neg(A \cap B)$$

x_1	x_2	$A \oplus B$
0	0	0
1	0	1
0	1	1
1	1	0

Thus, it requires the input of two different two-input perceptrons to implement: either one of the options mentioned above. Since we already completed the part of the first option, let's continue with this:

With $\theta_0 + \theta_1 x_1 + \theta_2 x_2$ and picking $\theta_0 = 0$, $\theta_1 = 10$, $\theta_2 = 20$, the hypothesis will be:

$$h_{\theta}(x) = \begin{cases} 1 & \text{if } \theta_0 + \theta_1 x_1 < \theta_2 x_2 \text{ OR } \theta_0 + \theta_2 x_2 < \theta_1 x_1 \\ -1 & \text{otherwise} \end{cases}$$