CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q1. In quadrilateral $CBAD$ as shon in figure 1,

$$CA = AD \tag{1}$$

and BA bisect $\angle A$. Show that $\triangle CAB \cong \triangle DAB$. What can you say about BC and BD?

Construction

The input parameters for construction:

Symbol	**Values**	**Description**
θ	$30°$	$\angle BAD = \angle BAC$
a	9	AB
c	5	AC
e1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	basis vector

$$A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, B = \begin{pmatrix} a \\ 0 \end{pmatrix}, C = \begin{pmatrix} c.cos\theta \\ c.sin\theta \end{pmatrix}, D = \begin{pmatrix} c.cos\theta \\ -c.sin\theta \end{pmatrix} \tag{2}$$

Solution:

It is given that AC and AD are equal i.e.,

$$CA = AD \tag{3}$$

and the line segment AB bisects $\angle A$.

To Prove:

The triangles ACB and ADB are similar i.e., $\triangle ACB \cong \triangle ADB$

Proof:

Consider the triangles $\triangle CAB$ and $\triangle CAD$

1. Now, consider equation of AB as $y = 0$ which can be written as

$$\mathbf{n}^\top X = 0 \tag{4}$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{5}$$

2. Next, we can consider the vertex B as

$$B = a(e1) \tag{6}$$

where

$$\mathbf{e1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

3. From the above assumptions, we get the coordinates of C and D as

$$\mathbf{C} = \begin{pmatrix} ccos\theta \\ csin\theta \end{pmatrix} \mathbf{D} = \begin{pmatrix} ccos\theta \\ -csin\theta \end{pmatrix} \tag{8}$$

4. Finding the angles(according to assumptions):

$$cos\angle CBA = \frac{(B-C)^\top (B-A)}{\|B-C\| \, \|B-A\|} \tag{9}$$

$$\implies ((\mathbf{B}-\mathbf{C})^\top)(\mathbf{B}-\mathbf{A}) = \begin{pmatrix} 9-5cos30^\circ \\ -5sin30^\circ \end{pmatrix}^\top \cdot \begin{pmatrix} 9 \\ 0 \end{pmatrix} \tag{10}$$

$$= 9^2 - 9.5.cos30^\circ - 9.5.sin30^\circ \tag{11}$$

$$= 81 - 45(\frac{1+\sqrt{3}}{2}) \tag{12}$$

$$\implies \|B-C\| \, \|B-A\| = (\sqrt{(9-5.cos30^\circ)^2 + (-5.sin30^\circ)^2})(9) \tag{13}$$

$$= (\sqrt{9^2+5^2-2.9.5.cos30^\circ})(9) \tag{14}$$

$$= 9\sqrt{106-45\sqrt{3}} \tag{15}$$

$$\tag{16}$$

From equations (12) and (14),

$$cos\angle CBA = \frac{9-5.cos30^\circ - 5.sin30^\circ}{\sqrt{9^2+5^2-2.9.5.cos30^\circ}} \tag{17}$$

$$= 65.8^\circ \tag{18}$$

$$\tag{19}$$

$$
\begin{aligned}
cos\angle ABD &= \frac{(B-D)^\top(B-A)}{\|B-D\|\,\|B-A\|} && (20)\\
Put\theta as-\theta in above equation & && (21)\\
\implies ((\mathbf{B}-\mathbf{D})^\top)(\mathbf{B}-\mathbf{A}) &= \begin{pmatrix} 9-5cos(-30)^\circ \\ 5sin(-30)^\circ \end{pmatrix}^\top \cdot \begin{pmatrix} 9 \\ 0 \end{pmatrix} && (22)\\
&= 9^2 - 9.5.cos(-30)^\circ + 9.5.sin(-30)^\circ && (23)\\
&= 81 - 45(\frac{\sqrt{3}+1}{2}) && (24)\\
\implies \|B-D\|\,\|B-A\| &= (\sqrt{(9-5.cos(-30)^\circ)^2 + (5.sin(-30)^\circ)^2})(9) && (25)\\
&= (\sqrt{9^2+5^2-2.9.5.cos(-30)^\circ})(9) && (26)\\
&= 9\sqrt{106-45\sqrt{3}} && (27)\\
& && (28)
\end{aligned}
$$

From equations (18) and (20),

$$
\begin{aligned}
cos\angle ABD &= \frac{9-5.cos30^\circ+5.sin30^\circ}{\sqrt{9^2+5^2-2.9.5.cos30^\circ}} && (29)\\
&= 65.8 && (30)\\
&= cos\angle CBA && (31)\\
& && (32)
\end{aligned}
$$

5. We know that sum of the angles in a triangle is 180° ,

$$
\begin{aligned}
\angle BAC &= 180^\circ - \angle CBA - \angle BCA && (33)\\
&= 180^\circ - \angle ABD - \angle DCA && (34)\\
&= 180^\circ - 65.8^\circ - 30^\circ && (35)\\
&= 84.2^\circ \text{ (From Eq.21 and given)} && (36)
\end{aligned}
$$

Since all the angles and sides of triangles CAB and CAD are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$
\begin{aligned}
\triangle CAB &\cong \triangle CAD && (37)\\
AB &= AD && (38)
\end{aligned}
$$

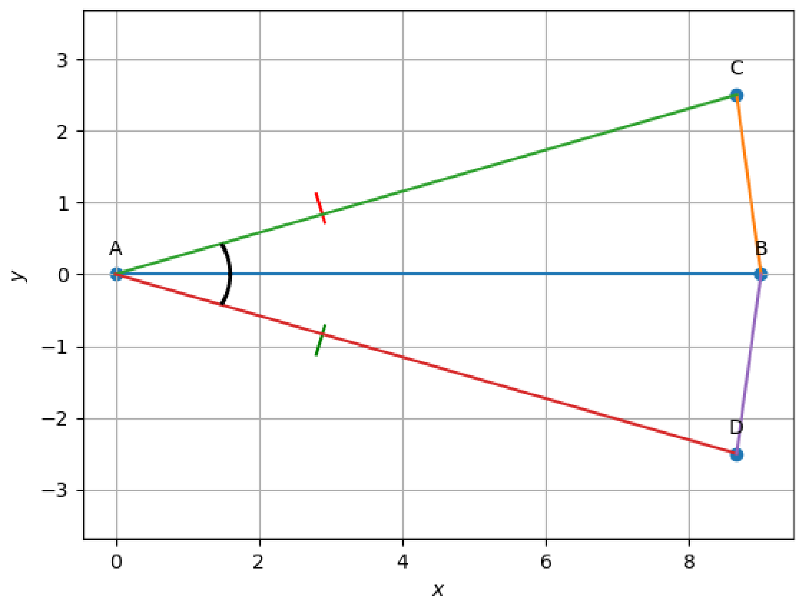

Figure 1: Quadrilateral CBAD