CHAPTER-7
TRIANGLES

1 Exercise 7.1

Q1. In quadrilateral $CBAD$,$CA = AD$ and BA bisect $\angle A$. Show that $\triangle CAB \cong \triangle DAB$. What can you say about BC and BD?

Construction

The input parameters for construction are shown in 1:

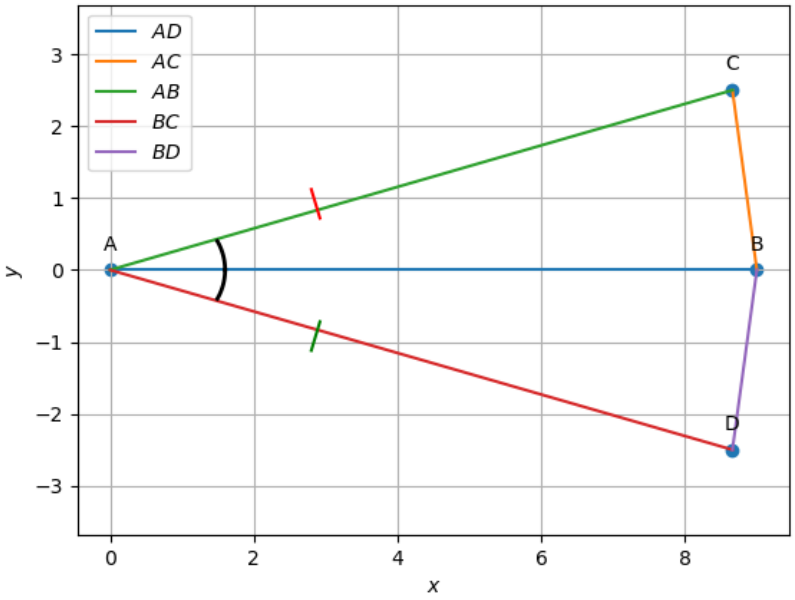

Figure 1: Quadrilateral CBAD

Symbol	Values	Description
θ	$30°$	$\angle BAD = \angle BAC$
a	9	AB
c	5	AC
$\mathbf{e}_1$	$\begin{pmatrix}1\\0\end{pmatrix}$	basis vector

Table 1: Parameters

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = a\mathbf{e_1}, \mathbf{C} = \begin{pmatrix} c\cos\theta \\ c\sin\theta \end{pmatrix}, \mathbf{D} = \begin{pmatrix} c\cos\theta \\ -c\sin\theta \end{pmatrix} \tag{1}$$

Solution:

$$C - A = A - D \tag{2}$$
$$\angle CAB = \angle DAB \tag{3}$$

To Prove:

$$\triangle ACB \cong \triangle ADB \tag{4}$$

Proof:
In $\triangle CAB$ and $\triangle CAD$
Let equation of AB be $y = 0$, which can be written as:

$$\mathbf{n}^\top X = 0, \tag{5}$$
$$\tag{6}$$

where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7}$$

Finding the angles(according to assumptions):

$$\text{Let } \theta_1 = \angle CBA \tag{8}$$
$$\mathbf{m_1} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} 4.7 \\ -2.5 \end{pmatrix}, \mathbf{m_2} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \tag{9}$$
$$\theta_1 = \cos^{-1} \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\| \|\mathbf{m_2}\|} \tag{10}$$
$$\implies \theta_1 = \cos^{-1} \frac{\begin{pmatrix} 4.7 & -2.5 \end{pmatrix} \begin{pmatrix} 9 \\ 0 \end{pmatrix}}{(9.2)(9)} = 59.3^\circ \tag{11}$$
$$\text{Let } \theta_2 = \angle ABD \tag{12}$$
$$\mathbf{n_1} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} -4.7 \\ 2.5 \end{pmatrix}, \mathbf{n_2} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} \tag{13}$$
$$\theta_2 = \cos^{-1} \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\| \|\mathbf{n_2}\|} \tag{14}$$
$$\implies \theta_2 = \cos^{-1} \frac{\begin{pmatrix} -4.7 & 2.5 \end{pmatrix} \begin{pmatrix} -9 \\ 0 \end{pmatrix}}{(9.2)(9)} = 59.3^\circ \tag{15}$$
$$\tag{16}$$

from (11) and (15)

$$\angle \text{ BAC} = \angle \text{ BAD} \quad (\text{Sum of the angles in a triangle} = 180^\circ)$$

Since all the angles and sides of triangles CAB and CAD are equal , from the definition of congruency both the triangles are said to be congruent to each other.

$$\triangle CAB \cong \triangle DAB \tag{17}$$

$$AB = AD \tag{18}$$