CHAPTER-7
TRIANGLES

EXERCISE 7.4

1. Find all the angles of an equilateral triangle.

2. The image of an object placed at a point A before a plane mirror LM is seen at the point B by an observer at D as shown in (1) Prove that the image is as far behind the mirror as the object is in front of the mirror.

   [Hint: CN is normal to the mirror. Also, angle of incidence = angle of reflection].

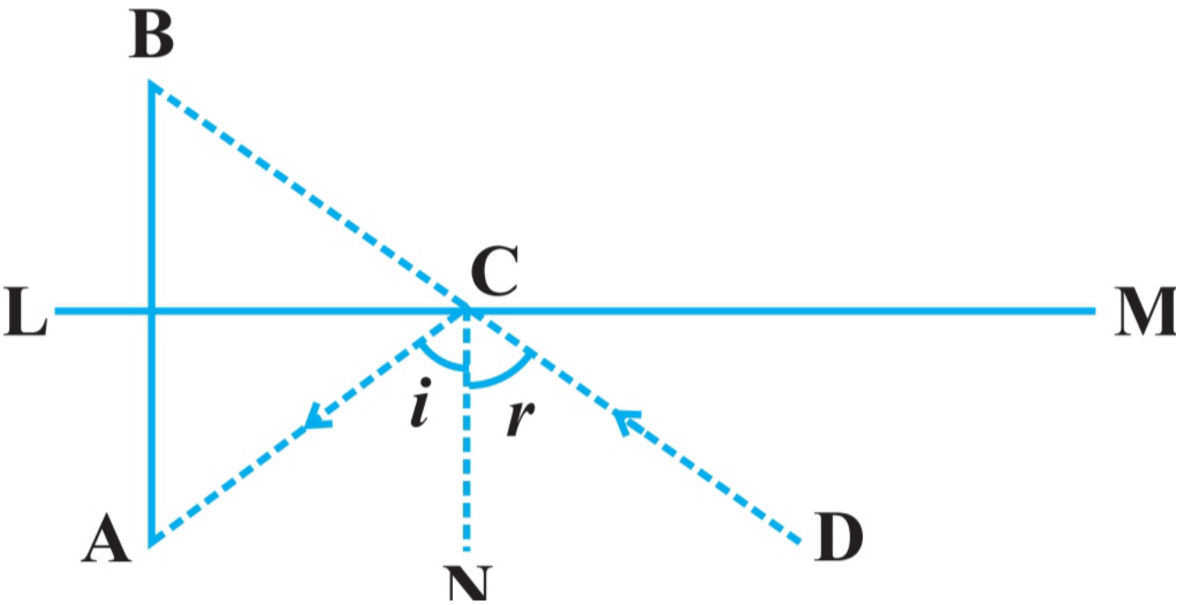

Figure 1

3. ABC is an isosceles triangle with $AB = AC$ and D is a point on BC such that $AD \perp BC$ (2). To prove that $\angle BAD = \angle CAD$, a student proceeded as follows:
   In $\triangle ABD$ and $\triangle ACD$,

$$AB = AC \quad \text{(Given)} \quad (1)$$
$$\angle B = \angle C \quad \text{(because } AB = AC\text{)} \quad (2)$$
$$\text{and } \angle ADB = \angle ADC \quad (3)$$
$$\therefore \triangle ABD \cong \triangle ACD \quad \text{(AAS)} \quad (4)$$
$$\text{So, } \angle BAD = \angle CAD \quad \text{(CPCT)} \quad (5)$$

   What is the defect in the above arguments?

[Hint: Recall how $\angle B = \angle C$ is proved when $AB = AC$].

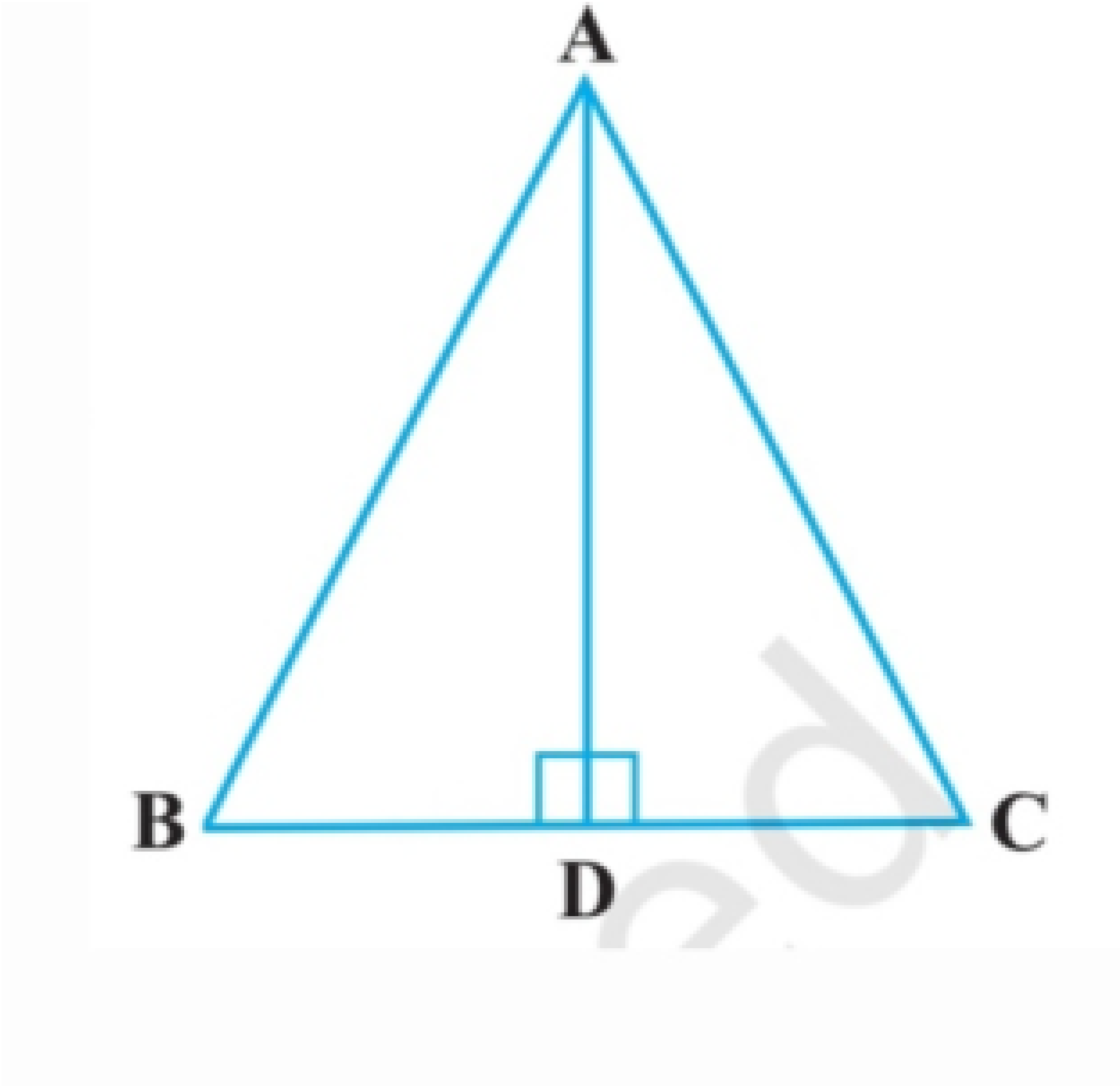

Figure 2

4. P is a point on the bisector of $\angle ABC$. If the line through P, parallel to BA meet BC at Q, prove that BPQ is an isosceles triangle.
5. $ABCD$ is a quadrilateral in which $AB = BC$ and $AD = CD$. Show that BD bisects both the angles ABC and ADC.
6. ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D. Prove that $BC = 2AD$
7. O is a point in the interior of a square $ABCD$ suchthat OAB is an equilateral triangle. Show that $\triangle OCD$ is an isosceles triangle.
8. ABC and DBC are two triangles on the same base BC such that A and D lie on the opposite sides of BC, $AB = AC$ and $DB = DC$. Show that AD is the perpendicular bisector of BC.

9. ABC is an isosceles triangle in which $AC = BC$. AD and BE are respectively two altitudes to sides BC and AC. Prove that $AE = BD$.
10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.
11. Show that in a quadrilateral $ABCD$,
$$AB + BC + CD + DA < (BD + AC) \tag{6}$$
12. Show that in a quadrilateral $ABCD$,
$$AB + BC + CD + DA > AC + BD \tag{7}$$
13. In a triangle ABC, D is the mid-point of side AC such that $BD = \frac{1}{2}AC$. Show that $\angle ABC$ is a right angle.
14. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.
15. Two lines l and m intersect at the point O and P is a point on a line n passing through the point O such that P is equidistant from l and m. Prove that n is the bisector of the angle formed by l and m.
16. Line segment joining the mid-points M and N of parallel sides AB and DC, respectively of a trapezium $ABCD$ is perpendicular to both the sides AB and DC. Prove that $AD = BC$.
17. $ABCD$ is a quadrilateral such that diagonal AC bisects the angles A and C. Prove that $AB = AD$ and $CB = CD$.
18. ABC is a right triangle such that $AB = AC$ and bisector of angle C intersects the side AB at D. Prove that $AC + AD = BC$.
19. AB and CD are the smallest and largest sides of a quadrilateral $ABCD$. Out of $\angle B$ and $\angle D$ decide which is greater.
20. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than $\frac{2}{3}$ of a right angle.
21. $ABCD$ is quadrilateral such that $AB = AD$ and $CB = CD$. Prove that AC is the perpendicular bisector of BD.