

## CHAPTER-7 TRIANGLES

### EXERCISE 7.4

- Find all the angles of an equilateral triangle.
- The image of an object placed at a point  $A$  before a plane mirror  $LM$  is seen at the point  $B$  by an observer at  $D$  as shown in (1) Prove that the image is as far behind the mirror as the object is in front of the mirror.

[Hint:  $CN$  is normal to the mirror. Also, angle of incidence = angle of reflection].

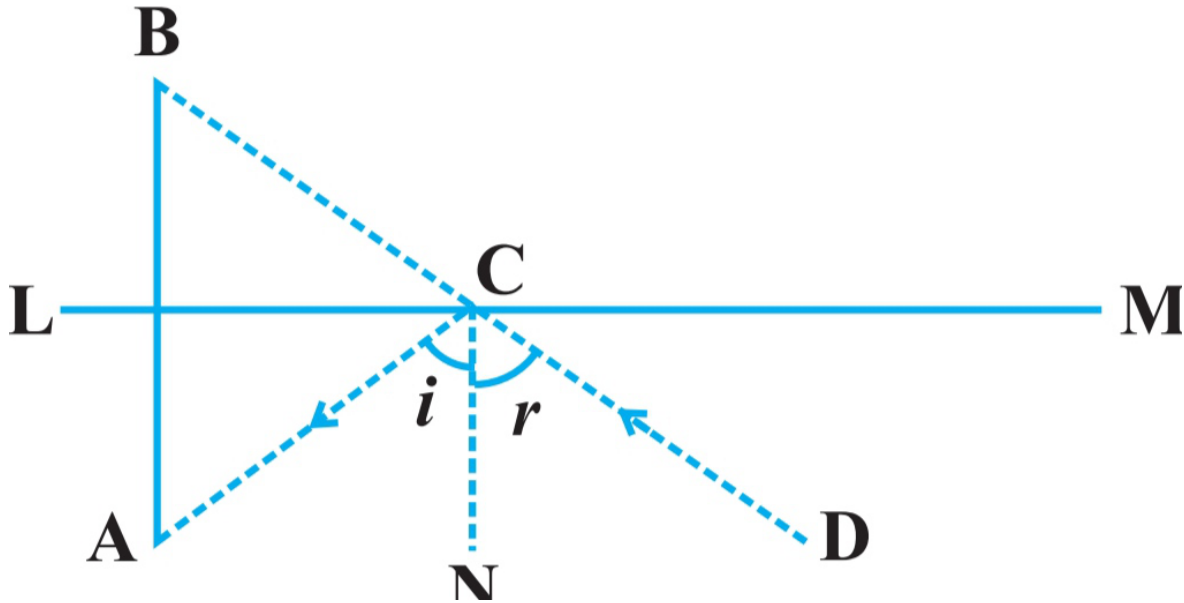


Figure 1

- $ABC$  is an isosceles triangle with  $AB = AC$  and  $D$  is a point on  $BC$  such that  $AD \perp BC$  (2). To prove that  $\angle BAD = \angle CAD$ , a student proceeded as follows:  
In  $\triangle ABD$  and  $\triangle ACD$ ,

$AB = AC$	(Given)	(1)
$\angle B = \angle C$	(because $AB = AC$ )	(2)
and $\angle ADB = \angle ADC$		(3)
$\therefore \triangle ABD \cong \triangle ACD$	(AAS)	(4)
So, $\angle BAD = \angle CAD$	(CPCT)	(5)

What is the defect in the above arguments?

[Hint: Recall how  $\angle B = \angle C$  is proved when  $AB = AC$ ].

- $P$  is a point on the bisector of  $\angle ABC$ . If the line through  $P$ , parallel to  $BA$  meet  $BC$  at  $Q$ , prove that  $BPQ$  is an isosceles triangle.
- $ABCD$  is a quadrilateral in which  $AB = BC$  and  $AD = CD$ . Show that  $BD$  bisects both the angles  $ABC$  and  $ADC$ .
- $ABC$  is a right triangle with  $AB = AC$ . Bisector of  $\angle A$  meets  $BC$  at  $D$ . Prove that  $BC = 2AD$

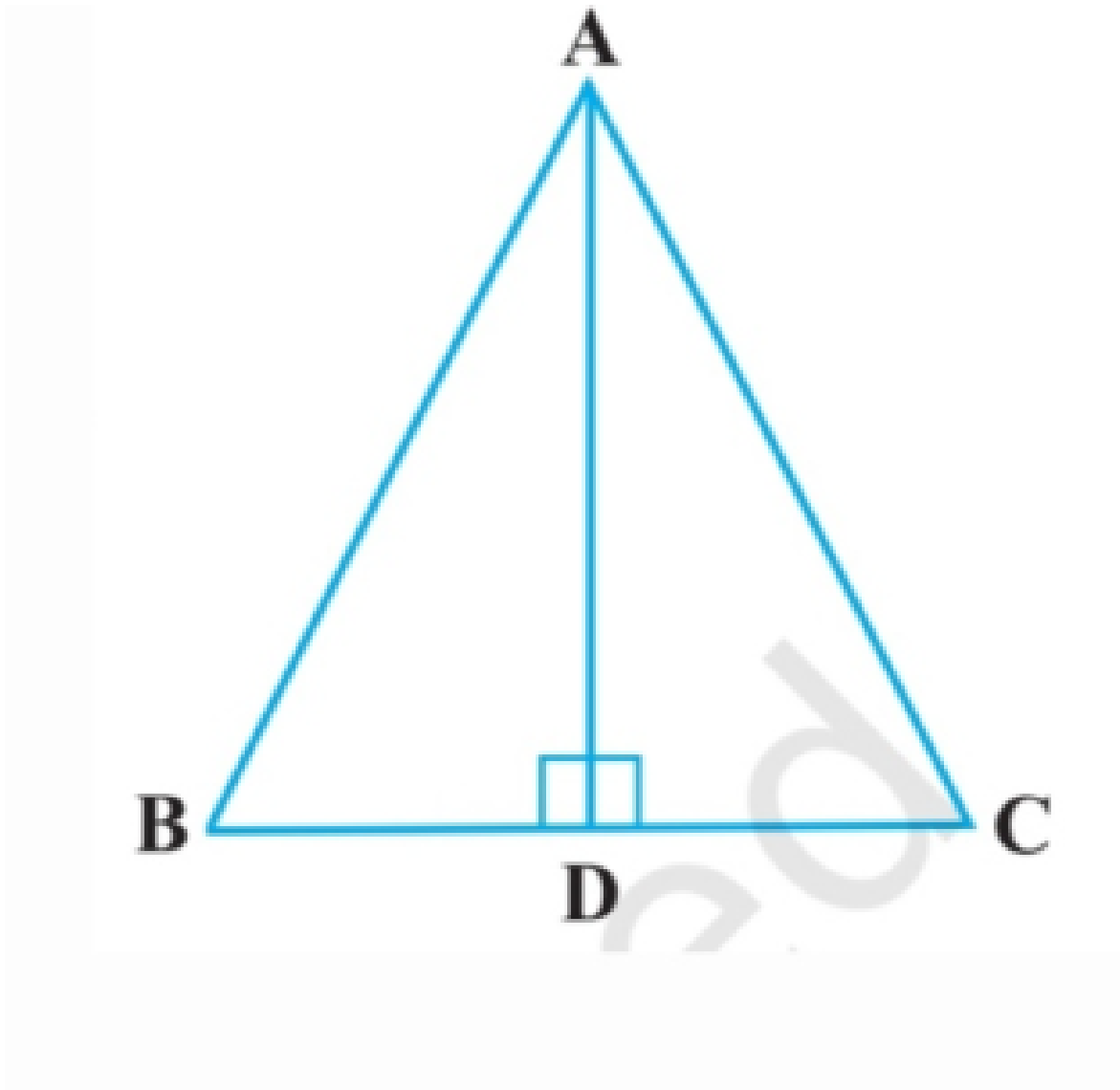


Figure 2

7.  $O$  is a point in the interior of a square  $ABCD$  such that  $OAB$  is an equilateral triangle. Show that  $\triangle OCD$  is an isosceles triangle.
8.  $ABC$  and  $DBC$  are two triangles on the same base  $BC$  such that  $A$  and  $D$  lie on the opposite sides of  $BC$ ,  $AB = AC$  and  $DB = DC$ . Show that  $AD$  is the perpendicular bisector of  $BC$ .
9.  $ABC$  is an isosceles triangle in which  $AC = BC$ .  $AD$  and  $BE$  are respectively two altitudes to sides  $BC$  and  $AC$ . Prove that  $AE = BD$ .
10. Prove that sum of any two sides of a triangle is greater than twice the median with respect to the third side.
11. Show that in a quadrilateral  $ABCD$ ,

$$AB + BC + CD + DA < (BD + AC) \quad (6)$$

12. Show that in a quadrilateral  $ABCD$ ,

$$AB + BC + CD + DA > AC + BD \quad (7)$$

13. In a triangle  $ABC$ ,  $D$  is the mid-point of side  $AC$  such that  $BD = \frac{1}{2}AC$ . Show that  $\angle ABC$  is a right angle.
14. In a right triangle, prove that the line-segment joining the mid-point of the hypotenuse to the opposite vertex is half the hypotenuse.
15. Two lines  $l$  and  $m$  intersect at the point  $O$  and  $P$  is a point on a line  $n$  passing through the point  $O$  such that  $P$  is equidistant from  $l$  and  $m$ . Prove that  $n$  is the bisector of the angle formed by  $l$  and  $m$ .
16. Line segment joining the mid-points  $M$  and  $N$  of parallel sides  $AB$  and  $DC$ , respectively of a trapezium  $ABCD$  is perpendicular to both the sides  $AB$  and  $DC$ . Prove that  $AD = BC$ .
17.  $ABCD$  is a quadrilateral such that diagonal  $AC$  bisects the angles  $A$  and  $C$ . Prove that  $AB = AD$  and  $CB = CD$ .
18.  $ABC$  is a right triangle such that  $AB = AC$  and bisector of angle  $C$  intersects the side  $AB$  at  $D$ . Prove that  $AC + AD = BC$ .
19.  $AB$  and  $CD$  are the smallest and largest sides of a quadrilateral  $ABCD$ . Out of  $\angle B$  and  $\angle D$  decide which is greater.
20. Prove that in a triangle, other than an equilateral triangle, angle opposite the longest side is greater than  $\frac{2}{3}$  of a right angle.
21.  $ABCD$  is quadrilateral such that  $AB = AD$  and  $CB = CD$ . Prove that  $AC$  is the perpendicular bisector of  $BD$ .