Difference equations and Yule-Walker equations

6/6 points (100.00%)

Quiz, 6 questions

Congratulations! You passed!

Next Item

1 / 1 points

1.
The following difference equation is given: $a_n = 4a_{n-1} - 3a_{n-2}$. What is the auxiliary or the characteristic equation?

- [x] $\lambda^2 - 4\lambda + 3 = 0$

Correct
Correct! $a_n = \lambda^n$ will give us the above equation.

- [ ] $\lambda^2 + 4\lambda - 3 = 0$
- [ ] $a_n - 4a_{n-1} + 3a_{n-2} = 0$

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1 / 1 points

2.
Solve the difference equation $a_n = 4a_{n-1} - 3a_{n-2}$.

- [x] $a_n = c_1 + c_2 3^n$

Correct
Correct! We take linear combination of $1^n = 1$ and 3^n.

- [ ] $a_n = 1 + 3^n$

Un-selected is correct

Difference equations and Yule-Walker equations

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Quiz, 6 questions

- [x] $a_n = c_1 3^n + c_2$

Correct

Correct! We take linear combination of 3^n and $1^n = 1$.

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1 / 1 points

3.

Solve the difference equation $a_n = 4a_{n-1} - 3a_{n-2}$ with initial data $a_0 = 2$ and $a_1 = -2$.

- [ ] $a_n = 6 - 4(2^n)$
- [x] $a_n = 4 - 2(3^n)$

Correct

Correct! For sanity check, we see that $a_0 = 4 - 2 = 2$ and $a_1 = 4 - 2(3) = -2$.

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1 / 1 points

4.

Stationary AR(1) process is given: $X_t = 0.4X_{t-1} + Z_t$. Find the Yule-Walker equations.

- [ ] $(1 - 0.4B)X_t = Z_t$
- [x] $\rho(k) = 0.4\rho(k-1)$ when $k \geq 1$.

Correct

Correct!

AR(1) process gives us Yule-Walker equations starting at lag 1.

- [ ] $\rho(k) = 0.4\rho(k-1)$ for all $k \in \mathbb{Z}$.

Difference equations and Yule–Walker equations

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Quiz, 6 questions

1 / 1 points

5.
Find the solution of the Yule-Walker equations of the process $X_t = 0.4X_{t-1} + Z_t$.

- [ ] $\rho(k) = c0.4^k$ for $k \geq 1$.

Un-selected is correct

- [x] $\rho(k) = 0.4^k$ for $k \geq 0$.

Correct
Correct! We get a geometric sequence that is decreasing to zero.

- [x] $\rho(k) = 0.4^k$ for $k \geq 0$, and $\rho(k) = \rho(-k)$ for $k \in \mathbb{Z}^-$.

Correct
Correct! $\rho(k)$ is an even function.

1 / 1 points

6.
Find the Yule-Walker equations and general solutions of them that govern autocorrelation coefficients of the AR(3) process

$X_t = \frac{1}{2}X_{t-1} + \frac{1}{9}X_{t-2} - \frac{1}{18}X_{t-3} + Z_t$

- [ ] $\rho(k) = \frac{1}{2}\rho(k-1) + \frac{1}{9}\rho(k-2) - \frac{1}{18}\rho(k-3)$

  $\rho(k) = c_1(\frac{1}{2})^k + c_2(\frac{1}{9})^k + c_3(-\frac{1}{18})^k$

- [ ] $18\rho(k) = 9\rho(k-1) + 2\rho(k-2) - \rho(k-3)$

  $\rho(k) = c_1(2)^k + c_2(9)^k + c_3(-18)^k$

$\rho(k) = \frac{1}{2}\rho(k-1) + \frac{1}{9}\rho(k-2) - \frac{1}{18}\rho(k-3)$

$\rho(k) = c_1(\frac{1}{2})^k + c_2(\frac{1}{3})^k + c_3(-\frac{1}{3})^k$

Difference equations and Yule–Walker equations

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Correct

Correct!

Characteristic equation can be written as $18\lambda^3 - 9\lambda^2 - 2\lambda + 1 = 0$. Factorization $(2\lambda - 1)(3\lambda - 1)(3\lambda + 1) = 0$ gives us the roots $\frac{1}{2}$, $\frac{1}{3}$ and $-\frac{1}{3}$.