Series, Backward Shift Operator, Invertibility and Duality

10/10 points (100.00%)

Quiz, 10 questions

✔ Congratulations! You passed!

Next Item

✔ 1 / 1 points

1.
Determine if the geometric series is convergent or divergent, and find the sum of the series if it is convergent.

$-3+\frac{3}{2}-\frac{3}{4}+\frac{3}{8}-\ldots$

- It is convergent, and the sum is $\frac{1}{2}$.
- It is divergent.
- ◉ It is convergent, and the sum is -2.

Correct
Correct! It is convergent since $r=-\frac{1}{2}$, and $\left|-\frac{1}{2}\right|<1$, and the sum is $\frac{a}{1-r}=\frac{-3}{1+\frac{1}{2}}=-2$.

✔ 1 / 1 points

2.
Express the rational function as a geometric series: $\frac{4}{1+x}$

- ☑ $4-4x+4x^2-4x^3+4x^4-\ldots$

Correct
Correct! We know $\frac{a}{1-r}=a+ar+ar^2+ar^3+\ldots$. In this case, $a=4$ and $r=-x$.

- ☑ $4(1-x+x^2-x^3+\ldots)$

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Correct

Correct! We know $\frac{a}{1-r} = a + ar + ar^2 + ar^3 + ...$ In this case, $a = 4$ and $r = -x$.

- [x] $4\sum_{n=1}^{n=\infty}(-1)^{n-1}x^{n-1}$

Correct

Correct! We know $\frac{a}{1-r} = a + ar + ar^2 + ar^3 + ...$ In this case, $a = 4$ and $r = -x$.

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1 / 1 points

3.

Express the following model by utilizing Backward shift operator.

$X_t = 0.5X_{t-1} + Z_t + 0.7Z_{t-1}$

- [ ] $(1 + 0.5B)X_t = (1 - 0.7B)Z_t$

Un-selected is correct

- [x] $(1 - 0.5B)X_t = (1 + 0.7B)Z_t$

Correct

Correct! We write $X_{t-1} = BX_t$ and $Z_{t-1} = BZ_t$.

- [x] $(1 - 0.5B)X_t = Z_t + 0.7Z_{t-1}$

Correct

Correct! We write $X_{t-1} = BX_t$. We can continue using B to write $Z_{t-1} = BZ_t$.

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1 / 1 points

4.

We write the model $X_t = X_{t-1} + 2X_{t-2} + Z_t$ as $\phi(B)X_t = Z_t$. What is $\phi(B)$?

$\phi(B) = (1 - B)(1 + 2B)$.

Un-selected is correct

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Quiz, 10 questions

$\phi(B) = (1 + B)(1 - 2B)$.

Correct

Correct! $1 - B - 2B^2 = (1 + B)(1 - 2B)$.

$\phi(B) = 1 - B - 2B^2$.

Correct

Correct! We obtain this since $X_t - X_{t-1} - 2X_{t-2} = Z_t$.

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1 / 1 points

5.

Is the following process invertible?

$X_t = Z_t + 3Z_{t-1}$

It is an invertible process since the coefficient 3 is larger than 1.

It is not an invertible process.

Correct

Correct! The root of the polynomial $1 + 3B$, $-\frac{1}{3}$, is not outside of a unit circle.

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1 / 1 points

6.

For what values of the θ, the process $X_t = Z_t - \theta Z_{t-1} - 6\theta^2 Z_{t-2}$ is an invertible process.

$|\theta| > \frac{1}{3}$

$|\theta| < \frac{1}{3}$

Series, Backward Shift Operator, Invertibility and Duality

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Quiz, 10 questions

Correct

Correct! Since $X_t = (1 - 3\theta B)(1 + 2\theta B)Z_t$, we need $|\theta| < \frac{1}{3}$ and $|\theta| < \frac{1}{2}$. But that means $|\theta| < \frac{1}{3}$.

$|\theta| < \frac{1}{2}$

1 / 1 points

7.

Is the AR(2) process $X_t = X_{t-1} + 2X_{t-2} + Z_t$ stationary?

It is a stationary process.

It is not a stationary process.

Correct

Correct! The roots of the AR polynomial are -1 and $\frac{1}{2}$, none of which are outside of the unit circle.

1 / 1 points

8.

Find all possible values of β so that the AR(2) process $X_t = 2\beta X_{t-1} - \beta^2 X_{t-2} + Z_t$ is stationary.

$|\beta| > 1$

$|\beta| < 1$

Correct

Correct! AR polynomial, $(1 - \beta B)^2$, has root $B = 1/\beta$. This root is outside of the unit circle if $|\beta| < 1$.

$|\beta| = 1$

1 / 1 points

Series, Backward Shift Operator, Invertibility and Duality

Quiz, 10 questions

10/10 points (100.00%)

9.

Determine if the process is stationary, invertible or both: $X_t = 0.5X_{t-1} + Z_t + 4Z_{t-1}$

- Invertible but not stationary.
- Neither stationary nor invertible.
- Stationary but not invertible.

Correct

Correct!

AR polynomial has root of 2 (outside of the unit circle) $\rightarrow$ stationary

MA polynomial has root of -0.25 (inside of the unit circle) $\rightarrow$ not invertible

- Stationary and invertible.

1 / 1 points

10.

Find all values of β and θ such that duality exists for the following process, i.e., it is stationary an invertible: $X_t = \beta^2 X_{t-1} + Z_t + 8\theta^3 Z_{t-1}$.

- $|\beta| > 1$ and $|\theta| > \frac{1}{2}$
- $|\beta| < 1$ and $|\theta| < \frac{1}{2}$

Correct

Correct! Roots of the polynomials are $\frac{1}{\beta^2}$ and $\frac{1}{8\theta^3}$. Thus we want them to be outside of the unit circle.

- $|\beta| < 1$ and $|\theta| > \frac{1}{2}$

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Quiz, 10 questions