Branching dynamics

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1 Introduction

To run your own simulations varying the parameters, or see other sets of parameters already simulated see this Github repository
We establish the ACL model and then make a section for each of the following questions.

Does Branching dynamics split the energy eigenspace between the branches in a particular way? Do the worlds share it equally? Proportional to their respective probabilities?

Having established that they share the eigenstates weirdly. Why? What drives this dynamics? Does it generalize beyond the ACL model in which we observe this dynamics?

Can we get it all from the energy spectrum and the distribution of the global state in it? Energy spectrum + state defined in energy basis → sharing of Hilbert space by worlds → explain the phenomena we observed

2 A measurement model

2.1 The Adapted Caldeira-Leggett dynamics

To answer these questions, we consdier the adapted Caldeira-Leggett model, a system $\mathcal{S}$ + environment $\mathcal{E}$ where the system is a truncated SHO. Its Hamiltonian takes the form,

$$H_{tot} = H_s + H_e + E_I H_I \tag{1}$$

We choose no self interaction for the truncated SHO, so $H_s = \mathbb{1} \otimes \mathbb{1}$, and we choose a random hermitian matrix for the environment self interaction $\mathbb{1} \otimes H_r$, with H_r a random hermitian matrix. The interaction Hamiltonian is $H_I = H_q \otimes H'_r$, with $(H_q)_{ij} = q_i \delta_{ij}$ and H'_r another random hermitian matrix. This makes it so that the environment acts on the system depending on its state.
Taking the eigenstates of H_q to be $\{|q_i\rangle\}\forall$, we note that $\{|q_i\rangle|k\rangle\}$, which form the energy eigenbais of H_{tot}, are also a basis in which H_e and H_I take the a block diagonal form. $i \in [0, d_1 - 1], j \in [0, d_2 - 1]$, d_1 and d_2 the dimensions of the system and environment respectively.

$$\begin{aligned} H_s &= \mathbb{1} \\ H_e &= H_r \otimes H_r \otimes H_r \otimes ... \\ H_I &= E_I q_0 H'_r \otimes ... \otimes E_I q_0 H'_r \otimes ... \otimes E_I q_1 H'_r \otimes ... \end{aligned}$$

We note the d_1 degeneracy of the spectrum of H_e and the no degeneracy of H_I. By our constrution, there is no degeneracy for H_{tot}

(a) Heat map of H_e (b) Heat map of H_I

Figure 1

Figure 2: Heat map of the Hamiltonians H_e and H_I in the eigenbasis of H_tot, for $d_1 = 10, d_2 = 200$.

2.2 The state

For $|q_i>$ the eigenstates of H_q and $\{|q_i\rangle|e_k>\}$ the eigenstates of H_e, we choose our initial state as being,

$$|\psi(0)\rangle = (\frac{|q_1\rangle}{\sqrt{w}} + \frac{|q_2\rangle}{\sqrt{1-w}})|e_i\rangle \tag{2}$$

which corresponds to a superposition of energy eigenstates not yet entangled with an environment, which we start in a random state. We also define here the eigensates of H_I to be ... and the eigenstates of H_{tot} to be $\{|q_i\rangle|k\rangle\}$

2.3 Time evolution and decoherence

With this in mind, our initial state is only defined in $2d_2$ out of $d_1.d_2$ energy eigenstates. Thus to understand the action of the $e^{-itH_{tot}}$ on our state, we only need to consider 3 different $d_2 * d_2$ matrices, $H_r, E_I q_1 H'_r, E_I q_2 H'_r$ (or 2 if we consider H_{tot}). We can write the time evolution,

$$\begin{aligned}
|\psi(t)\rangle &= e^{-itH_{tot}}|\psi(0)\rangle \\
&= e^{-itH_{tot}}\frac{|q_1\rangle}{\sqrt{w}}|e_i\rangle + e^{-itH_{tot}}\frac{|q_2\rangle}{\sqrt{1-w}}|e_i\rangle \\
&= \frac{|q_1\rangle}{\sqrt{w}}e^{-it(H_e^{(1)})}|e_i\rangle + \frac{|q_2\rangle}{\sqrt{1-w}}e^{-it(H_e^{(2)})}|e_i\rangle \\
&= |q_1\rangle|E_1(t)\rangle + |q_2\rangle|E_2(t)\rangle
\end{aligned}$$

We defined, $H_e^{(1)} = H_r + E_I q_1 H'_r$ and $H_e^{(2)} = H_r + E_I q_2 H'_r$, two matrices acting on $\mathcal{E}$. Note that $|\langle q_i|q_i\rangle|^2 = 1$, and so we make the $|E_i(t)\rangle = A_i(t)|\mathcal{E}_i(t)\rangle$ carry the amplitudes ($\langle\mathcal{E}_i(t)|\mathcal{E}_i(t)\rangle|^2 = 1$).

2.3.1 Existence of pointer states

Block diagonal Hamiltonians always allow for certain pointer states (See "A Search for Classical Subsystems in Quantum Worlds" Appendix A)

We find that $|q_i\rangle|E_i(t)\rangle = |p_i(t)\rangle$ are pointer states since the time evolution leaves the state as a separable state. The evolution of the pointer state, is only in the environment space and it is restrained to the subspace of the particular choice of pointer state. (ie it will generally be d_2 dimensional for us, but it can technically be less.).

2.3.2 Decoherence

For an arbitrary initial pure state, the system density matrix becomes diagonalin the pointer basis. SEE MY INITIAL STATE then find the reduced density matrix note that it depends on $c_{ij}(t) = \langle\phi_i(t)|\phi_j(t)\rangle$, where, using the fact that the environment drives the environment states into approximately orthogonal states, we have $c_{ij} \approx \delta_{ij}$

Given a system and an environment $\mathcal{S}+\mathcal{E}$ and a pure state defined over both $|\psi(t)\rangle$. It can always be decomposed in its Schmidt basis,

$$|\psi(t)\rangle = \sum_i \sqrt{p_i(t)}|a_i(t)\rangle|b_i(t)\rangle \tag{3}$$

The important points being that the sum is over one index, that the sum is over dimension of the smaller of both systems ($\mathcal{S}$ or $\mathcal{E}$) at most, and that the basis itself depends on the state, which implies basis varying in time.

Decoherence has it that $\langle b_j(t)|b_i(t)\rangle \to \delta_{ij}$ for $t \to \infty$. Pure decoherence, has the evolution of the system be,

$$U(t) = \sum_j |a_j\rangle\langle a_j| \otimes U_j^{\mathcal{E}}(t) \tag{4}$$

Starting the system in $|b\rangle$, then $U_j^{\mathcal{E}}|b >= |b_j(t)\rangle$. This means that the reduced density matrix of the system $\mathcal{S}$, evovlves as,

$$\rho_{ij} = \langle b_i(t)|b_i(t)\rangle\rho_{ij}(0) \tag{5}$$

This results in the evolution of the system density matrix to a diagonal form. This diagonal form, explains the apparent classicality of the unique measurment result. The 2 results, after some decoherence time, belong to 2 different orthogonal states.

Decoherence induced pointer states for our particular block diagonal case are such that they are defined over a limited subspace of the hilbert space. it is usefull to think of the Hilert space in the total energy eigenbasis, since in it, we see that the pointer states are defined in $d_{tot}/d_1 = d_2$ energy eigenbasis states. And importantly, 2 pointer states defined with different $|q_i\rangle$ have no energy eigenstate in common (supposing no degeneracy.)

2.3.3 Decoherence induces convergence of Schmidt to pointer states

What the Schmidt to pointer convergence shows is that it selects a generic pointer state, not a specifically choosen one. And from the relative weight of $|S_0\rangle$ in $|q_2\rangle$ (and vice versa) goes down over time, we have the convergence of the Schmidt states to the pointer states.

Schmidt states converge to pointer states because. pointer states are the diagonal terms of the density matrix, and the matrix goes to being only diagonal by decoherence... something like that. ====

In practice, the Hilbert space in which the pointer states are defined is d_2 dimensional and is $\{|q_1\rangle|v_k\rangle\}$ and $\{|q_2\rangle|v_k\rangle\}$ respectively. We can also individually look at the spectrum of $E_I q_i H'_r$, which for $i = 1$ corresponds to a thin distribution in the energy spectrum, like the blue spectrum in Fig. (**??**). For $i = 2$ it corresponds to one of the wide spectrums of Fig. (**??**). So Schmidt states converging to pointer states should be visible in their distribution in the interaction energy eigenbasis.

TODO explain convergence of Schmidt states to pointer states by some perturbation theory.

This also has deep ramifications. Our Schmidt states converge to living in totally orthogonal subspaces of the Hilbert space. This is visible by the fact that the Schmidt states converge to pointer states. And that theses pointer states can be looked at in the H_I eigenbasis, and they do not share a no-zero term in this basis.

The quantum states of

!!!!! this is a super particular case where the system states define the first term in the total energy eigenbasis. !!!!

===== We use Schmidt states as our object of study, since they are known to converge to Pointer states, which are an attempt at defining classical Branches of the wavefunction.

The initial state is separable state with,

$$\begin{aligned}\rho(0) &= |\psi(0)\rangle\langle\psi(0)| \\ \rho_s(0) &= Tr_e[\rho(0)] \\ \rho_e(0) &= Tr_s[\rho(0)]\end{aligned}$$

(a) Distribution before decoherence

(b) Distribution after decoherence

Figure 3

Figure 4: Freeze frame on the distribution of a Schmidt state in the energy eigenbasis of H_I before and after decoherence. We see the distribution converge to that of a pointer state.

We have in the $\{|q_i\rangle\}$ basis,

$$\rho_s(0) = (\frac{|q_1\rangle}{\sqrt{w}} + \frac{|q_2\rangle}{\sqrt{1-w}})(\frac{\langle q_1|}{\sqrt{w}} + \frac{\langle q_2|}{\sqrt{1-w}}) = \begin{pmatrix} 0 & ... & ... & 0 \\ ... & \frac{1}{w} & ... & \frac{1}{\sqrt{w(1-w)}} & ... \\ ... & 0 & ... & 0 & ... \\ ... & \frac{1}{\sqrt{w(1-w)}} & ... & \frac{1}{(1-w)} & ... \\ 0 & ... & ... & 0 \end{pmatrix} \tag{6}$$

In the basis $\{\frac{|q_1\rangle}{\sqrt{w}} + \frac{|q_2\rangle}{\sqrt{1-w},...}$ it is diagonal with only non-zero 1 eigenvalue $\lambda_0 = 1$. After some time evolution,

$$\rho_s(t) = (|q_1\rangle A(t) \sum_j c_j(t)|e_j\rangle + |q_2\rangle B(t) \sum_j c'_j(t)|e_j >) * (...)^\dagger$$

$$= \begin{pmatrix} 0 & ... & ... & 0 \\ ... & |A(t)|^2 \sum_k |c_k(t)|^2 & ... & A(t)B^*(t) \sum_k c_k(t)c'^*_k(t) & ... \\ ... & 0 & ... & 0 & ... \\ ... & A^*(t)B(t) \sum_k c^*_k(t)c'_k(t) & ... & |B(t)|^2 \sum_k |c'_k(t)|^2 & ... \\ 0 & ... & ... & 0 \end{pmatrix}$$

What happens is that as interaction starts, the density matrix de-diagonalizes as it becomes non-classical (ie increaes entanglement). And then becomes diagonal again, from 1 to 2 non-zero eigenvalues.

Anyways to understand that the off diagonal terms become first highly non-zero, and then go to zero again?

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2.4 Schmidt states in the ACL

It can no longer be diagonalized with only 1 non-zero eigenvalue, it now has 2 non-zero eigenvalues (max). The eigenvectors of this density matrix are the Schmitd states of the system, and take the form,

$$|s_0(t)\rangle = u_1(t)|q_1\rangle + u_2(t)|q_2\rangle$$
$$|s_1(t)\rangle = v_1(t)|q_1\rangle + v_2(t)|q_2\rangle$$

By unitary evolution our state stay pure and can always be reduced to the superposition of 2 separable states (in this particular bipartide decomposition, and because of the fact that the system has at most 2 non-zero Schmidt values). We can express our $|q_i\rangle$'s from $|s_i\rangle$'s and get the following,

$$\begin{aligned}|\psi(t)\rangle &= A(t)|q_1\rangle|E_1(t)\rangle + B(t)|q_2\rangle|E_2(t)\rangle \\ &= A'(t)|s_0\rangle|\mathcal{E}_0(t)\rangle + B'(t)|s_1\rangle|\mathcal{E}_1(t)\rangle\end{aligned}$$

To find $|\mathcal{E}_i(t)\rangle$, we consider $P_i|\psi(t)\rangle$, with $P_i(t) = |s_i(t)\rangle\langle s_i(t)|$ using the orthogonality $\langle s_0|s_1\rangle = 0$. And finding the first nonzero term in $|s_i(t)\rangle$ we isolate the vector $|\mathcal{E}_i(t)\rangle$.
We then define the following objects as the Schmidt states of our system + environment,

$$|S_0(t)\rangle = |s_0\rangle|\mathcal{E}_0(t)\rangle$$
$$|S_1(t)\rangle = |s_1\rangle|\mathcal{E}_1(t)\rangle$$

They are our primary object of consideration.

Figures/Wigner_sum.png

Figure 5: The set of wigner semi-circles who's sum gives us the spectrum of H_I. The wider ones are for higher q_i's.

2.5 Perturbation theory

Need to figure out what I am looking for...

perturbation theory to figure out the spectrum? Yes Perturbation theory to explain the convergence? No need Pertubration theory to find an effective Hamiltonian?

The perturbation is as a function of EI. A little weird because q1*EI is not really at a smaller scale than the other hamiltonian.

2.6 Ramification of sharing the total energy eigenstate

The branches of the wavefunction loose access to half of the energy eigenbasis. Implication about loss of access to some dimension of the Hilbert space? This is the very special case where the total energy basis states can be written a product states of the system energy basis states and some environment state, AND there being no self interaction of the system. But this begs the question. Everytime we are able to define a measurement with this model, then the dimension of our accessible Hilbert space in the totla energy eigenbasis is halfed? (Simple example of no self interaction superposition of state, 2 photons in a cavity.)
Note that post decoherence the branches have a different effective Hamiltonian, ie one made of the sum of 2 block diagonal terms (one of the block diagonal si different for diff branches). So the total Hamiltonian wrt which you loose access to part of the Hilbert space changes after each measurement. This I think, makes the continuous loss of Hilbert space narrative no longer true.

$H_{tot,1} \rightarrow$ branching of state defined over $2 * d_2$ energy eigenstates of H_{tot} leads to 2 worlds $|p_1\rangle$ and $|p_2\rangle$ defined over d_1 eigenstates of H_{tot} and evolving under

2.7 Worlds

Pointer states as worlds? Because for now I underdtand it as such, and use it as such in this paper. See paper saying no.

2.8 Characterizing the probability distribution of the global state

The state, and Schmidts are define as non-zero on only part of the energy eigenspace because of our model. The Hamiltonian has random matrices acting on only the environment subspace, and no self interaction for the system. By starting and staying defined over 2 out of 10 energy eigenstates of the system, the global state is always defined over 1/5th the space (for $d_1 = 10$). Is that important at all?

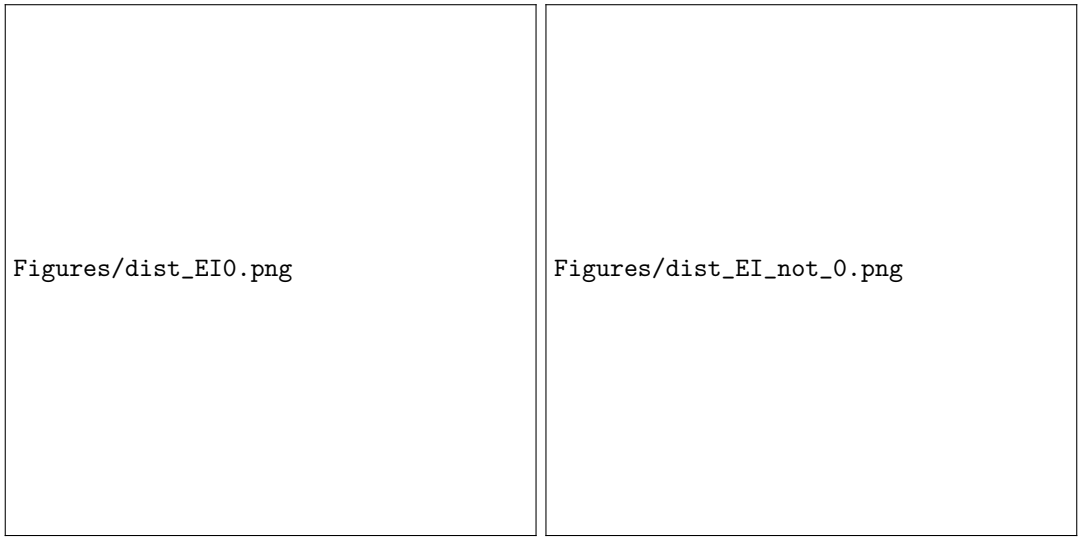


Figure 6

Impact of E_I: Varying E_I changes the total energy eigenbasis. Taking $E_I = 0$ we have as expected a delta, as the environment starts and stays as a total energy eigenstate. As we vary E_I, it takes a bell shape and its standard deviation rises. See Fig. (**??**), which shows us how an H_e eigenstate is defined in the eigenbasis of $H_e + H'_e$.

Impact of w: It is a factor in front of the 2 branches, so changes the relative importance of each branches when we vary it.

3 Does Branching dynamics split the energy eigenspace between the branches in a particular way?

This question is motivated by the behavior of $Neff_{schmidt,i}/Neff_{total} = \delta_i$, with $Neff_{state} = \frac{1}{\sum_i p_i^2}$, p_i the probability of finding the state we are considering in total energy eigenstate i. See Fig. (**??**). The idea of $Neff$ being to measure the spread of the Schmidts over the energy eigenspace. Neff is maximized by a uniform distribution and minimized by $p_i = 1$ for $i = a$ for some integer a. δ_i is going to depend on how the distribution of Schmidt i evolves in the energy eigenspace, which is what we want to study.

Figure 7: In this specific run, the curves join and stay together. With different parameters, we see the 2 curves stabilize with some distance, positive or negative

What we saw looking at the graphs for multiple runs of the simulation with varied parameters, is that sometimes both δ_i's would match up, sometimes after decoherence $\delta_1 > \delta_2$ or $\delta_2 > \delta_1$.

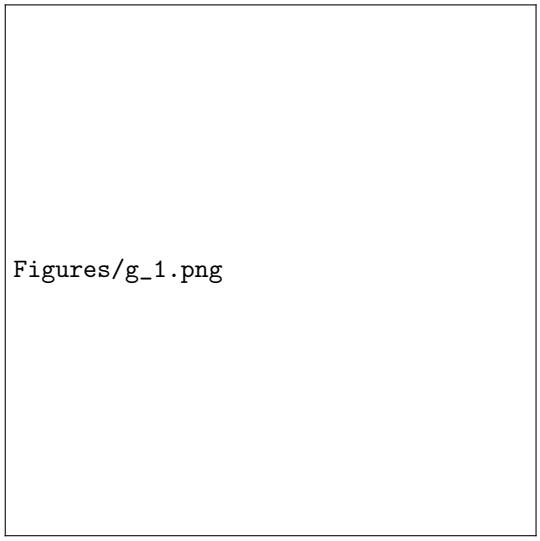
It would be interesting to characterize $Neff$ to understand its behavior. Are there necessary conditions for the convergence of the 2 curves? Why does Neff have different behaviors for different E_I, w or the initial environment state?

Figure 8: Graphs of δ for varying parameters E_I and w. Top left $w = 0.1, E_I = 0.03$, $\rightarrow$ rising E_I, $\downarrow$ rising w.

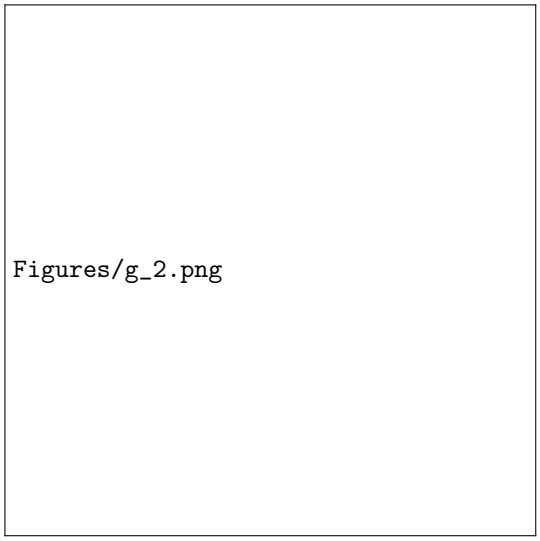
We see in Fig. (**??**) that δ_1 always starts at 1 since it is the global state at $t = 0$. It then varies a little from its base value to some equilibrium value slightly above or under 1. The fact that δ_1 barely varies comes from the fact that the shape of the distribution of Schmidt 1 also barely varies. δ_2 varies (from initial to equilibrium) by a factor from 10 to 100.

3.1 Other hints that eigenspace is shared in an interesting way

Looking at the difference between how things are a little after interaction starts and after all has settled, Fig. (**??**) we see that the Schmidt states seem to have shared the energy eigenspace.



(a) Zoomed view of the probability distribution of Schmidt 1 and 2 in the total energy eigenbasis at frame 11/100



(b) Zoomed view of the probability distribution of Schmidt 1 and 2 in the total energy eigenbasis at frame 89/100

Figure 9: There is less large probability overlap at frame 89 than 11

3.2 Are probabilities not just shared following a conservation law?

Since we have unitary evolution, wont the probability of being in an energy eigenstate for both Schmidt states just sum up to what it is for the global state? They wont because of interferences.

At some time t:

$$|\psi\rangle = \sqrt{s_0}|q_1\rangle|E_1\rangle > +\sqrt{s1}|q_2\rangle|E_2\rangle$$

Taking a total energy eigenstate $|Ai\rangle$

The probability that the state be in this energy eigenstate is

$$P(|\psi\rangle \text{ in } |A_i\rangle) = |\langle A_i|\psi\rangle|^2 = s_0|\langle A_i|q_1E_1\rangle|^2 + s1|\langle A_i|q_2E_2\rangle|^2 + \sqrt{s0s1}(\langle A_i|q_1E_1\rangle^* + \langle A_i|q_2E_2\rangle^*)$$

This can also be written,

$$P(|\psi\rangle \text{ in } |A_i\rangle) = s_0P(|Schmidt_1\rangle \text{ in } |A_i\rangle) + s_1P(|Schmidt_2\rangle \text{ in } |A_i\rangle) + \sqrt{s0s1}(\langle A_i|q_1E_1\rangle^* + \langle A_i|q_2E_2\rangle^*)$$

Where the leftover are quantum interference terms. Interestingly, $\sqrt{s_0 s_1}$ is maximal for $s_0 = s_1$ at maximal entanglement, which goes against at least my intuition that the worlds are supposed to be at maximal non-interference then.

We can visualize this interference in Fig. (**??**) and Fig. (**??**).

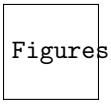


Figure 10: In this plot we see the importance of interference effects for the value of probability that a Schmidt has to be in an energy eigenstate. (Chosen for its high interference.)

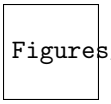


Figure 11: Graph of the total interference between Schmidt states

3.3 Other metrics to describe the sharing of eigenspace

We want to confirm that there is indeed interesting dynamics to find in the sharing of energy eigenspace + we want the right tool to characterize it.

Mean and standard deviation To characterize the shape of the distributions, we look at the variation of the mean and standard deviation of both distributions in time. The question is "How do the shapes of the distributions evolve in time?"

Figure 12: Graphs of the means and standard deviations of the probability distributions of Schmidt 1 and 2 for parameters $E_I = [0.05, 0.06, 0.07, 0.08]$ and $w = [0.2,0.25,0.3,0.35]$.$\rightarrow$ rising E_I, $\downarrow$ rising w.

What this confirms is that the shape of the distribution of Schmidt 1 varies little in time, while Schmidt 2 varies much at the beginning.

Overlap of the probability distributions The overlap is the projected value of the square root of the probability vectors of Schmidt 1 and 2 with themselves and with the global state. The question investigated is "Are Schmidt 1 and 2 defined over the same eigenbasis?". In more detail, they are of course defined in the same Hilbert space, but do they both have high probability in the same eigenbasis? The green curve, shows that the Schmidt

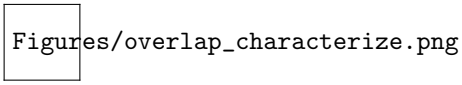


Figure 13: Graphs of the overlap of the probability distributions of the Schmidt states and the global state. Parameters $w = [0.1, 0.2, 0.3, 0.4], EI = [0.03, 0.05, 0.07, 0.09]$. $\rightarrow$ rising E_I, $\downarrow$ rising w.

states start by becoming similar likely as Schmidt 2 catch's up quickly to the shape it is supposed to have, but then give way to each other, do not overlap much in the energy eigenbasis. The distributions evolve to similar shapes, but do not occupy the same eigenstates.

Occupation of the Hilbert space For the occupation measure of the Hilbert space, we count the number of total energy eigenstates in which Schmidts have probability above a threshold ϵ. The question investigated is, "Does Schmidt 1 gives some of its space to Schmidt 2?". No. All we see, is that no the number of eigenbasis states "occupied" by Schmidt 1 does not really change. And for Schmidt 2 we see the expected behavior, knowing that it goes from a uniform distribution to a more concentrated one. **??**

Figure 14: Graphs of the occupation of Hilbert space by Schmidt 1 and 2 ($\epsilon = 0.001$).Parameters $w = [0.1, 0.2, 0.3, 0.4], EI = [0.03, 0.05, 0.07, 0.09]$. $\rightarrow$ rising E_I, $\downarrow$ rising w.

Gifs Gifs of how the distributions of both Schmidts changes in time. Cannot put them in a pdf but we can discuss them here.

The distribution of Schmidt 1 starts as the distribution of the global state. As the second Schmidt value goes from 0 to not 0, Schmidt 2 gains in importance, it takes over some of the energy eigenstates on which Schmidt 1 is defined and takes a similar shape. So we have 2 distributions of the same shape but not defined over the same space.

3.3.1 General characterization of these metrics

Interference and w We generally see these metrics vary in unison with the amount of interference between Schmidt 1 and 2. We also note an inversion in behavior at equilibrium for a change of w to 1-w. As an example of this law, is the plots of occupation with w=0.03 and 0.07 (which also have underneath the plots of interference, and we clearly see the same shape.) **??**

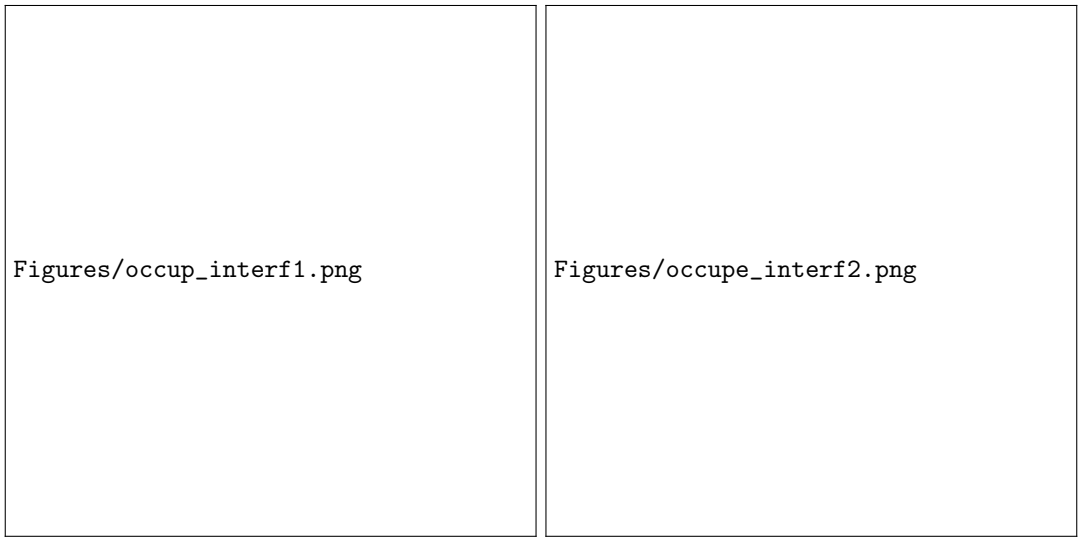


Figure 15: We see that the occupation number varies with total interference. We also see that we have an inversion of equilibrium behavior for w=0.3 and w=0.7

Interference makes sense of the common behavior.
Inverting w the probability, inverts teh equilibrium behavior.

General solution The weight in front of both states are equal if,

$$q_1 = 2(\frac{\sqrt{w}}{\sqrt{1-w}} - 1) + q_2 \frac{\sqrt{w}}{\sqrt{1-w}} \tag{7}$$

Then,

$$|\psi(t)\rangle = |q_1\rangle(A) \sum_j c_j(t)|e_j\rangle + |q_2\rangle(A) \sum_j c'_j(t)|e_j\rangle \tag{8}$$

We can make that happen while keeping E_I and w invariant by changing either the energy between the eigenstates of the SHO, or the states we choose for the superposition (which is what we do in practice). The idea being that we eliminated the asymmetry between the 2 Schmidt states, which makes δ_1 and δ_2 behave the same at equilibrium (as well as other metrics).

We verify this by making a plot of δ where the curves converge and another where they are inversed while keeping $w = 0.41$ and $E_I = 0.06$, typically values where we get distinct equilibrium behaviors for Schmidt 1 and 2.
See notebook "The same"

3.4 Conclusion:

The "energy" of the superimposed states, and their probability (in our case $q_1, q_2, w, (w-1)$) has an impact on the distributions of Schmidts in the energy eigenbasis.

(a) For weight difference of 0.01 (b) For a weight difference of 10

Figure 16: We see a that by making Schmidt 1 and 2 as "important" we can get them to agree on Neff. And we see the opposite happen when we maximize their weight difference.

Figure 17: The distribution of Schmidts has the possibility under certain conditions of going from uniformly distributed to ending and staying in this distribution.

3.5 Pseudo pointer in the interaction spectrum

We find an interesting evolution of the distribution in this basis. See Fig. (**??**). The motivation for looking in this basis was to look for pointer like behavior of converging towards an energy eigenstate of H_I. We see that it has a behavior like that, only not so complet.

3.5.1 Characterizing the behavior of the distributions in the interaction energy basis.

env does not matter

characterize behavior to see if it is weight dependant. interestingly you can get both to do so by macthing the weihghts. can you get neither to do so?

We look back at the equation,

$$|\psi(t)\rangle = |q_1\rangle(\frac{2+q_1}{\sqrt{w}})\sum_j c_j(t)|e_j\rangle + |q_2\rangle(\frac{2+q_2}{\sqrt{1-w}})\sum_j c_j(t)'|e_j\rangle \tag{9}$$

We have that the eigenstates of H_e are $\{|e_i\rangle\}$, the eigenstates of H_I are $\{|q_i\rangle|A_j\rangle\}$ We can re-write $|e_j\rangle = \sum_k a_{kj}|A_k\rangle$, meaning,

$$|\psi(t)\rangle = |q_1\rangle(\frac{2+q_1}{\sqrt{w}})\sum_{jk} c_j(t)a_{kj}|A_k\rangle + |q_2\rangle(\frac{2+q_2}{\sqrt{1-w}})\sum_{jk} c_j(t)'a_{kj}|A_k\rangle \tag{10}$$

Which leads to ,

$$P(|s_2(t)\rangle \text{ is in } |A_l\rangle) = |\frac{2+q_2}{\sqrt{1-w}}\sum_{jk} c'_j(t)a_{kj}\delta_{lk}|^2 = |\frac{2+q_2}{\sqrt{1-w}}|^2|\sum_{jl} c'_j(t)a_{lj}|^2 \tag{11}$$

Somehow, we have that $|\sum_{jl} c'_j(t)a_{lj}|^2$ is suppressed for a certain set of l's. The l's that correspind to outside the bounds we can see in Fig. (**??**). To study this, we need to understand the time evolution of the $c_j(t)$'s.

This is not explained by our numerical or model (at least trivially), since we do see that in principle the a_{lj}'s are not zero.

What could explain the supression? Not interferences. For multiple reasons. interferences go to 0 on the out

We would like to see how it evolves in the basis of H_I

To do that theoretically, we would need to Haussdorf approximate out unitary of non-commuting op.

Figure 18: Graphs of the similarity between s1, parameters $E_I = 0.06$ and $w = 0.3$.

4 Why do they share the energy eigenstates in this way?

What explains this way of sharing the eigenspace? Interferences between Schmidt states?

Do the probabilities in individual eigenstates vary less once equilibrium is reached? Do near zeros stay at near zero? Could it be that they spread in the eigenbasis and since they dont have many high probabilities, they dont overlap meaningfully simply by chance?

5 Can we get it all from the energy spectrum and the distribution of the global state in it?

Energy spectrum + state defined in energy basis -¿ sharing of Hilbert space by worlds -¿ explain the phenomenas we observed

6 Other

Measure of fluctuation in time of the distributions Similarity in time, a sort of measure of fluctuation. Maybe useful to characterize the behaviors of s1 and s2 distribution themselves.

Potential selection mechanism for schmidt states that fluctuate less, ie are stable in time?

6.1 ...

We have a certain sharing of the Hilbert space under measurement This sharing implies certain difference in behavior for quantities such as Neff. And if possible, it would be amazing to recover this sharing from the shape of the spectrum and the distribution of the global state! mad dog evrietian style.

Something that troubles me. q and w have a large impact on the "shape "of the distribution. The way I used to see things. Is that the shape of the distribution was the determining factor to how the world would be, ie it would define it classically, ie macroscopically. And to me the 2 worlds couldn't me so different. But, this macro looking factor (distribution shape), might just be irrelevant.

Could there be something to say about the fact that we are looking at overlap in the probability space? This is because of course the 2 worlds will be orthogonal in the Hilbert space. So our fundamental object of study is really their distribution in the total energy eigenbasis.

Pointer states: found using predicatibility sieve. They are the states that devellope the least entanglement with the environment in a robust manner.

So I guess we dont have pointer states in our case, since we dont have self interaction to vary the system state.

This dynamics of Schmdit basis states towards the pointer states might be worth studying. In the case of no self interaction for the system, the pointer states are eigenstates of the interaction Hamiltonian. Might be interesting to check.

Note the Schmidt basis does not have to be orthogonal. we have always orthogonal schmidt basis in our ACL case due to orthogonal system state + no self interaction.

(Andreas Albrecht study of Schmidt states dynamics under measurement: https://journals.aps.org/prd/abstract/10.1103/Phy https://journals.aps.org/prd/abstract/10.1103/PhysRevD.48.3768)

6.1.1 Distinction between Schmidt and Pointers

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.40.1071

6.1.2 A discussion of Schmidt might not be enough

How pointers might not really be branches https://arxiv.org/pdf/gr-qc/9610028.pdf Is a paper, looking at the possibility of s=using the schmidt decomposition to seelct the physics set of consistent histories. The find is that it would not be enought. The schmidt basis is induced by ...

The pointer basis is induced by decoherence, or more precisely is einselected by the environment-system interaction

issue, isn't einselection only defined for S+M+E? while we are working here with S+E?

looking at the schmidt states has the nice property of always looking at 2 orthogonal states.

Are pointer states worlds? https://arxiv.org/abs/gr-qc/9610028 says that no

A Spectrum of H_{tot}

A.1 Characterizing the spectrum of the Hamiltonian

The Hamiltonian who's spectrum we are trying to characterize is composed of 3 parts, $\mathbb{1}, \mathbb{1} \otimes H_e, E_I H_q \otimes H'_e$. We are interested in the shape that the distribution of eigenvalues will take. $\mathbb{1}$ will simply shift that distribution by $+1$, since it commutes with the other 2.

We will use the fact that the random matrices that appear in these Hamiltonians, are Gaussian Unitary Ensemble (GUE) matrices, and are known to have semi-circle spectrum distribution. Wigner semi-circle law:

$$f(E) = \frac{2}{\pi R^2}\sqrt{E^2 - R^2} \tag{12}$$

distribution between $[-R, R]$ bounds.

Considering $\mathbb{1} \otimes H_e$, H_e has spectrum $\sigma_2 = \{\lambda_1, \lambda_2, ...\lambda_{d_1}\}$ and the identity is here defined over Hilbert space of dimension d_1. This makes the final spectrum $\sigma_{1,2} = \lambda_1, \lambda_1, ..., \lambda_1, \lambda_2, ..., \lambda_2, \lambda_3, ...,$ where each element of the spectrum is repeated d_1 times. The distribution of the spectrum of the first term is then an elevated Wigner semi-circle.

$$f_{env}(E) = \frac{2}{\pi R^2}\sqrt{R^2 - E^2} + d_1 \tag{13}$$

Considering $H_q \otimes H'_e$, with the spectrum of the random matrix $\sigma_2 = \{\lambda'_1, \lambda'_2, ...\lambda'_{d_1}\}$. We get a new spectrum, $\sigma_{1,2} = 0 * \sigma_2 \bigcup \sigma_2 \bigcup 2\sigma_2 \bigcup ... \bigcup (d_1 - 1)\sigma_2$ ie $\sigma_{1,2} = \{0, ..., 0, \lambda'_1, \lambda'_2, ..., 2\lambda'_1, 2\lambda'_2, ..., (d_1 - 1)\lambda'_1, ...\}$. This produces a sum of Wigner semi-circles with $R_i = i * R_1$ for $i = 0, 1.., d - 1$.

$$f_{int}(E) = E_I \sum_{j=1}^{d_1 - 1} \frac{2}{\pi j^2 R^2}\sqrt{j^2 R^2 - E^2} + \delta_{E,0} d_2 \tag{14}$$

The $\delta_{E,0} d_2$ comes from our SHO having ground state energy $E_0 = 0$. The E_I factor is added to the distribution here. Since these distributions are centered on 0, the factor E_I acts as a spread factor. (ie the max and min are split $2E_I * R_{effective}$)

We verify that we do indeed get this in Fig. (**??**).

Figures/Wigner_sum.png

Figure 19: The set of wigner semi-circles who's sum gives us the spectrum of H_I. The wider ones are for higher q_i's.

With our simulation we can also get them numerically as seen in Fig. (**??**), we choose $E_I = 0.06$ and $w = 0.3$.

About Fig. (**??**), it looks like $\sigma_{total} = \sigma_{env} + E_I \sigma_{int} + 1$ (+1 from identity matrix). But that would only be the case if both Hamiltonians where diagonalizable in a common basis, which they are not, being different independant random matrices. Is it still possible to consider the final spectrum as such? I do not see another way to characterize

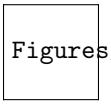


Figure 20: First is the distribution of the state in the total energy eigenbasis. Second is the spectrum of that total energy eigenbasis. Third is the distribution of ythe spectrum of $\mathbb{1} \otimes H_e$ in its own energy eigenbasis. We do see a Wigner semi-circle lifted up by 10. Finally is the distribution of the spectrum of $H_q \otimes H'_e$, which also matches a sum of 10 Wigner semi-circles with growing R. We also see the $200 * \delta_{E,0}$ predicted at the center. Note that this last distribution has to be scaled down by the factor $E_I = 0.06$ (in this case)

it.

The spectra look like they sum, even though the Hamiltonians do not commute

If we take the summing of the spectra as a correct approximation, we have characterized the shape of the total spectrum depending on E_I.

$$f_{total}(E) = \frac{2}{\pi R^2}\sqrt{R^2 - E^2} + E_I \sum_{j=1}^{d_1-1} \frac{2}{\pi j^2 R^2}\sqrt{j^2R^2 - E^2} + \delta_{E,0}d_2 + d_1\theta(R^2 - E^2) \tag{15}$$

$\theta(x)$ is the Heaviside function. For the $H_{rand,d=200}$ $R \approx 10$.

Potential change to the model. The identity spectrum does not add anything interesting we could remove it. It might also be beneficial to make the GS of the SHO different from 0.

A.1.1 We use bounds on the values of the sum of 2 matrices.

For 2 hermitian matrices A, B such that $A + B = C$. with ordered eigenvalues, we have,

$$c_{i+j-1} \leq a_i + b_j \quad \text{and} \quad c_{n-i-j} \geq a_{n-i} + b_{n-j} \tag{16}$$

This means for us that

$$f_{total}(E) \leq \frac{2}{\pi R^2}\sqrt{R^2 - (E-e)^2} + d_1 + E_I \sum_{j=1}^{d_1-1} \frac{2}{\pi j^2 R^2}\sqrt{j^2R^2 - e^2} + \delta_{E,0}d_2 \tag{17}$$