Branching dynamics

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February 2024

1 Introduction

To run your own simulations varying the parameters, or see other sets of parameters already simulated see this Github repository
We establish the ACL model and then make a section for each of the following questions.

Does Branching dynamics split the energy eigenspace between the branches in a particular way? Do the worlds share it equally? Proportional to their respective probabilities?

Having established that they share the eigenstates weirdly. Why? What drives this dynamics? Does it generalize beyond the ACL model in which we observe this dynamics?

Can we get it all from the energy spectrum and the distribution of the global state in it? Energy spectrum + state defined in energy basis → sharing of Hilbert space by worlds → explain the phenomena we observed

2 A measurement model

2.1 The Adapted Caldeira-Leggett dynamics

To answer these questions, we consdier the adapted Caldeira-Leggett model, a system $\mathcal{S}$ + environment $\mathcal{E}$ where the system is a truncated SHO. Its Hamiltonian takes the form,

$$H_{tot} = H_s + H_e + E_I H_I \tag{1}$$

We choose no self interaction for the truncated SHO, so $H_s = \mathbb{1} \otimes \mathbb{1}$, and we choose a random hermitian matrix for the environment self interaction $\mathbb{1} \otimes H_r$, with H_r a random hermitian matrix. The interaction Hamiltonian is $H_I = H_q \otimes H'_r$, with $(H_q)_{ij} = q_i \delta_{ij}$ and H'_r another random hermitian matrix. This makes it so that the environment acts on the system depending on its state.
Taking the eigenstates of H_q to be $\{|q_i\rangle\}\forall$, there is a basis like $\{|q_i\rangle|k_i\rangle\}$, such that H_{tot}, H_e and H_I take the a block diagonal form. $i \in [0, d_1 - 1], j \in [0, d_2 - 1]$, d_1 and d_2 the dimensions of the system and environment respectively, giving a total dimension $d = d_1 * d_2$.

2.2 The state

We choose our initial state to be a separble state with $|e\rangle$ an environment state,

$$|\psi(0)\rangle = (\sqrt{w}|q_1\rangle + \sqrt{1-w}|q_2\rangle)|e\rangle \tag{2}$$

this corresponds to a superposition of energy eigenstates not yet entangled with an environment. w is the probability of being in $|q_1\rangle$. We also define here the eigenstates of H_{tot} to be $\{|q_i\rangle|k_i\rangle\}$. Note the fact that $|k_i\rangle$ depends on q_i. This is because each block of H_{tot} is different, meaning that it diagonalizes in different basis. More precisely, we can isolate the d_1 Hamiltonians that act on our environment depending on q_i, but since they do not commute, they do not share a common eigenbasis. Note that we can use a basis defined without q_i dependance in the k terms in the H_I dominated limit or the $E_I = 0$ limit (that last one is less interesting).

(a) Heat map of H_e (b) Heat map of H_I

Figure 1

Figure 2: Heat map of the Hamiltonians H_e and H_I in the eigenbasis of H_{tot}, for $d_1 = 10, d_2 = 200$. We note the d_1 degeneracy of the spectrum of H_e and the no degeneracy of H_I. By our constrution, there is no degeneracy for H_{tot}

2.3 Time evolution and decoherence

With this in mind, our initial state is only defined in $2d_2$ out of d energy eigenstates. Thus to understand the action of the $e^{-itH_{tot}}$ on our state, we only need to consider 3 different $d_2 * d_2$ matrices, $H_r, E_I q_1 H'_r, E_I q_2 H'_r$ (or 2 if we consider H_{tot}). We can write the time evolution,

$$
\begin{aligned}
|\psi(t)\rangle &= e^{-itH_{tot}}|\psi(0)\rangle && (3)\\
&= e^{-itH_{tot}}|q_1\rangle\sqrt{w}|e\rangle + e^{-itH_{tot}}|q_2\rangle\sqrt{1-w}|e\rangle && (4)\\
&= |q_1\rangle\sqrt{w}e^{-it(H_e^{(1)})}|e\rangle + |q_2\rangle\sqrt{1-w}e^{-it(H_e^{(2)})}|e\rangle && (5)\\
&= |q_1\rangle\sqrt{w}|\mathcal{E}_1(t)\rangle + |q_2\rangle\sqrt{1-w}|\mathcal{E}_2(t)\rangle && (6)\\
& && (7)
\end{aligned}
$$

We defined, $H_e^{(1)} = H_r + E_I q_1 H'_r$ and $H_e^{(2)} = H_r + E_I q_2 H'_r$, two matrices acting on $\mathcal{E}$.

2.3.1 Existence of pointer states

Block diagonal Hamiltonians always allow for certain pointer states (See "A Search for Classical Subsystems in Quantum Worlds" Appendix A)

We find that $|q_i\rangle|E_i(t)\rangle = |p_i(t)\rangle$ are global pointer states since the time evolution leaves the state as a separable state. In the total energy eigenbasis, a pointer state is defined in a d/d_1 subspace at most (and would need to be fine tuned to be defined in a smaller state). Technically, pointer states would be defined on the system, but we consider here an extension to the full Hilbert space, since our system has no self interaction.

2.3.2 Decoherence

The initial state is separable with,

$$\rho(0) = |\psi(0)\rangle\langle\psi(0)|$$
$$\rho_s(0) = Tr_e[\rho(0)]$$
$$\rho_e(0) = Tr_s[\rho(0)]$$

We have in the $\{|q_i\rangle\}$ basis,

$$\rho_s(0) = (|q_1\rangle\sqrt{w} + |q_2\rangle\sqrt{1-w})(\langle q_1|\sqrt{w} + \langle q_2|\sqrt{1-w}) = \begin{pmatrix} 0 & ... & ... & 0 \\ ... & w & ... & \sqrt{w(1-w)} & ... \\ ... & 0 & ... & 0 & ... \\ ... & \sqrt{w(1-w)} & ... & (1-w) & ... \\ 0 & ... & ... & 0 \end{pmatrix} \tag{8}$$

In a basis $\{|q_1\rangle\sqrt{w} + |q_2\rangle\sqrt{1-w}, ...\}$ it is diagonal with only one non-zero eigenvalue $\lambda_0 = 1$. After some time we get,

$$\rho(t) = (|q_1\rangle\sqrt{w}|\mathcal{E}_1(t)\rangle + |q_2\rangle\sqrt{1-w}|\mathcal{E}_2(t) >) * (\cdots)^\dagger \tag{9}$$

$$\implies \rho_s(t) = Tr_e[\rho(t)] = \begin{pmatrix} 0 & ... & ... & 0 \\ ... & w & ... & \sqrt{w-w^2}\sum_k \langle k|\mathcal{E}_2(t)\rangle\langle\mathcal{E}_1(t)|k\rangle & ... \\ ... & 0 & ... & 0 & ... \\ ... & \sqrt{w-w^2}\sum_k \langle k|\mathcal{E}_1(t)\rangle\langle\mathcal{E}_2(t)|k\rangle & ... & 1-w & ... \\ 0 & ... & ... & 0 \end{pmatrix} \tag{10}$$

$$\tag{11}$$

We used here $\{|k\rangle\}$ a basis for the environment that is not one of the $\{|k_i\rangle\}$. This means that $|\mathcal{E}_1\rangle = \sum_k c_k^{\mathcal{E}_1}(t)|k\rangle$ with the $c_k^{\mathcal{E}_1}(t)$'s not just phases and actually varying in amplitude. We see this using the expression of the $|k_1\rangle$ basis states in the $|k\rangle$ basis,

$$|\mathcal{E}_1\rangle = \sum_{k_1} e^{-itk_1} c_{k_1}|k_1\rangle \tag{12}$$
$$= \sum_{k_1} e^{-itk_1} c_{k_1} \sum_k a_k^{(k_1)}|k\rangle \tag{13}$$
$$= \sum_k (\sum_{k_1} e^{-itk_1} c_{k_1} a_k^{(k_1)})|k\rangle \tag{14}$$

The term $(\sum_{k_1} e^{-itk_1} c_{k_1} a_k^{(k_1)}) = c_k^{\mathcal{E}_1}(t)$ has varying norm.

Or density matrix is already in the pointer basis $|p_i\rangle = |q_i\rangle$ so decoherence would be diagonalization in this basis, ie $|\langle\mathcal{E}_1|\mathcal{E}_2\rangle|^2 = |r(t)|^2 \to 0$. We get for the diagonal term,

$$\langle\mathcal{E}_1|\mathcal{E}_2\rangle(t) = \sum_k (c_k^{\mathcal{E}_1})^\dagger(t) c_k^{\mathcal{E}_2}(t) \tag{15}$$

2.3.3 Decoherence by variance

What do we know about $c_k^{\mathcal{E}_i}$? It is a complex number with norm varying between 0 and 1 with a normalization condition $\sum_k |c_k^{\mathcal{E}_i}|^2 = 1$. Suppose after some time t, they are Gaussian distributed $\mathbb{E}(c_k^{\mathcal{E}_i}) = 0$. Then $\mathbb{E}(\sum_k c_k^{\mathcal{E}_i}) = 0$, and we know that $|Var(\sum_k c_k^{\mathcal{E}_i})| \leq 1$ .We can also suppose that after some time, the $c_k^{\mathcal{E}_1}$ and $c_k^{\mathcal{E}_2}$ are independant. Then,
FIND WAY TO BOUND VARIANCE EXPONENTIALLY IN D_2..
Note, we neglected the independance of the $c_k^{\mathcal{E}_i}$ (leading to probabilities).

2.3.4 Decoherence by unit sphere

Other way of explaining away decoherence, we give each term the same amplitude and just look at phases. We have a sum of a bunch of random phases. Or simply, we can see it as the overlap of 2 random quantum states, ie 2 random vectors on the unit sphere of the Hilbert space (of dimension $2d_2-1$). Doing that, we set $|\mathcal{E}_1\rangle = (1, 0, ..., 0)$, meaning we now only need to care about the zero value of a random vector on the unit sphere ($|\mathcal{E}_2\rangle = (r_1, ...r_2)$). It turns out that the distribution of r_i for points randomly selected on the sphere (the r_i's are points in $2n-1$ real space) is such that it is distributed according to approximately $(1+r_1)^{d_2}(1-r_1)^{d_2}$, which means that the mean of r_1 is approximately $B(d_2+1/2, d_2+1/2) \approx \frac{\Gamma(d_2-1/2)\Gamma(d_2+1/2)}{\Gamma(2*d_2)}$.

2.3.5 Decoherence other

If we had a bosonic heat bath then the issue is already solved by Zurek in "Coherent States via Decoherence".

a way to explain the decoherence is as a washing away of a delicate balance. keeping, particularly, $|\mathcal{E}_1(t)\rangle$ and $|\mathcal{E}_2(t)\rangle$ start defined by the same set of set of amplitudes and phases in the $|k\rangle$ basis, but then each starts essentially evolving independantly of each other.

What is the delicate balance I am washing away here??

A coherent state is the specific quantum state of the quantum harmonic oscillator, often described as a state that has dynamics most closely resembling the oscillatory behavior of a classical harmonic oscillator.Mathematically, a coherent state is defined to be the (unique) eigenstate of the annihilation operator â. This leads me to a confusion. I am starting my state in a superposition of 2 eigenstates of the SHO. The $|q_i\rangle$'s are not position states but coherent states, ie eigenstates of the SHO ie eigenstates of $\hat{a}\hat{a^\dagger}$. So as Zurek says, the ACL model leads in its non-trivial limit, to pointer states that are coherent states.

What happens is that as interaction starts, the density matrix de-diagonalizes as it becomes non-classical (ie increaes entanglement). And then becomes diagonal again, from 1 to 2 non-zero eigenvalues.

Anyways to understand that the off diagonal terms become first highly non-zero, and then go to zero again?

For an arbitrary initial pure state, the system density matrix becomes diagonal in the pointer basis. SEE MY INITIAL STATE then find the reduced density matrix note that it depends on $c_{ij}(t) = \langle\phi_i(t)|\phi_j(t)\rangle$, where, using the fact that the environment drives the environment states into approximately orthogonal states, we have $c_{ij} \approx \delta_{ij}$

Decoherence induced pointer states for our particular block diagonal case are such that they are defined over a limited subspace of the hilbert space. it is usefull to think of the Hilert space in the total energy eigenbasis, since in it, we see that the pointer states are defined in $d_{tot}/d_1 = d_2$ energy eigenbasis states. And importantly, 2 pointer states defined with different $|q_i\rangle$ have no energy eigenstate in common (supposing no degeneracy.)

2.3.6 Decoherence time

We study a simple case, the limit where the interaction strength dominates. This implies that the blocks in the Hamiltonian are diagonalizable together. We get from this, that $|\mathcal{E}_i\rangle = \frac{1}{\sqrt{d_2}}\sum_k e^{-itE_k^{(i)}}|k\rangle$. (Where we also assumed that we started in a uniformly distributed state). This makes the overlap easier to study,

$$r(t) = \langle\mathcal{E}_1|\mathcal{E}_2\rangle = \frac{1}{d_2}\sum_k e^{-it(E_k - E'_k)} \tag{16}$$

$$\tag{17}$$

We study the case where the energy spectrums of $H_e^{(i)}$ is uniform, $E_k = q_1 k$ and $E'_k = q_2 k$, also defining $q_1 - q_2 = f$. We apply the next simplification. We now look at $|r(t)|^2$,

$$|r(t)|^2 = \frac{1}{d_2^2} |\sum_k e^{-itkf}|^2 \tag{18}$$

$$= \frac{1}{d_2^2} \sum_{kj} e^{-itk(1-f)+itjf} \tag{19}$$

$$= \frac{1}{d_2^2} (d_2 + \sum_{k \neq j} e^{-it(k-j)f}) \tag{20}$$

$$= \frac{1}{d_2^2} (d_2 + 2 \sum_{k=1}^{d_2} (d_2 - k) cos(ktf)) \tag{21}$$

$$\tag{22}$$

Where what we end up with is a sum that has terms of higher importance than others (the low k's have a higher weight.). Fully time symmetric as expected of unitary evolution.

For this function, we can look for its poincare recurrence time, but this will not provid us with an accurate esrtiamte of the decoherence time. We could look at the T_{min}, which minimizes the function, but again decoherence is likely before that.

Decoherence time dependance on $\frac{q_2}{q_1}$: By time symmetry of the solution, we choose to consider q_2 superior to q_1 yet keep f positive. Suppose we have a T_{dec}, such that $|r(T_{dec})|^2 = \epsilon$, where ϵ is choosen to satisfy our decoherence condition. Then varying the difference $q_2 - q_1$ rescales time in the above equation, making the decoherence time depend on them as,

$$T_{dec}(f) = \frac{T_{dec}(1)}{f} \tag{23}$$

This is to interpret as $T_{dec}(q_1, q_2) = \frac{T_dec(q'_2 = q'_1 + 1)}{q_2 - q_1}$. We see this as meaning that increasing q_2 and leaving q_1 unchanged re-scales the decoherence time by their difference. In other words, the decoherence time depends on the scale of energy difference of the Hamiltonian by which each pointer state evolves.... kinda.

what I find interesting here. depends on energy scal diff, not ratio.

About the relative importance of the terms of this function: Note that,

$$\sum_{k=1}^{n} = \frac{n(n+1)}{2} \tag{24}$$

from which we can look at,

$$\Delta = \frac{d_2(d_2 - 1)}{2} - \frac{a(a-1)}{2} \tag{25}$$

Δ represents some measure of the error coming from neglecting the $d_2 - a$ terms. For large d_2 and small a (we want to neglect as much as we can), we get,

$$\Delta \propto \frac{d_2^2}{2} - \frac{a^2}{2} \tag{26}$$

This leads to,

$$a \propto d_2 \sqrt{1 - \frac{\Delta}{d_2^2}} \tag{27}$$

Where we find that taking $\Delta = 0$ leads to $a = d_2$, meaning that to get no error, we need to consider the full sum, as expected. Inverting the formula,

$$\Delta' = \frac{2\Delta}{d_2^2} = \frac{d_2^2}{2} \sqrt{1 - \frac{a^2}{d_2^2}} \tag{28}$$

Defining This has Δ' grows faster closer to $a = d_2$ than $a = 0$, which is again an argument that the error is less sensitive to neglecting the small a's. We can use this formula to get a bound on the error for a certain precision.

Note, we care about having to put less of the terms in the sum to 0, because they imply havig a lower T_0 for the equation.

Note that the ultimate goal would be to use the form of the spectrum distribution to figure out the decoherence time...

2.4 Schmidt states in the ACL

2.4.1 Schmidt states

Given a system and an environment $\mathcal{S}+\mathcal{E}$ and a pure state defined over both $|\psi(t)\rangle$. It can always be decomposed in its Schmidt basis,

$$|\psi(t)\rangle = \sum_i \sqrt{p_i(t)}|a_i(t)\rangle|b_i(t)\rangle \tag{29}$$

The important points being that the sum is over one index, that the sum is over dimension of the smaller of both systems ($\mathcal{S}$ or $\mathcal{E}$) at most, and that the basis itself depends on the state, which implies basis varying in time.

While $\rho_s(0)$ Eq. (**??**) can be diagonalized and have only 1 eigenvalue $\lambda = 0$, after some time t, we get $\rho_s(t)$ Eq. (**??**) which now has 2 eigenvalues. This is due to the fact that the environment evolves differently for the 2 states in superposition, we get entanglement between the evolving environment and the state of the system. The eigenvectors of this density matrix are the Schmitd states of the system, and take the form,

$$|s_1(t)\rangle = u_1(t)|q_1\rangle + u_2(t)|q_2\rangle \tag{30}$$
$$|s_2(t)\rangle = v_1(t)|q_1\rangle + v_2(t)|q_2\rangle \tag{31}$$
$$\implies \tag{32}$$
$$|q_1\rangle = (\frac{v_2}{\alpha})|s_1\rangle - (\frac{u_2}{\alpha})|s_2\rangle \tag{33}$$
$$|q_2\rangle = -(\frac{v_1}{\alpha})|s_1\rangle + (\frac{u_1}{\alpha})|s_2\rangle \tag{34}$$

With $\alpha = (u_1v_2 - u_2v_1)$ By unitary evolution our global state stay pure and the Schmidt decomposition tells us that it can always be seen as the superposition of 2 separable states (in this particular bipartide decomposition, and because of the fact that the system has at most 2 non-zero Schmidt values). Going from the time evolution of the global state Eq. (**??**), we can express our $|q_i\rangle$'s from $|s_i\rangle$'s and get the following,

$$|\psi(t)\rangle = \sqrt{w}|q_1\rangle|\mathcal{E}_1(t)\rangle + \sqrt{1-w}|q_2\rangle|\mathcal{E}_2(t)\rangle \tag{35}$$
$$= \frac{1}{\alpha}[|s_1(t)\rangle(\sqrt{w}v_2(t)|\mathcal{E}_1(t)\rangle - \sqrt{1-w}v_1(t)|\mathcal{E}_2(t)\rangle) + |s_2(t)\rangle(-\sqrt{w}u_2(t)t|\mathcal{E}_1(t)\rangle + \sqrt{1-w}u_1(t)|\mathcal{E}_2(t)\rangle)] \tag{36}$$
$$= |s_1(t)\rangle|s_1^{(e)}(t)\rangle + |s_2(t)\rangle|s_2^{(e)}(t)\rangle \tag{37}$$

We define the following objects as the Schmidt states of our system + environment,

$$|S_1(t)\rangle = |s_1(t)\rangle|s_1^{(e)}(t)\rangle$$
$$|S_2(t)\rangle = |s_2(t)\rangle|s_2^{(e)}(t)\rangle$$

To find $|s_i^{(e)}\rangle$ we can find A_i, u_i, v_i and $|\mathcal{E}_i\rangle$ and use them to express it. Diagonalize ρ_s and ρ_e, find their eigenvector and associate them to each other by matching their eigenvalue. Or we can do the following (which is implemeted numerically.)

To find $|s_i^{(e)}(t)\rangle$, we consider $P_i|\psi(t)\rangle$, with $P_i(t) = |s_i(t)\rangle\langle s_i(t)|$ using the orthogonality $\langle s_1|s_2\rangle = 0$. And finding the first nonzero term in $|s_i(t)\rangle$ we isolate the vector $|s_i^{(e)}(t)\rangle$.

2.4.2 Decoherence induces convergence of Schmidt to pointer states

Here we explain how $|\vec{S_i}\rangle \approx |p_i\rangle$ for post-decoherence times. Note that the pointer states are defined by their weights in the total energy eigenbasis, the phase associated to each weight is not important to their property of staying separable.

Potential confusion: Pointer states are states that are stable in their separability, but our Schmidt states as defined above are already separable. Pointer states are stable in their separability **in a set basis.** Which means that our Schmidt states, are not pointer states by definition.

By the form of pointer states being $|q_j\rangle|$any environment state$\rangle$, we can show that the Schmidt states converge to pointer states simply by showing $|s_i\rangle \to a_i|q_j\rangle$. This mean that we dont actually care about the form of $|s_i^{(e)}\rangle$ of Eq. (**??**) and all we are asking is that either $|u_1(t)|$ or $|u_2(t)|$ tends to 0. And vice versa for v.

We start at $|u_1(0)| = \frac{1}{\sqrt{w}}$ and $|u_2(0)| = \frac{1}{\sqrt{1-w}}$.

We construct $\rho_s(t)$ from its Schmidt form, and express it in the $\{|q_i\rangle\}$ basis

$$= \begin{pmatrix} 0 & ... & ... & 0 \\ ... & |u_1|^2 + |v_1|^2 + u_1v_1^* + v_1u_1^* & ... & u_1u_2^* + v_1v_2^* + u_1v_2^* + v_1u_2^* & ... \\ ... & 0 & ... & 0 & ... \\ ... & u_2u_1^* + v_2v_1^* + u_2v_1^* + v_2u_1^* & ... & |u_2|^2 + |v_2|^2 + u_2v_2^* + v_2u_2^* & ... \\ 0 & ... & ... & 0 \end{pmatrix} \tag{38}$$

Matching eq. (**??**) and Eq. (**??**), we find the following equalities,

$$\begin{aligned} |u_1|^2 + |v_1|^2 + u_1v_1^* + v_1u_1^* &= w \\ |u_2|^2 + |v_2|^2 + u_2v_2^* + v_2u_2^* &= 1 - w \\ u_1u_2^* + v_1v_2^* + u_1v_2^* + v_1u_2^* &= \sqrt{w - w^2}\langle\mathcal{E}_1(t)|\mathcal{E}_2(t)\rangle \\ u_2u_1^* + v_2v_1^* + u_2v_1^* + v_2u_1^* &= \sqrt{w - w^2}\langle\mathcal{E}_2(t)|\mathcal{E}_1(t)\rangle \end{aligned}$$

Supposing from a certain time t, we have $\langle\mathcal{E}_1(t)|\mathcal{E}_2(t)\rangle \approx 0$. Then we have, that without a complicated balancing act, taking $u_2 = v_1 = 0$ and $u_1 = \sqrt{w}$ $v_2 = \sqrt{1-w}$ makes the equations work. Does it become the only solution if we add the 2 conditions $|u_1| + |u_2| = |v_1| + |v_2| = 1$ and remove the phase of u_1 and v_2 for not being physical? (8 eqs for 8 unknowns.).

Ok, so we see that post decoherence (ie orthogonality of environment states), we have that the Schmidt states are pointer states,

$$\begin{aligned} |S_1(t)\rangle &= |s_1(t)\rangle|s_1^{(e)}(t)\rangle \\ &= |q_1\rangle|\mathcal{E}_1(t)\rangle \\ |S_2(t)\rangle &= |s_2(t)\rangle|s_2^{(e)}(t)\rangle \\ &= |q_2\rangle|\mathcal{E}_2(t)\rangle \end{aligned}$$

We used for this Eq. (**??**,**??**) (DEAL WITH FACTOR)

In this part we take for granted that the density matrix of the system and environment converge to diagonals in the pointer basis. Now we need to understand this as the convergence of the Schmdit states to the pointer states.

What the Schmidt to pointer convergence shows is that it selects a generic pointer state, not a specifically chosen one. And from the relative weight of $|S_0\rangle$ in $|q_2\rangle$ (and vice versa) goes down over time, we have the convergence of the Schmidt states to the pointer states.

orthogonality of $\mathcal{E}_i$ for i 0 and 1 is interestingly not needed to expalin the non-overlap. but it is to explain decoherence??? not needed for overlap, because non-overlap is due to s0 and s1 going to being just a function of q1 and q2 respectively, ie this makes them orthongonal by defaulf (there seems to be a double decoherence happening here...).

Schmidt states converge to pointer states because. pointer states are the diagonal terms of the density matrix, and the matrix goes to being only diagonal by decoherence... something like that. ====

We can, in general terms take our unitary evolution as,

$$U(t) = \sum_j |a_j\rangle\langle a_j| \otimes U_j^{\mathcal{E}}(t) \tag{39}$$

Starting the system in $|b\rangle$, then $U_j^{\mathcal{E}}|b >= |b_j(t)\rangle$. This means that the reduced density matrix of the system $\mathcal{S}$, evovlves as,

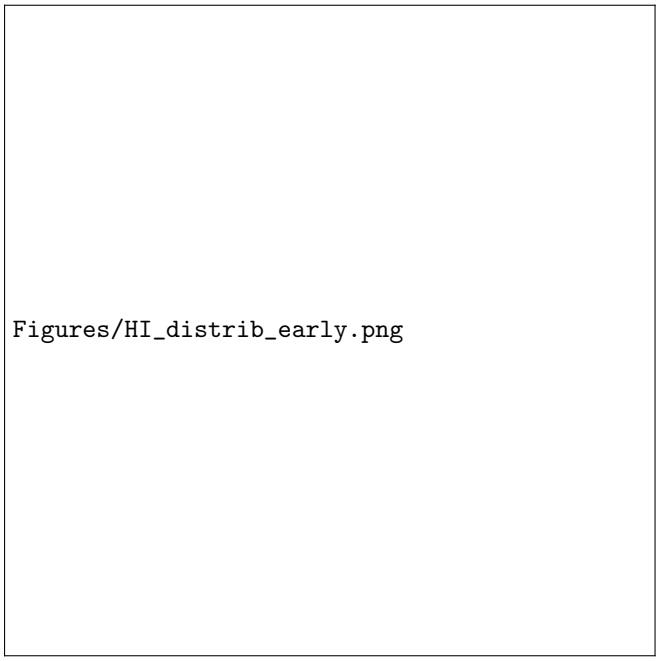
$$\rho_{ij} = \langle b_i(t)|b_i(t)\rangle\rho_{ij}(0) \tag{40}$$

This results in the evolution of the system density matrix to a diagonal form. This diagonal form, explains the apparent classicality of the unique measurement result. The 2 results, after some decoherence time, belong to 2 different orthogonal states.

Decoherence has it that $\langle b_j(t)|b_i(t)\rangle \to \delta_{ij}$ for $t \to \infty$.

Exaplain how A(t) goes from 1 to not 1.

In practice, the Hilbert space in which the pointer states are defined is d_2 dimensional and is $\{|q_1\rangle|v_k\rangle\}$ and $\{|q_2\rangle|v_k\rangle\}$ respectively. We can also individually look at the spectrum of $E_I q_i H'_r$, which for $i = 1$ corresponds to a thin distribution in the energy spectrum, like the blue spectrum in Fig. (**??**). For $i = 2$ it corresponds to one of the wide spectrums of Fig. (**??**). So Schmidt states converging to pointer states should be visible in their distribution in the interaction energy eigenbasis.



(a) Distribution before decoherence

(b) Distribution after decoherence

Figure 3

Figure 4: Freeze frame on the distribution of a Schmidt state in the energy eigenbasis of H_I before and after decoherence. We see the distribution converge to that of a pointer state.

TODO explain convergence of Schmidt states to pointer states by some perturbation theory.

This also has deep ramifications. Our Schmidt states converge to living in totally orthogonal subspaces of the Hilbert space. This is visible by the fact that the Schmidt states converge to pointer states. And that theses pointer states can be looked at in the H_I eigenbasis, and they do not share a no-zero term in this basis.

The quantum states of

!!!!! this is a super particular case where the system states define the first term in the total energy eigenbasis. !!!!

===== We use Schmidt states as our object of study, since they are known to converge to Pointer states, which are an attempt at defining classical Branches of the wavefunction.

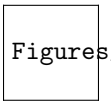


Figure 5: The set of wigner semi-circles who's sum gives us the spectrum of H_I. The wider ones are for higher q_i's.

2.5 Perturbation theory

Need to figure out what I am looking for...

perturbation theory to figure out the spectrum? Yes Perturbation theory to explain the convergence? No need Pertubration theory to find an effective Hamiltonian?

The perturbation is as a function of EI. A little weird because q1*EI is not really at a smaller scale than the other hamiltonian.

The point of the Caldeira-Leggett model is to study the intermediate regime of measurement $H_s \approx H_I$. So i do not think I can do a perturbation theory on the interaction energy

2.6 Ramification of sharing the total energy eigenstate

The branches of the wavefunction loose access to half of the energy eigenbasis. Implication about loss of access to some dimension of the Hilbert space? This is the very special case where the total energy basis states can be written a product states of the system energy basis states and some environment state, AND there being no self interaction of the system. But this begs the question. Everytime we are able to define a measurement with this model, then the dimension of our accessible Hilbert space in the totla energy eigenbasis is halfed? (Simple example of no self interaction superposition of state, 2 photons in a cavity.)

Note that post decoherence the branches have a different effective Hamiltonian, ie one made of the sum of 2 block diagonal terms (one of the block diagonal si different for diff branches). So the total Hamiltonian wrt which you loose access to part of the Hilbert space changes after each measurement. This I think, makes the continuous loss of Hilbert space narrative no longer true.

$H_{tot,1} \rightarrow$ branching of state defined over $2 * d_2$ energy eigenstates of H_{tot} leads to 2 worlds $|p_1\rangle$ and $|p_2\rangle$ defined over d_1 eigenstates of H_{tot} and evolving under

2.7 Worlds

Pointer states as worlds? Because for now I underdtand it as such, and use it as such in this paper. See paper saying no.

2.8 Explaining the energy splitting

We need to explain why worlds do not share energy eigenstaes. ie, why the amplitudes of the state in an energy eigenstate is fully given to a world and not shared. Indeed, if we only ask that worlds be orthogonal states, we get that there are ways to do this without sharing the energy eigenstates.

Very generally we can define worlds as, (TODO IS THAT REALLY APT? ARE WORLDS STATICALLY DEFINED AS A SUM OF CONSTANT TERMS IN THE TOTAL ENERGY EIGENBASIS?)

$$|w_i(t)\rangle = \sum_{k \in O_i} c_k e^{e^{-itE_k}} |k\rangle \tag{41}$$

O_i defines the set of total hamiltonian eigenstates in which world i is defined. If we take that we have 2 worlds and that they share an energy eigenstate (j), then the overlap is constant and worth $\langle w_1|w_2\rangle(t) = |c_j|^2$. If we have that the overlap is in n eigenstates, it is still constant. Basically, the only way for our worlds to not have an overlap is that they do not share an energy eigenbasis.

2.8.1 Note on the branching event and why we look at Schmidt states

For an global state,

$$|\psi(t)\rangle \sum_k u_k e^{-itE_k} |k\rangle \tag{42}$$

we can always extract from this orthogonal worlds 2 pointer states that do not mix, and stay orthogonal. ie there is not visible branching event to be found by just looking at the global state like this.

Thus we look at SChmidt states. TODO detail more why Schmidt states are legitimate objects to look at to witness decoherence. other than being states the converge to objects with world like properties at the rate of decoherence (pointer states).

The explanation

2.9 Characterizing the probability distribution of the global state

The state, and Schmidts are define as non-zero on only part of the energy eigenspace because of our model. The Hamiltonian has random matrices acting on only the environment subspace, and no self interaction for the system. By starting and staying defined over 2 out of 10 energy eigenstates of the system, the global state is always defined over 1/5th the space (for $d_1 = 10$). Is that important at all?

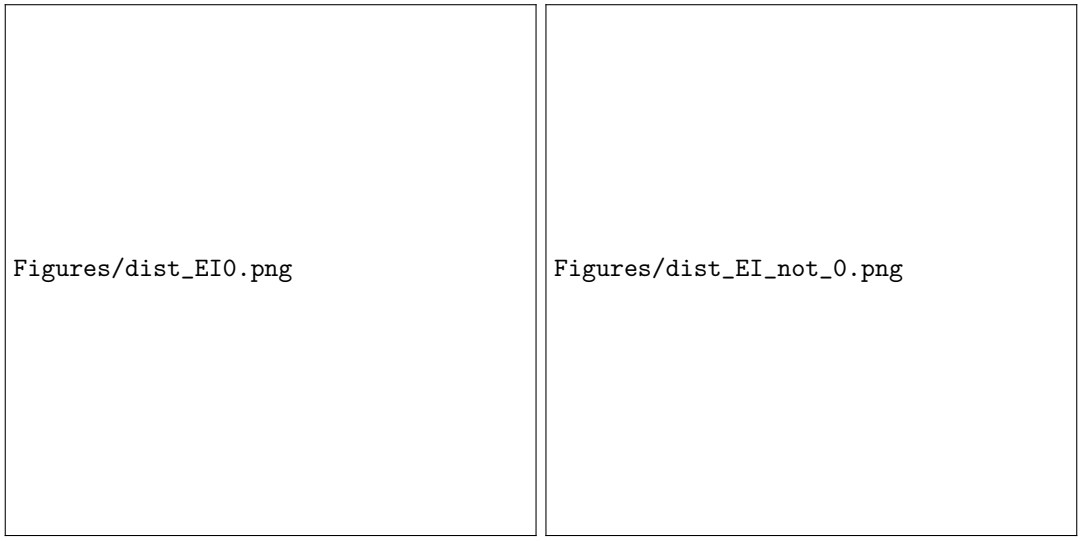


Figure 6

Impact of E_I: Varying E_I changes the total energy eigenbasis. Taking $E_I = 0$ we have as expected a delta, as the environment starts and stays as a total energy eigenstate. As we vary E_I, it takes a bell shape and its standard deviation rises. See Fig. (**??**), which shows us how an H_e eigenstate is defined in the eigenbasis of $H_e + H'_e$.

Impact of w: It is a factor in front of the 2 branches, so changes the relative importance of each branches when we vary it.

3 Does Branching dynamics split the energy eigenspace between the branches in a particular way?

This question is motivated by the behavior of $Neff_{schmidt,i}/Neff_{total} = \delta_i$, with $Neff_{state} = \frac{1}{\sum_i p_i^2}$, p_i the probability of finding the state we are considering in total energy eigenstate i. See Fig. (**??**). The idea of $Neff$ being to measure the spread of the Schmidts over the energy eigenspace. Neff is maximized by a uniform distribution and minimized by $p_i = 1$ for $i = a$ for some integer a. δ_i is going to depend on how the distribution of Schmidt i evolves in the energy eigenspace, which is what we want to study.

Figure 7: In this specific run, the curves join and stay together. With different parameters, we see the 2 curves stabilize with some distance, positive or negative

What we saw looking at the graphs for multiple runs of the simulation with varied parameters, is that sometimes both δ_i's would match up, sometimes after decoherence $\delta_1 > \delta_2$ or $\delta_2 > \delta_1$.

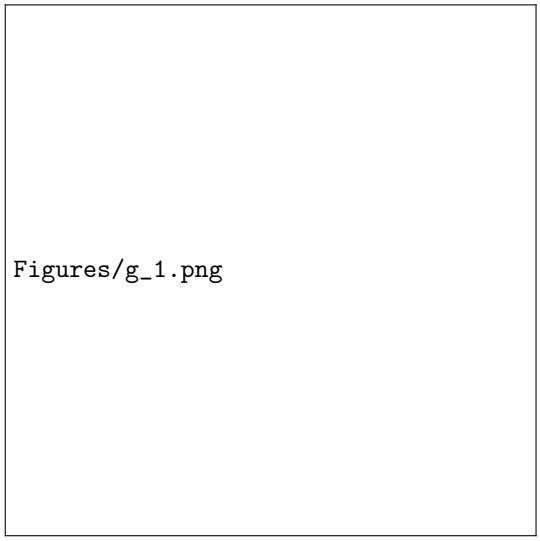
It would be interesting to characterize $Neff$ to understand its behavior. Are there necessary conditions for the convergence of the 2 curves? Why does Neff have different behaviors for different E_I, w or the initial environment state?

Figure 8: Graphs of δ for varying parameters E_I and w. Top left $w = 0.1, E_I = 0.03$, $\rightarrow$ rising E_I, $\downarrow$ rising w.

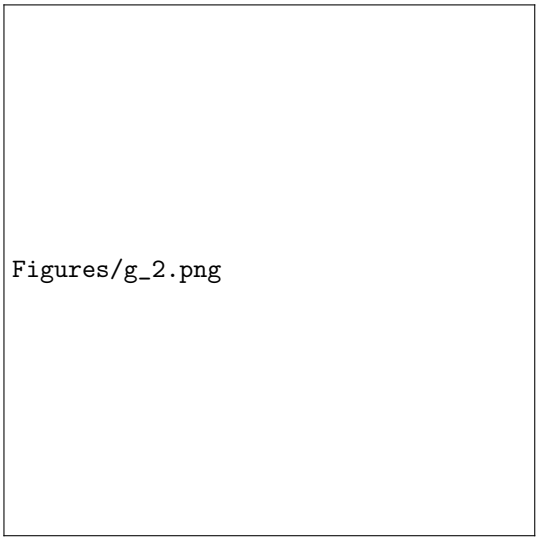
We see in Fig. (**??**) that δ_1 always starts at 1 since it is the global state at $t = 0$. It then varies a little from its base value to some equilibrium value slightly above or under 1. The fact that δ_1 barely varies comes from the fact that the shape of the distribution of Schmidt 1 also barely varies. δ_2 varies (from initial to equilibrium) by a factor from 10 to 100.

3.1 Other hints that eigenspace is shared in an interesting way

Looking at the difference between how things are a little after interaction starts and after all has settled, Fig. (**??**) we see that the Schmidt states seem to have shared the energy eigenspace.



(a) Zoomed view of the probability distribution of Schmidt 1 and 2 in the total energy eigenbasis at frame 11/100



(b) Zoomed view of the probability distribution of Schmidt 1 and 2 in the total energy eigenbasis at frame 89/100

Figure 9: There is less large probability overlap at frame 89 than 11

3.2 Are probabilities not just shared following a conservation law?

Since we have unitary evolution, wont the probability of being in an energy eigenstate for both Schmidt states just sum up to what it is for the global state? They wont because of interferences.

At some time t:

$$|\psi\rangle = \sqrt{s_0}|q_1\rangle|E_1\rangle > +\sqrt{s1}|q_2\rangle|E_2\rangle$$

Taking a total energy eigenstate $|Ai\rangle$

The probability that the state be in this energy eigenstate is

$$P(|\psi\rangle \text{ in } |A_i\rangle) = |\langle A_i|\psi\rangle|^2 = s_0|\langle A_i|q_1E_1\rangle|^2 + s1|\langle A_i|q_2E_2\rangle|^2 + \sqrt{s0s1}(\langle A_i|q_1E_1\rangle^* + \langle A_i|q_2E_2\rangle^*)$$

This can also be written,

$$P(|\psi\rangle \text{ in } |A_i\rangle) = s_0P(|Schmidt_1\rangle \text{ in } |A_i\rangle) + s_1P(|Schmidt_2\rangle \text{ in } |A_i\rangle) + \sqrt{s0s1}(\langle A_i|q_1E_1\rangle^* + \langle A_i|q_2E_2\rangle^*)$$

Where the leftover are quantum interference terms. Interestingly, $\sqrt{s_0 s_1}$ is maximal for $s_0 = s_1$ at maximal entanglement, which goes against at least my intuition that the worlds are supposed to be at maximal non-interference then.

We can visualize this interference in Fig. (**??**) and Fig. (**??**).

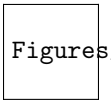


Figure 10: In this plot we see the importance of interference effects for the value of probability that a Schmidt has to be in an energy eigenstate. (Chosen for its high interference.)

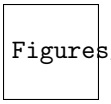


Figure 11: Graph of the total interference between Schmidt states

3.3 Other metrics to describe the sharing of eigenspace

We want to confirm that there is indeed interesting dynamics to find in the sharing of energy eigenspace + we want the right tool to characterize it.

Mean and standard deviation To characterize the shape of the distributions, we look at the variation of the mean and standard deviation of both distributions in time. The question is "How do the shapes of the distributions evolve in time?"

Figure 12: Graphs of the means and standard deviations of the probability distributions of Schmidt 1 and 2 for parameters $E_I = [0.05, 0.06, 0.07, 0.08]$ and $w = [0.2,0.25,0.3,0.35]$.$\rightarrow$ rising E_I, $\downarrow$ rising w.

What this confirms is that the shape of the distribution of Schmidt 1 varies little in time, while Schmidt 2 varies much at the beginning.

Overlap of the probability distributions The overlap is the projected value of the square root of the probability vectors of Schmidt 1 and 2 with themselves and with the global state. The question investigated is "Are Schmidt 1 and 2 defined over the same eigenbasis?". In more detail, they are of course defined in the same Hilbert space, but do they both have high probability in the same eigenbasis? The green curve, shows that the Schmidt

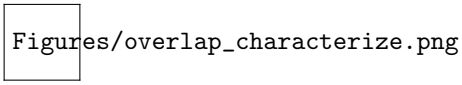


Figure 13: Graphs of the overlap of the probability distributions of the Schmidt states and the global state. Parameters $w = [0.1, 0.2, 0.3, 0.4], EI = [0.03, 0.05, 0.07, 0.09]$. $\rightarrow$ rising E_I, $\downarrow$ rising w.

states start by becoming similar likely as Schmidt 2 catch's up quickly to the shape it is supposed to have, but then give way to each other, do not overlap much in the energy eigenbasis. The distributions evolve to similar shapes, but do not occupy the same eigenstates.

Occupation of the Hilbert space For the occupation measure of the Hilbert space, we count the number of total energy eigenstates in which Schmidts have probability above a threshold ϵ. The question investigated is, "Does Schmidt 1 gives some of its space to Schmidt 2?". No. All we see, is that no the number of eigenbasis states "occupied" by Schmidt 1 does not really change. And for Schmidt 2 we see the expected behavior, knowing that it goes from a uniform distribution to a more concentrated one. **??**

Figure 14: Graphs of the occupation of Hilbert space by Schmidt 1 and 2 ($\epsilon = 0.001$).Parameters $w = [0.1, 0.2, 0.3, 0.4], EI = [0.03, 0.05, 0.07, 0.09]$. $\rightarrow$ rising E_I, $\downarrow$ rising w.

Gifs Gifs of how the distributions of both Schmidts changes in time. Cannot put them in a pdf but we can discuss them here.

The distribution of Schmidt 1 starts as the distribution of the global state. As the second Schmidt value goes from 0 to not 0, Schmidt 2 gains in importance, it takes over some of the energy eigenstates on which Schmidt 1 is defined and takes a similar shape. So we have 2 distributions of the same shape but not defined over the same space.

3.3.1 General characterization of these metrics

Interference and w We generally see these metrics vary in unison with the amount of interference between Schmidt 1 and 2. We also note an inversion in behavior at equilibrium for a change of w to 1-w. As an example of this law, is the plots of occupation with w=0.03 and 0.07 (which also have underneath the plots of interference, and we clearly see the same shape.) **??**

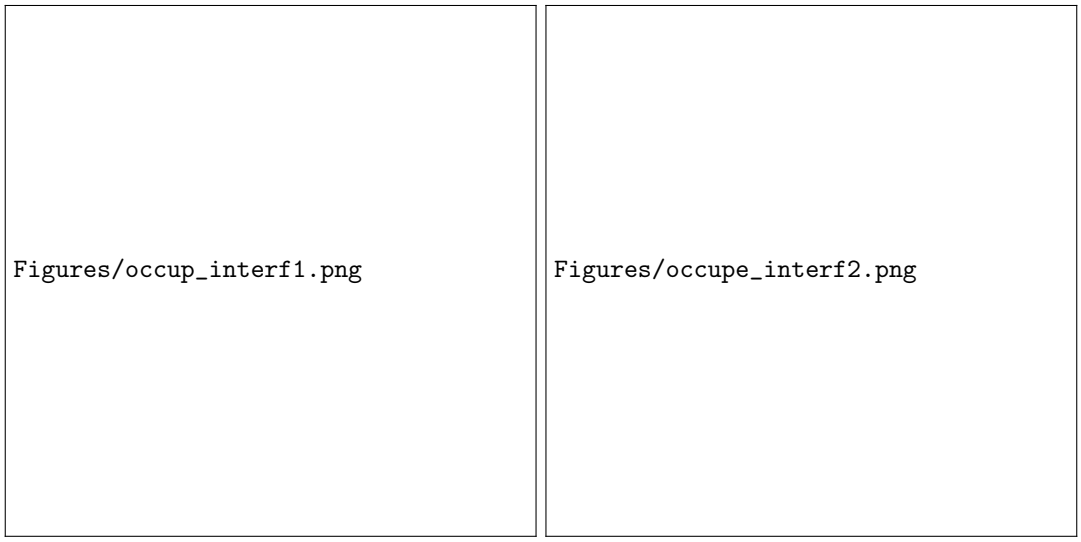


Figure 15: We see that the occupation number varies with total interference. We also see that we have an inversion of equilibrium behavior for w=0.3 and w=0.7

Interference makes sense of the common behavior.
Inverting w the probability, inverts teh equilibrium behavior.

General solution The weight in front of both states are equal if,

$$q_1 = 2(\frac{\sqrt{w}}{\sqrt{1-w}} - 1) + q_2 \frac{\sqrt{w}}{\sqrt{1-w}} \tag{43}$$

Then,

$$|\psi(t)\rangle = |q_1\rangle(A) \sum_j c_j(t)|e_j\rangle + |q_2\rangle(A) \sum_j c'_j(t)|e_j\rangle \tag{44}$$

We can make that happen while keeping E_I and w invariant by changing either the energy between the eigenstates of the SHO, or the states we choose for the superposition (which is what we do in practice). The idea being that we eliminated the asymmetry between the 2 Schmidt states, which makes δ_1 and δ_2 behave the same at equilibrium (as well as other metrics).

We verify this by making a plot of δ where the curves converge and another where they are inversed while keeping $w = 0.41$ and $E_I = 0.06$, typically values where we get distinct equilibrium behaviors for Schmidt 1 and 2.
See notebook "The same"

3.4 Conclusion:

The "energy" of the superimposed states, and their probability (in our case $q_1, q_2, w, (w-1)$) has an impact on the distributions of Schmidts in the energy eigenbasis.

(a) For weight difference of 0.01 (b) For a weight difference of 10

Figure 16: We see a that by making Schmidt 1 and 2 as "important" we can get them to agree on Neff. And we see the opposite happen when we maximize their weight difference.

Figure 17: The distribution of Schmidts has the possibility under certain conditions of going from uniformly distributed to ending and staying in this distribution.

3.5 Pseudo pointer in the interaction spectrum

We find an interesting evolution of the distribution in this basis. See Fig. (**??**). The motivation for looking in this basis was to look for pointer like behavior of converging towards an energy eigenstate of H_I. We see that it has a behavior like that, only not so complet.

3.5.1 Characterizing the behavior of the distributions in the interaction energy basis.

env does not matter

characterize behavior to see if it is weight dependant. interestingly you can get both to do so by macthing the weihghts. can you get neither to do so?

We look back at the equation,

$$|\psi(t)\rangle = |q_1\rangle(\frac{2+q_1}{\sqrt{w}})\sum_j c_j(t)|e_j\rangle + |q_2\rangle(\frac{2+q_2}{\sqrt{1-w}})\sum_j c_j(t)'|e_j\rangle \tag{45}$$

We have that the eigenstates of H_e are $\{|e_i\rangle\}$, the eigenstates of H_I are $\{|q_i\rangle|A_j\rangle\}$ We can re-write $|e_j\rangle = \sum_k a_{kj}|A_k\rangle$, meaning,

$$|\psi(t)\rangle = |q_1\rangle(\frac{2+q_1}{\sqrt{w}})\sum_{jk} c_j(t)a_{kj}|A_k\rangle + |q_2\rangle(\frac{2+q_2}{\sqrt{1-w}})\sum_{jk} c_j(t)'a_{kj}|A_k\rangle \tag{46}$$

Which leads to ,

$$P(|s_2(t)\rangle \text{ is in } |A_l\rangle) = |\frac{2+q_2}{\sqrt{1-w}}\sum_{jk} c'_j(t)a_{kj}\delta_{lk}|^2 = |\frac{2+q_2}{\sqrt{1-w}}|^2|\sum_{jl} c'_j(t)a_{lj}|^2 \tag{47}$$

Somehow, we have that $|\sum_{jl} c'_j(t)a_{lj}|^2$ is suppressed for a certain set of l's. The l's that correspind to outside the bounds we can see in Fig. (**??**). To study this, we need to understand the time evolution of the $c_j(t)$'s.

This is not explained by our numerical or model (at least trivially), since we do see that in principle the a_{lj}'s are not zero.

What could explain the supression? Not interferences. For multiple reasons. interferences go to 0 on the out

We would like to see how it evolves in the basis of H_I

To do that theoretically, we would need to Haussdorf approximate out unitary of non-commuting op.

4 Why do they share the energy eigenstates in this way?

What explains this way of sharing the eigenspace? Interferences between Schmidt states?

Do the probabilities in individual eigenstates vary less once equilibrium is reached? Do near zeros stay at near zero? Could it be that they spread in the eigenbasis and since they dont have many high probabilities, they dont overlap meaningfully simply by chance?

5 Can we get it all from the energy spectrum and the distribution of the global state in it?

Energy spectrum + state defined in energy basis -¿ sharing of Hilbert space by worlds -¿ explain the phenomenas we observed

Can we get the decoherence time from the spectrum?

5.1 Getting decoherence time from the spectrum

Note on getting things from the spectrum.... the decoherence time should be essentially defined by how fast we can consider the phases in front of the decomposition of $|\mathcal{E}_1(t)\rangle$ and $|\mathcal{E}_2(t)\rangle$ in $|k\rangle$, independent/randomised. $|\mathcal{E}_1(t)\rangle$ and $|\mathcal{E}_2(t)\rangle$ evolve according to $H_e^{(1)}$ and $H_e^{(2)}$ which have different spectrum. it might be possible to estimate from the spectrum distribution the time after which the relative phase of both vectors (compared in a common basis) look randomly selected, at which point we would have decoherence. Note that I need to prove that this is sufficient for decoherence. There is also a complication, which is that there is no common eigenbasis for $H_e^{(1)}$ and $H_e^{(2)}$. But it should be easy to look at in the H_I dominated limit.

5.1.1 H_I dominated limit

In this limit $H_e^{(1)} = E_I q_1 H_r$ and $H_e^{(2)} = E_I q_2 H_r$, meaning that they have a common eigenbasis. Taking this common eigenbasis to be $|k\rangle$, it means that we have,

$$|\mathcal{E}_1(t)\rangle = \frac{1}{\sqrt{d_2}} \sum_k e^{-itE_k} |k\rangle \tag{48}$$

$$|\mathcal{E}_2(t)\rangle = \frac{1}{\sqrt{d_2}} \sum_k e^{-itE'_k} |k\rangle \tag{49}$$

Since both environment states start in the same state, they have the same amplitudes in any basis. We simplify the problem by taking that amplitudes to be uniform in the basis we consider. The differentiation between both states then only comes from the phases.

The spectrum of $H_e^{(1)}$ and $H_e^{(2)}$ are simply different by a factor of $\frac{q_2}{q_1} = f$.

6 Other

6.0.1 Condition on there being an energy splitting

Bsically, if we have a Hamiltonian that can be split as the tensor product of a diagonal matrix and a complex Hamiltonian, and that in the same bassi have the state of the universe be a superposition product some random state, we will get this splitting happening. TODO specify better the conditions for the splitting happening, and look into if that is a common condition? if we go from a hamiltonian that has that, do the world afterwards and the effective hamltonians they live in also have that?

6.0.2 More on pointer states

Pointer states are states selected by the predictability sieve, which selects states of the system which deteriorate the least into mixtures over time. (or stay separable if we consider the global state.) By this condition, they are the state with the highest degree of reversibility in the system, whiich is a property of classical systems. (no growing entropy associated to reversibility). Fundamentally, we justify looking at global pointer states because they are

defiend by properties we ask of classical worlds. Can we get an useful defintion of worlds by developing our thinking on pointer states?

We ask of a world to be separable into many classical subsystems. This can be translated into the following condition. Choosing a scale d (ie number of dimension of Hilbert space above which we consider a system classical), and a subdivision of the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes ... \otimes \mathcal{H}_n$, with each $\mathcal{H}_i$ has dimension d we can alternatively consider each Hilbert space $\mathcal{H}_i$ as a subsystem, in which the state is a pointer state einselected by the rest of the Hilbert space. Furthermore, the evolution of the global state is separated (meaning you can see it like an operator phase in front of both term in the separable state). This must be true for all sub-system, meaning that there is a global pointer state basis in which the global state can be written like this,

$$|w\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes ... \otimes |\psi_n\rangle \tag{50}$$

$$|w(t)\rangle = e^{-itH_1}|\psi_1\rangle \otimes e^{-itH_2}|\psi_2\rangle \otimes ... \otimes e^{-itH_n}|\psi_n\rangle \tag{51}$$

If we go back to the table analogy and the subsystems as pieces of the table big enough to be considered classical, then we note that there are different ways to cut a table up in chunks of set size. That is we also need to consider $\mathcal{H} = \mathcal{H}_{1'} \otimes ... \otimes \mathcal{H}_{n'} \neq \mathcal{H}_1 \otimes ... \otimes \mathcal{H}_n$. And the conclusion is that we also need our world to have a global pointer state basis in which the world can be written,

$$|w\rangle = |\psi'_1\rangle \otimes |\psi'_2\rangle \otimes ... \otimes |\psi'_n\rangle \tag{52}$$

$$|w(t)\rangle = e^{-itH'_1}|\psi'_1\rangle \otimes e^{-itH'_2}|\psi'_2\rangle \otimes ... \otimes e^{-itH'_n}|\psi'_n\rangle \tag{53}$$

This imposes a condition on $|\psi\rangle$ and H_{tot}. What is that condition? is it that the world is separable in any possible subdivision of the Hilbert space? I dont know....

What has been discussed here is true for local subdivisions (pieces of table I can hold in my hands), but what if we consider $\mathcal{H} = \mathcal{H}_{1''} \otimes ... \otimes \mathcal{H}_{n''}$ a non-local decomposition of the Hilbert space.

the simple idea being that a table can be cut down in many "classical" pieces in many ways. The basic question is: What happens when I consider global pointer states defined by being product state of pointer states of many subsystems. And do I get anything else from asking for a variable decomposition (setting maybe the size of the subsytems to be classical).

6.1 Why do we get no overlape?

World overlape in the total energy eigenbasis. We defien our 2 worlds as $|w_1\rangle(t) = \sum_k c_k e^{-ite_k}|k\rangle$ and $|w_1\rangle = \sum_k c'_k e^{-ite_k}|k\rangle$. Then their overlap is,

$$\langle w_1|w_2\rangle(t) = \sum_k c^*_k c'_k \tag{54}$$

Note that the time dependance of the worlds is such that there is time independance of their overlap.

With only 1 eigenstate where the worlds are non-zero together, they have nonzero overlap $\langle w_1|w_2\rangle(t) = cst = c^*_i c'_i$. With more than one such eigenstate there exists a possible balance. ie, $\langle w_1|w_2\rangle(t) = \sum_{i\in nz} c^*_i c'_i = 0$ is possible.

6.2 What does it imply?

If the worlds are truly defined in different eigenstate of the total energy eigenbasis, what are the implications? Do we loose most of the space after each measurement?

What are the conditions for such measurement events happening? We need a Hamiltonian that can be decomposed in the form of the ACL model,.... what exactly is needed to reproduce theses effects... + condition that in this basis the state can be written in the manner we specify at the beginning. (I can work in the general case.)

Can there be an integrability condition for the Hamiltonian having this level of separability?

Anyways, once you have this Hamiltonian, you can imagine

Figure 18: Graphs of the similarity between s1, parameters $E_I = 0.06$ and $w = 0.3$.

6.2.1 Confusion regarding the loss of dimension

The state of the worlds are themselves defined in less dimensions, ie using less numbers in the total energy eigenbasis. Is that recoverable? Do they keep on loosing more? My confusion comes from the fact that $|0\rangle = \frac{|0\rangle}{2} + |1\rangle + \frac{|0\rangle}{2}|1\rangle = |+\rangle + |-\rangle$. and if somehow $|\pm\rangle$ happened to be the worlds post another measurement, then theses worlds have recovered 1 dimension... But I think, the point is mostly that my world is defined in 1 dim (here) but also only time evolves according to a Hamiltonian that is a scalar, which is incapable of creating worlds defined in more dimensions.

Measure of fluctuation in time of the distributions Similarity in time, a sort of measure of fluctuation. Maybe useful to characterize the behaviors of s1 and s2 distribution themselves.

Potential selection mechanism for schmidt states that fluctuate less, ie are stable in time?

6.3 ...

We have a certain sharing of the Hilbert space under measurement This sharing implies certain difference in behavior for quantities such as Neff. And if possible, it would be amazing to recover this sharing from the shape of the spectrum and the distribution of the global state! mad dog evrietian style.

Something that troubles me. q and w have a large impact on the "shape "of the distribution. The way I used to see things. Is that the shape of the distribution was the determining factor to how the world would be, ie it would define it classically, ie macroscopically. And to me the 2 worlds couldn't me so different. But, this macro looking factor (distribution shape), might just be irrelevant.

Could there be something to say about the fact that we are looking at overlap in the probability space? This is because of course the 2 worlds will be orthogonal in the Hilbert space. So our fundamental object of study is really their distribution in the total energy eigenbasis.

Pointer states: found using predicatibility sieve. They are the states that devellope the least entanglement with the environment in a robust manner.

So I guess we dont have pointer states in our case, since we dont have self interaction to vary the system state.

This dynamics of Schmdit basis states towards the pointer states might be worth studying. In the case of no self interaction for the system, the pointer states are eigenstates of the interaction Hamiltonian. Might be interesting to check.

Note the Schmidt basis does not have to be orthogonal. we have always orthogonal schmidt basis in our ACL case due to orthogonal system state + no self interaction.

(Andreas Albrecht study of Schmidt states dynamics under measurement: https://journals.aps.org/prd/abstract/10.1103/Phy https://journals.aps.org/prd/abstract/10.1103/PhysRevD.48.3768)

6.3.1 Distinction between Schmidt and Pointers

https://journals.aps.org/prd/abstract/10.1103/PhysRevD.40.1071

6.3.2 A discussion of Schmidt might not be enough

How pointers might not really be branches https://arxiv.org/pdf/gr-qc/9610028.pdf Is a paper, looking at the possibility of s=using the schmidt decomposition to seelct the physics set of consistent histories. The find is that it would not be enought. The schmidt basis is induced by ...

The pointer basis is induced by decoherence, or more precisely is einselected by the environment-system interaction

issue, isn't einselection only defined for S+M+E? while we are working here with S+E?

looking at the schmidt states has the nice property of always looking at 2 orthogonal states.

Are pointer states worlds? https://arxiv.org/abs/gr-qc/9610028 says that no

A Spectrum of H_{tot}

A.1 Characterizing the spectrum of the Hamiltonian

The Hamiltonian who's spectrum we are trying to characterize is composed of 3 parts, $\mathbb{1}, \mathbb{1} \otimes H_e, E_I H_q \otimes H'_e$. We are interested in the shape that the distribution of eigenvalues will take. $\mathbb{1}$ will simply shift that distribution by $+1$, since it commutes with the other 2.

We will use the fact that the random matrices that appear in these Hamiltonians, are Gaussian Unitary Ensemble (GUE) matrices, and are known to have semi-circle spectrum distribution. Wigner semi-circle law:

$$f(E) = \frac{2}{\pi R^2}\sqrt{E^2 - R^2} \tag{55}$$

distribution between $[-R, R]$ bounds.

Considering $\mathbb{1} \otimes H_e$, H_e has spectrum $\sigma_2 = \{\lambda_1, \lambda_2, ...\lambda_{d_1}\}$ and the identity is here defined over Hilbert space of dimension d_1. This makes the final spectrum $\sigma_{1,2} = \lambda_1, \lambda_1, ..., \lambda_1, \lambda_2, ..., \lambda_2, \lambda_3, ...$, where each element of the spectrum is repeated d_1 times. The distribution of the spectrum of the first term is then an elevated Wigner semi-circle.

$$f_{env}(E) = \frac{2}{\pi R^2}\sqrt{R^2 - E^2} + d_1 \tag{56}$$

Considering $H_q \otimes H'_e$, with the spectrum of the random matrix $\sigma_2 = \{\lambda'_1, \lambda'_2, ...\lambda'_{d_1}\}$. We get a new spectrum, $\sigma_{1,2} = 0 * \sigma_2 \bigcup \sigma_2 \bigcup 2\sigma_2 \bigcup ... \bigcup (d_1 - 1)\sigma_2$ ie $\sigma_{1,2} = \{0, ..., 0, \lambda'_1, \lambda'_2, ..., 2\lambda'_1, 2\lambda'_2, ..., (d_1 - 1)\lambda'_1, ...\}$. This produces a sum of Wigner semi-circles with $R_i = i * R_1$ for $i = 0, 1.., d - 1$.

$$f_{int}(E) = E_I \sum_{j=1}^{d_1 - 1} \frac{2}{\pi j^2 R^2}\sqrt{j^2 R^2 - E^2} + \delta_{E,0} d_2 \tag{57}$$

The $\delta_{E,0} d_2$ comes from our SHO having ground state energy $E_0 = 0$. The E_I factor is added to the distribution here. Since these distributions are centered on 0, the factor E_I acts as a spread factor. (ie the max and min are split $2E_I * R_{effective}$)

We verify that we do indeed get this in Fig. (**??**).

Figures/Wigner_sum.png

Figure 19: The set of wigner semi-circles who's sum gives us the spectrum of H_I. The wider ones are for higher q_i's.

With our simulation we can also get them numerically as seen in Fig. (**??**), we choose $E_I = 0.06$ and $w = 0.3$.

About Fig. (**??**), it looks like $\sigma_{total} = \sigma_{env} + E_I \sigma_{int} + 1$ ($+1$ from identity matrix). But that would only be the case if both Hamiltonians where diagonalizable in a common basis, which they are not, being different independant random matrices. Is it still possible to consider the final spectrum as such? I do not see another way to characterize it.

Figures/Spectra.png

Figure 20: First is the distribution of the state in the total energy eigenbasis. Second is the spectrum of that total energy eigenbasis. Third is the distribution of ythe spectrum of $\mathbb{1} \otimes H_e$ in its own energy eigenbasis. We do see a Wigner semi-circle lifted up by 10. Finally is the distribution of the spectrum of $H_q \otimes H'_e$, which also matches a sum of 10 Wigner semi-circles with growing R. We also see the $200 * \delta_{E,0}$ predicted at the center. Note that this last distribution has to be scaled down by the factor $E_I = 0.06$ (in this case)

The spectra look like they sum, even though the Hamiltonians do not commute

If we take the summing of the spectra as a correct approximation, we have characterized the shape of the total spectrum depending on E_I.

$$f_{total}(E) = \frac{2}{\pi R^2}\sqrt{R^2 - E^2} + E_I \sum_{j=1}^{d_1-1} \frac{2}{\pi j^2 R^2}\sqrt{j^2R^2 - E^2} + \delta_{E,0}d_2 + d_1\theta(R^2 - E^2) \tag{58}$$

$\theta(x)$ is the Heaviside function. For the $H_{rand,d=200}$ $R \approx 10$.

Potential change to the model. The identity spectrum does not add anything interesting we could remove it. It might also be beneficial to make the GS of the SHO different from 0.

A.1.1 We use bounds on the values of the sum of 2 matrices.

For 2 hermitian matrices A, B such that $A + B = C$. with ordered eigenvalues, we have,

$$c_{i+j-1} \leq a_i + b_j \quad \text{and} \quad c_{n-i-j} \geq a_{n-i} + b_{n-j} \tag{59}$$

This means for us that

$$f_{total}(E) \leq \frac{2}{\pi R^2}\sqrt{R^2 - (E-e)^2} + d_1 + E_I \sum_{j=1}^{d_1-1} \frac{2}{\pi j^2 R^2}\sqrt{j^2R^2 - e^2} + \delta_{E,0}d_2 \tag{60}$$