Practice quiz on Sets

PUNTOS TOTALES DE 3

---

1. Let $A = \{1, 3, 5\}$. Is the following statement: $3 \in A$. True or false?

   ⬜ False

   🔘 True

   1 / 1 puntos

   ✓ **Correcto**

   The symbol $\in$ stands for "is an element of" and it is true that 3 is an element of A. The other two elements of A are 1 and 5.

2. Let $E = \{-1, -2, -3\}$. Compute the cardinality $|E|$ of E:

   ⬜ -3

   🔘 3

   ⬜ 0

   ⬜ E

   1 / 1 puntos

   ✓ **Correcto**

   Recall that the cardinality of a set is the number of elements in it. Since E has three elements (which are $-1, -2, -3$), the cardinality of E is $|E| = 3$.

3. Let $A = \{1, 3, 5\}$ and $B = \{3, 5, 10, 11, 14\}$.

   Which of the following sets is equal to the intersection $A \cap B$?

   ⬜ $\{3\}$

   ⬜ $\{1, 3, 5\}$

   ⬜ $\{3, 5, 10\}$

   1 / 1 puntos

Practice quiz on the Number Line, including Inequalities

PUNTOS TOTALES DE 8

---

1. Which of the following real numbers is **_not_** an integer?   `1/1 puntos`

   ○ 7

   ◉ 4.3

   ○ 0

   ○ −3

   ✓ Correcto
   4.3 is a decimal that is between two consecutive integers (4 and 5).

---

2. Which of the following is the absolute value $|-7|$ of the number -7?   `1/1 puntos`

   ○ 1

   ○ −7

   ◉ 7

   ○ 0

   ✓ Correcto
   The absolute value of a number x is the distance along the number line from x to 0. In this case,

---

3. Suppose I tell you that x and y are two real numbers which make the statement $x < y$ true. Which pair of   `1/1 puntos` numbers **_cannot_** be values for x and y?

   ○ $x = -17.3$ and $y = -17.1$

   ○ $x = 1$ and $y = 7.3$

   ◉ $x = 5$ and $y = 3.3$

   ○ $x = -1$ and $y = 0$

   ✓ Correcto
   The statement $x < y$ means that x is to the left of y on the real number line. Since 5 is to the right of 3.3, these cannot be values for x and y.

---

4. Suppose I tell you that w is a real number which makes both of the following   `1/1 puntos` statements true: $w > 1$ and $w < 1.2$. Which of the following numbers could be w?

   ◉ $w = 1.05$

   ○ $w = 11$

   ○ $w = 0$

   ○ $w = 1.2$

   ✓ Correcto
   $1.05 > 1$ is true since 1.05 is to the right of 1 on the real number line, and $1.05 < 1.2$ is also true, since 1.05 is to the left of 1.2 on the real number line.

5. Suppose that x and y are two real numbers which satisfy $x + 3 = 4y + 1$. **Which of the following statements are false?**

0 / 1 puntos

- ○ $x = 4y$
- ○ $x = 4y - 2$
- ○ $2x + 6 = 8y + 2$
- ◉ $x + 2 = 4y$

! **Incorrecto**
  If we subtract 1 from both sides of the given equation, we get this equation.

6. Which of the following real numbers is in the open interval $(2, 3)$?

0 / 1 puntos

- ○ 2.1
- ○ 1
- ◉ 2
- ○ 3

! **Incorrecto**
  The open interval $(2, 3)$ consists of
  all real numbers x which satisfy $2 < x < 3$. While $2 < 3$, it is not true that $2 < 2$.

7. Which of the following real numbers are in the open ray $(3.1, \infty)$?

1 / 1 puntos

- ○ 0
- ○ 3.1
- ◉ 4.75
- ○ −5

✓ **Correcto**
  Recall that $(3.1, \infty) = \{x \in \mathbb{R} \mid x > 3.1\}$. Since $4.75 > 3.1$ is true, $4.75 \in (3.1, \infty)$.

8. Which of the following values for x solves the equation $-3x + 2 = -4$

1 / 1 puntos

- ○ $x = -2$
- ◉ $x = 2$
- ○ All values of x such that $x \leq 2$
- ○ $x = \dfrac{2}{3}$

✓ **Correcto**
  First we subtract 2 from both sides of the given
  equation, to obtain $-3x = -6$. Finally, to isolate x we divide both sides of the
  equation by -3 to obtain $x = 2$.

Practice quiz on Simplification Rules and Sigma Notation

PUNTOS TOTALES DE 6

1. Which of the numbers below is equal to the following summation: $\Sigma_{i=1}^{3} i^2$? `1/1 puntos`

   ○ 30

   ◉ 14

   ○ 1

   ○ 9

   > ✓ Correcto
   >
   > We compute $\Sigma_{i=1}^{3} i^2 = 1^2 + 2^3 + 3^2 = 14$

2. Suppose that $A = \Sigma_{k=1}^{100} k^4$ and $B = \Sigma_{j=1}^{100} j^4$ `1/1 puntos`

   Which of the following statements is true?

   ◉ $A = B$

   ○ $B > A$

   ○ $A > B$

   ○ There is not enough information to do the problem

   > ✓ Correcto
   >
   > A = B. Both summations evaluate to the same number, since k and j are just dummy indices.

3. Which of the numbers below is equal to the summation $\sum_{i=1}^{10} 7$?

- ● 70
- ○ 7
- ○ 55
- ○ 0

✓ **Correcto**

According to one of our Sigma notation simplification rules, this summation is just equal to 10 copies of the number 7 all added together, and so we get $10 \times 7 = 70$.

4. Suppose that $X = \sum_{i=1}^{5} i^3$ and $Y = \sum_{i=1}^{5} i^4$.

Which of the following expressions is equal to the summation $\sum_{i=1}^{5} (2i^3 + 5i^4)$?

- ○ $X + Y$
- ● $2X + 5Y$
- ○ 3375
- ○ 7

✓ **Correcto**

To get here, you apply two of our Sigma notation simplification rules $\sum_{i=1}^{5} 2i^3 + 5i^4 = 2\left(\sum_{i=1}^{5} i^3\right) + 5\left(\sum_{i=1}^{5} i^4\right) = 2X + 5Y$.

5. Which of the following numbers is the mean μ_Z of the set $Z = \{-2, 4, 7\}$?

◉ 3

◯ $\dfrac{13}{3}$

◯ 4

◯ 9

✓ **Correcto**

To get the mean of a set of numbers, you need to perform two steps: first add them all up (in this case getting $-2 + 4 + 7 = 9$), and then divide by the number of elements in the set (in this case that number is 3).

So you should obtain $\mu_Z = \dfrac{9}{3} = 3$, which you did!

---

6. Suppose the set X has five numbers in it: $X = \{x_1, x_2, x_3, x_4, x_5\}$. Which of the following expression represents the mean of the set X?

◉ $\dfrac{1}{5}[\sum_{i=1}^{5} x_i]$

◯ $\sum_{i=1}^{5} x_i$

◯ $\dfrac{1}{N}[\sum_{i=1}^{N} x_i]$

◯ $\dfrac{1}{5}[\sum_{i=1}^{5}(x_i - \mu_X)^2]$

✓ **Correcto**

To obtain the mean of a set of numbers, you first add them all up (which is expressed here by the sigma operation inside the square brackets) and then you divide by the number of numbers in the set (which is expressed here by the $\dfrac{1}{5}$ outside the square brackets).

Graded quiz on Sets, Number Line, Inequalities, Simplification, and Sigma Notation

CALIFICACIÓN DEL ÚLTIMO ENVÍO

100%

---

1. Let $B = \{3, 5, 10, 11, 14\}$. Is the following statement true or false: $3 \notin B$ — **1 / 1 puntos**

○ True

◉ False

> ✓ **Correcto**
>
> The symbol $\notin$ stands for "is not an element of." Since 3 is in an element of the set B, the given statement is not true.

---

2. Let $A = \{1, 3, 5\}$ and $B = \{3, 5, 10, 11, 14\}$. Which of the following sets is equal to the union $A \cup B$? — **1 / 1 puntos**

○ $\{1, 10, 18\}$

○ $\{3, 5, 10, 11, 14\}$

◉ $\{1, 3, 5, 10, 11, 14\}$

○ $\{1, 3, 5, 3, 5, 10, 11, 14\}$

> ✓ **Correcto**
>
> The union of two sets consists precisely of the elements that are in at least one of the two sets. That is precisely what is listed here.

---

3. How many real numbers are there between the integers 1 and 4? — **1 / 1 puntos**

○ 4

○ 2

◉ Infinitely many

○ None

> ✓ **Correcto**
>
> There are in fact infinitely many real numbers between any pair of distinct integers, or indeed any pair of distinct real numbers!

---

4. Suppose I tell you that x and y are two real numbers which make the statement $x \geq y$ true. Which pair of numbers **_cannot_** be values for x and y? — **1 / 1 puntos**

○ $x = 2$ and $y = 1$

○ $x = 10$ and $y = 10$

◉ $x = -1$ and $y = 0$

○ $x = 5$ and $y = 3.3$

> ✓ **Correcto**
>
> Recall that the statement $x \geq y$ means that x is either equal to y or x is to the right of y on the real number line. Since -1 is actually to the left of 0, these cannot be values for x and y.

5. Suppose that z and w are two positive numbers with $z < w$. Which of the following inequalities is false?

○ $w - 7 > z - 7$

○ $-z > -w$

○ $z + 3 < w + 3$

◉ $-5z < -5w$

✓ **Correcto**

If we start with $z < w$ and multiply both sides by -5, we need to flip the less-than sign, which would give $-5z > -5w$. For an example, try $z = 1$ and $y = 2$ and see what happens!

6. Find the set of all x which solve the inequality $-2x + 5 \leq 7$

◉ $x \geq -1$

○ $x \leq -1$

○ $x \geq -6$

○ $x = -1$

✓ **Correcto**

Subtracting 5 from both sides of the given inequality gives $-2x \leq 2$. Then we divide both sides by -2, remembering to flip the inequality sign, and we obtain this answer

7. Which of the following real numbers is not in the closed interval $[2, 3]$

1/1 puntos

- ◉ 1
- ○ 2.1
- ○ 2
- ○ 3

✓ **Correcto**
Recall that the closed interval $[2, 3]$ consists of all
real numbers x which satisfy $2 \le x \le 3$. Since $2 \le 1$ is false, $1 \notin [2, 3]$

8. Which of the following intervals represents the set of all solutions to:

1/1 puntos

$-5 \le x + 2 < 10$?

- ○ $[-7, 8]$
- ○ $(7, 8)$
- ◉ $[-7, 8)$
- ○ $[-5, 10)$

✓ **Correcto**
Subtracting 2 from all sides of the inequalities gives $-7 \le x < 8$, and the set of all real numbers
x which make that true is exactly
the half-open interval $[-7, 8)$.

9. Which of the numbers below is equal to the following summation: $\sum_{k=2}^{5} 2k$?

1/1 puntos

- ○ 14
- ○ 4
- ◉ 28
- ○ 10

✓ **Correcto**
We compute $\sum_{k=2}^{5} 2k = 4 + 6 + 8 + 10 = 28$.

10. Suppose we already know that $\sum_{k=1}^{20} k = 210$. Which of the numbers below is equal to $\sum_{k=1}^{20} 2k$?

1/1 puntos

- ○ 210
- ○ 2
- ◉ 420
- ○ 40

✓ **Correcto**
By applying one of our Sigma notation simplification rules, we can rewrite the summation in
question as $2 \left(\sum_{k=1}^{20} k \right) = 2 \times 210 = 420$.

11. Which of the numbers below is equal to the summation $\sum_{i=2}^{10} 7$?

○ 63
○ 70
○ 7
○ 48

✓ **Correcto**

According to one of our Sigma notation simplification rules, this summation is just equal to 9 copies of the number 7 all added together, and so we get $9 \cdot 7 = 63$.

12. Which of the following numbers is the variance of the set $Z = \{-2, 4, 7\}$?

○ 69
○ 42
◉ 14
○ $\sqrt{14}$

✓ **Correcto**

To get the variance of a set of numbers, you need to perform four steps:

First compute the mean (which is 3)

Then calculate all the squared differences between the numbers in the set and this mean (here you get $25, 1, 16$)

Then add all these up (here you get 42)

Then divide by the number of elements in the set (which is 3).

Therefore, the variance of Z

13. Which of the following sets does *not* have zero variance? (hint: don't do any calculation here, just think!)

○ $\{1, 1, 1, 1\}$
○ $\{0, 0, 0, 0, 0, 0, 0\}$
◉ $\{2, 5, 9, 13\}$
○ $\{5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5\}$

✓ **Correcto**

Intuitively, the numbers in this set are spread out.

Practice quiz on the Cartesian Plane

PUNTOS TOTALES DE 5

1. Which of the following points in the Cartesian Plane is on the y-axis?

- ○ $(-5, 0)$
- ○ $(1, 1)$
- ○ $(5, 0)$
- ◉ $(0, -5)$

✓ Correcto

The y-axis is defined to be all points in the Cartesian plane with zero as x-coordinate. The point $(0, -5)$ meets that requirement.

2. Find the distance between the points $A = (2, 2)$ and $C = (3, 3)$:

- ○ 1
- ○ 2
- ◉ $\sqrt{2}$
- ○ 0

✓ Correcto

Recall that the distance between points (a, b) and (c, d) is $\sqrt{(c - a)^2 + (d - b)^2}$.

In this case $(a, b) = (2, 2)$ and $(c, d) = (3, 3)$, so the distance is $\sqrt{(3 - 2)^2 + (3 - 2)^2} = \sqrt{2}$.

3. Find the point-slope form of the equation of the line that goes between $A = (1, 1)$ and $B = (5, 3)$:

○ $y - 1 = \frac{1}{2}(x - 5)$

◉ $y - 1 = \frac{1}{2}(x - 1)$

○ $y = \frac{1}{2}x$

○ $y - 3 = \frac{1}{2}(x - 1)$

✓ Correcto

The point-slope form for the equation of a line with slope m that goes through the point (x_0, y_0) is $y - y_0 = m(x - x_0)$

In this case, the slope $m = \dfrac{3 - 1}{5 - 1} = \dfrac{1}{2}$

We can choose either A or B for the point on the line, but in neither case do we get this chosen answer.

4. Which of the following points is on the line with equation:

$y - 1 = 2(x - 2)$?

○ $(3, 2)$

○ $(0, 0)$

○ $(2, 3)$

◉ $(2, 1)$

✓ Correcto

If we plug in 1 for y and 2 for x in the equation of the line, we make a true statement, $0 = 0$, so this point lies on the line.

5. Suppose that a line ℓ has slope 2 and goes through the point $(-1, 0)$. What is the y-intercept of ℓ?

◉ 2

○ 0

○ −1

○ 1

✓ Correcto

Recall that the y-intercept of ℓ is the y-coordinate of where ℓ hits the y-axis.

Since $(-1, 0) \in \ell$, the point on ℓ with $x = 0$ is obtained by running one unit from $(-1, 0)$ while rising two units.

This gives $y = 2$ as the y-intercept.

1. Suppose that $A = \{1, 2, 10\}$ and $B = \{4, 8, 40\}$. Which of the following formulae do **not** define a function $f : A \to B$?

- ⊙ $f(1) = 5$, $f(2) = 8$, and $f(10) = 40$.
- ○ $f(1) = 4$, $f(2) = 40$, and $f(10) = 8$.
- ○ $f(1) = 4$, $f(2) = 4$, and $f(10) = 4$.
- ○ $f(a) = 4a$, for each $a \in A$

✓ **Correcto**

A function $f : A \to B$ is a rule which assigns an element $f(a) \in B$ to each $a \in A$. In this case, unfortunately, $f(1) = 5 \notin B$.

2. Suppose that A contains every person in the VBS study (see the second video in the course if you're confused here!). Suppose that $Y = \{+, -\}$ and $Z = \{H, S\}$

Suppose that $T : A \to Y$ is the function which gives $T(a) = +$ if person a tests positive and $T(a) = -$ if they test negative.

Suppose that $D : A \to Z$ is the function which gives $D(a) = H$ does not actually have VBS and $D(a) = S$ if the person actually has VBS.

Which of the following must be true of person a if we have a false positive?

- ○ $T(a) = -$ and $D(a) = H$
- ⊙ $T(a) = +$ and $D(a) = H$
- ○ $T(a) = +$ and $D(a) = S$
- ○ $T(a) = -$ and $D(a) = S$

✓ **Correcto**

Recall that a false positive is a positive test result (so $T(a) = +$) which is misleading because the person actually does not have the disease ($D(a) = H$)

3. Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2 - 1$. Which of the following points are *not* on the graph of g? **1/1 puntos**

○ $(-1, 0)$

◉ $(2, -1)$

○ $(0, -1)$

○ $(1, 0)$

✓ Correcto

Recall that the graph of g consists of all points (x, y) such that $y = g(x)$. Here $g(2) = 3 \neq -1$, so the point $(2, -1)$ is \emph{not} on the graph of g.

4. Let the point $A = (2, 4)$. Which of the following graphs does *not* contain the point A? **1/1 puntos**

○ The graph of $g(x) = x + 2$

◉ The graph of $h(x) = x - 1$

○ The graph of $s(x) = x^2$

○ The graph of $f(x) = 2x$

✓ Correcto

The graph of h consists of all points (x, y) such that $y = h(x)$. Here $h(2) = 1 \neq 4$, so the point $(2, 4)$ is *not* on the graph of h.

5. Suppose that $h(x) = -3x + 4$. Which of the following statements is true?

○ h is neither a strictly increasing function nor a strictly decreasing function.

○ All statements are correct

○ h is a strictly increasing function

◉ h is a strictly decreasing function

✓ **Correcto**

A function h is called strictly decreasing if whenever $a < b$, then $h(a) > h(b)$

Since the graph of h is a line with negative slope, this is in fact true!

6. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a strictly increasing function, with $f(3) = 15$

Which of the following is a possible value for $f(3.7)$?

○ 14.7

○ -3

◉ 17

○ 3

✓ **Correcto**

A function f is called strictly increasing if whenever $a < b$, then $f(a) < f(b)$.

Since $f(3) = 15$ is given and $3 < 3.7$, it must be that $15 < f(3.7)$, and this answer satisfies that.

Graded quiz on Cartesian Plane and Types of Function

CALIFICACIÓN DEL ÚLTIMO ENVÍO

92.3%

1. Which of the following points in the Cartesian Plane have positive x-coordinate and negative y-coordinate? **1 / 1 puntos**

   ○ $(5, 7)$

   ○ $(-4, 5)$

   ⦿ $(7, -1)$

   ○ $(0, 0)$

   > ✓ **Correcto**
   > The x-coordinate, 7, is positive, and the y-coordinate, -1, is negative.

2. Which of the following points is in the first quadrant of the Cartesian Plane? **1 / 1 puntos**

   ○ $(-5, 1)$

   ⦿ $(7, 11)$

   ○ $(-4, -7)$

   ○ $(5, -1)$

   > ✓ **Correcto**
   > The first quadrant is defined to be all points in the Cartesian plane whose coordinates are both positive.

3. Let A, B, C, D be points in the Cartesian Plane, and let the set $S = \{B, C, D\}$

1/1 puntos

Suppose that the distances from A to B, C, D are $5.3, 2.1$, and 11.75, respectively.

Which of the following points is the nearest neighbor to the point A in the set S?

- ◯ D
- ⦿ C
- ◯ A
- ◯ B

✓ **Correcto**
The distance from A to C is 2.1 and that is smaller than the distance from A to any other element of S.

4. Find the distance between the points $A = (2, 2)$ and $B = (-1, -2)$.

1/1 puntos

- ◯ -25
- ⦿ 5
- ◯ 25
- ◯ 1

✓ **Correcto**
Recall that the distance between points (a, b) and (c, d) is $\sqrt{(c-a)^2 + (d-b)^2}$

In this case we have:

$$\sqrt{(-1-2)^2 + (-2-2)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$$

5. Find the slope of the line segment between the points $A = (0,1)$ and $B = (1,0)$.

○ -1

◉ 1

○ $\sqrt{2}$

○ 0

> ! **Incorrecto**
>
> If you got here, you probably made a mistake with negative signs. If you draw the line segment, you will notice that the line slopes downwards, so the slope needs to be negative.

6. Find the point-slope form of the equation of the line with slope -2 that goes through the point $(5,4)$.

◉ $y - 4 = -2(x - 5)$

○ $y - 4 = 2(x - 5)$

○ $(5,4)$

○ $y - 5 = -2(x - 4)$

> ✓ **Correcto**
>
> The point-slope form for the equation of a line with slope m that goes through the point (x_0, y_0) is $y - y_0 = m(x - x_0)$.
>
> In this case, the slope $m = -2$ is given and the point $(5,4)$ on the line is given.

7. Which of the following equations is for a line with the same slope as $y = -3x + 2$?

○ $y = 8x - 3$

○ $y = 5x$

◉ $y = -3x - 8$

○ $y = 5x + 2$

✓ **Correcto**
The slope-intercept formula for a line is $y = mx + b$, where m is the slope and b is the y-coordinate of the point where the line hits the y-axis.

This line has slope $m = -3$ which is the same slope as the given line.

8. Which of the following equations is for a line with the same y-intercept as $y = -3x + 2$?

○ $y = 8x - 3$

◉ $y = 5x + 2$

○ $y = -3x - 8$

○ $y = 5x$

✓ **Correcto**
The the slope-intercept formula for a line is $y = mx + b$, where m is the slope and b is the y-coordinate of the point where the line hits the y-axis. This line has a y-intercept of 2 which is the same as the given line.

9. How many lines contain both the point $A = (1, 1)$ and the point $B = (2, 2)$?

○ infinitely many

○ 2

◉ 1

○ None

✓ **Correcto**

The line with equation $y = x$ is the one and only line that meets the stated requirements.

10. Suppose that we have two sets, $A = \{a, b\}$ and $Z = \{x, y\}$. How many different functions $F : A \to Z$ are possible?

○ There are none

○ There are infinitely many

◉ 4

○ 1

✓ **Correcto**

A function $F : A \to Z$ is a rule which assigns an element $F(a) \in Z$ to each element $a \in A$.

There are two elements in A; namely, a and b. For each of these elements, there are two assignment choices we could make: x and y.

Here are the four possible functions:

$F(a) = x, F(b) = y$, OR

$F(a) = y, F(b) = x$, OR

$F(a) = x, F(b) = x$, OR

11. How many graphs contain both the point $A = (0,0)$ and the point $B = (1,1)$

○ None

○ 1

○ 2

◉ Infinitely many

✓ Correcto

The graphs of $f(x) = x, g(x) = x^2, h(x) = x^3, s(x) = x^4, \ldots$ all contain both A and B

12. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is a continuous function whose graph intersects the x-axis more than once. Which of the following statements is true?

○ g is strictly decreasing.

○ All of the above.

◉ g is neither strictly increasing nor strictly decreasing.

○ g is strictly increasing.

✓ Correcto

The function g fails the horizontal line test, so it can neither be strictly increasing nor strictly decreasing.

13. Find the slope of the line segment between the points $A = (1,1)$ and $B = (5,3)$.

○ $\sqrt{20}$

○ 4

○ 2

◉ $\dfrac{1}{2}$

✓ Correcto

The slope of this line segment is $\dfrac{3-1}{5-1} = \dfrac{1}{2}$, where $3-1$ is the rise and $5-1$ is the run.

Practice quiz onTangent Lines to Functions

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following expressions corresponds to $f'(2)$, the slope of the tangent line to the graph of $f(x)$ at $x = 2$?   **1 / 1 puntos**

    ⦿ $f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

    ○ $f'(2) = 2$

    ○ $f'(2) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

    ○ $f'(2) = mx + b$

    ✓ **Correcto**
    This expression can be obtained from the first screen of our video by plugging in 2 for a.

2. Suppose that $h : \mathbb{R} \to \mathbb{R}$ is a function whose graph is shown as the blue curve in the figure. For how many values of a is $h'(a) = 0$?   **1 / 1 puntos**
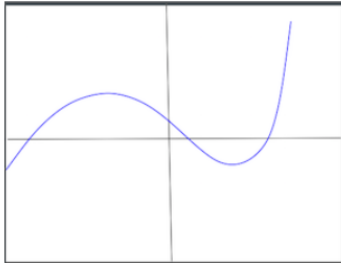
    ○ 3

    ○ Never

    ○ Always

    ⦿ 2

    ✓ **Correcto**
    $h'(a)$ gives the slope of the tangent line to the graph of h at the point $x = a$.

    When $h'(a) = 0$, this means that the tangent line is horizontal.

Practice quiz on Exponents and Logarithms

PUNTOS TOTALES DE 12

1. Re write the number $784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$ using exponents.

   1 / 1 puntos

   ⦿ $(2^4)(7^2)$

   ○ $(16^4)(49^2)$

   ○ $(2^6)(7^6)$

   ○ $(2 \times 7)^6$

   ✓ **Correcto**

   For this type of problem, count the number of times each relevant factor
   appears in the product. That number is the exponent for that factor.

2. What is $(x^2 - 5)^0$?

   1 / 1 puntos

   ⦿ 1

   ○ $(x^2) - 5$

   ○ (x^2)

   ○ -4

   ✓ **Correcto**

   Any real number (except zero) raised to the"zeroith" power $= 1$.

3. Simplify $((x - 5)^2)^{-3}$

   1 / 1 puntos

   ⦿ $(x - 5)^{-6}$

   ○ $(x - 5)^{-5}$

   ○ $(x - 5)$

   ○ $(x - 5)^{-1}$

   ✓ **Correcto**

   By Rule 2, "Power to a Power," multiply the exponents and get:

4.

Simplify $\left(\dfrac{8^2}{8^7}\right)^2$

○ 8^{-5}

◉ 8^{-10}

○ 8^{-4}

○ 8^{-1}

✓ **Correcto**

We can first simplify what is inside the parenthesis to 8^{-5} using the Division and Negative Powers Rule.

Then apply division and negative powers-- the result is the same. $\dfrac{8^4}{8^{14}} = 8^{-10}$

5. $\log 35 = \log 7 + \log x$

Solve for x

○ 7

○ 4

○ 28

◉ 5

✓ **Correcto**

$\log(x) = \log 35 - \log 7$

$\log(x) = \log\left(\dfrac{35}{7}\right)$

By the Quotient Rule $\log x = \log 5$

6. $\log_2 (x^2 + 5x + 7) = 0$

    Solve for x

    1/1 puntos

    ○ $x = 3$

    ○ $x = 2$

    ○ $x = 2$ or $x = 3$

    ● $x = -2$ or $x = -3$

    ✓ **Correcto**

    We use the property that $b^{\log_b a} = a$

    Use both sides as exponent for 2.

    $2^{\log_2 x^2 + 5x + 7} = 2^0$

    $x^2 + 5x + 7 = 1$

    $x^2 + 5x + 6 = 0$

    $(x + 3)(x + 2) = 0$

7. Simplify $\log_2 72 - \log_2 9$

    1/1 puntos

    ○ $\log_2 4$

    ○ 4

    ● 3

    ○ $\log_2 63$

    ✓ **Correcto**

    By the quotient rule, this is $\log_2 \dfrac{72}{9} = \log_2 2^3 = 3$

8. Simplify $\log_3 9 - \log_3 3 + \log_3 5$

    1/1 puntos

    ○ 15

    ● $\log_3 15$

    ○ $\log_3 8$

    ○ 8

    ✓ **Correcto**

    9×5

9. Simplify $\log_2(3^8 \times 5^7)$

   - ○ $(5 \times \log_2 3) + (8 \times \log_2 5)$
   - ◉ $(8 \times \log_2 3) + (7 \times \log_2 5)$
   - ○ $15 \times \log_2 56$
   - ○ $56 \times \log_2 15$

   ✓ **Correcto**
   We first apply the Product Rule to convert to the sum: $\log_2(3^8) + \log_2(5^7)$. Then apply the power and root rule.

10. If $\log_{10} y = 100$, what is $\log_2 y =$?

    - ○ 500
    - ◉ 332.19
    - ○ 301.03
    - ○ 20

    ✓ **Correcto**
    Use the change of base formula, $\log_a b = \dfrac{\log_x b}{\log_x a}$

    Where the "old" base is x and the "new" base is a.

11. A tree is growing taller at a continuous rate. In the past 12 years it has grown from 3 meters to 15 meters. What is its rate of growth per year?

    - ○ 12.41%
    - ◉ 13.41%
    - ○ 10.41%
    - ○ 11.41%

    ✓ **Correcto**
    $$\frac{\ln \frac{15}{3}}{12} = 0.1341$$

12. Bacteria can reproduce exponentially if not constrained. Assume a colony grows at a continually compounded rate of 400% per day. How many days before a colony with initial mass of 6.25×10^{-10} grams weights 1000 Kilograms?

    - ◉ 8.75 days
    - ○ 875 days
    - ○ 0.875 days
    - ○ 87.5 days

    ✓ **Correcto**
    $6.25 \times 10^{-10} \times e^{4t} \quad 10^6$

Graded quiz on Tangent Lines to Functions, Exponents and Logarithms

84.61%

---

1. Convert $\dfrac{1}{49}$ to exponential form, using 7 as the factor.

   1 / 1 puntos

   ○ 49^{-1}

   ◉ 7^{-2}

   ○ (7^2)

   ○ $\dfrac{7}{7^3}$

   ✓ **Correcto**
   The rule for a factor to a Negative exponent is to divide by the same factor to a positive exponent with the same absolute value.

2. A light-year (the distance light travels in a vacuum in one year) is $9,460$ trillion meters. Express in scientific notation.

   0 / 1 puntos

   ○ 0.946×10^{16}

   ○ 9460×10^{12} meters

   ○ 9.46×10^{15} meters.

   ◉ 9.46×10^{15} kilometers

   ! **Incorrecto**
   $9,460$ is (9.4×10^3) meters and one trillion meters is 10^{12} meters. $(9.4 \times 10^3)(10^{12}) = 9.4 \times 10^{15}$. A kilometer is 1000 meters.

3. Simplify $(x^8)(y^3)(x^{-10})(y^{-2})$

- ◉ $(x^{-2})(y)$
- ○ $(x^2)(y)$
- ○ $(x^{-80})(y^{-6})$
- ○ $(x)(y^{-2})$

✓ **Correcto**

By the Division and Negative Powers Rule, this is $(x^{(8-10)})(y^{(3-2)})$

4. Simplify $[(x^4)(y^{-6})]^{-1}$

- ○ $\dfrac{(x^4)}{(y^{-6})}$
- ○ $(x^3)(y^{-7})$
- ◉ $(x^{-4})(y^6)$
- ○ $\dfrac{(x^-4)}{(y^6)}$

✓ **Correcto**

By the Power to a Power Rule, each of the exponents is multiplied by (-1)

5. Solve for x:

$$\log_2 (39x) - \log_2 (x - 5) = 4$$

○ $\dfrac{80}{38}$

◉ $\dfrac{-80}{23}$

○ $\dfrac{39}{23}$

○ $\dfrac{23}{80}$

✓ Correcto

$\log_2 \dfrac{39x}{(x - 5)} = 4$ by the Quotient Rule.

Since both sides are equal, we can use them as exponents in an equation.

$$2^{\log_2 \frac{39x}{(x - 5)}} = 2^4$$

$$\frac{39x}{(x - 5)} = 16$$

$$39x = 16 \times (x - 5)$$

$$39x = 16x - 80$$

$$23x = -80$$

$$x = \frac{-80}{23}$$

6. Simplify this expression:

$$\left(x^{\frac{1}{2}}\right)^{\frac{-3}{2}}$$

○ $x^{\frac{1}{3}}$

○ $x^{\frac{-3}{4}}$

○ x^{-1}

◉ $x^{\frac{4}{3}}$

! **Incorrecto**
We use the Power to a Power Rule -- multiply exponents.

$$x^{\frac{1}{2} \times \frac{-3}{2}} = ?$$

Don't give up! Try again.

---

7. Simplify $\log_{10} 1000 + \log_{10} \frac{1}{10000}$

◉ -1

○ $\frac{1}{10}$

○ 1

○ $\log_{10} -10$

✓ **Correcto**
By the Product Rule, this is:

$$\log_{10}\left(\frac{1000}{10000}\right) = \log_{10}\left(\frac{1}{10}\right) = -1$$

8. If $\log_3 19 = 2.680$, what is $\log_9 19$?

◉ 1.304

○ 0.8934

○ 5.216

○ 0.4347

✓ Correcto

To convert from $\log_3$ to $\log_9$, divide by $\log_3 9$. Which is equal to 2, so the answer is 1.34

9. If $\log_{10} b = 1.8$ and $\log_a b = 2.5752$, what is a?

○ 6

◉ 5

○ 3

○ 4

✓ Correcto

To solve for a in the formula;

$$\log_a b = \frac{\log_x b}{\log_x a}$$

$\log_a b = 2.5752$ and $\log_{10} b = 1.8$

Therefore, $\log_{10} a$ must equal to $\dfrac{1.8}{2.5752} = 0.69897$

10. An investment of $1,600$ is worth $7,400$ after 8.5 years. What is the continuously compounded rate of return of this investment?

- ⦿ 18.02%
- ◯ 19.01%
- ◯ 17.01%
- ◯ 20.01

> ✓ Correcto
>
> $$\frac{\ln \frac{7400}{1600}}{8.5} = 0.18017$$

11. A pearl grows in an oyster at a continuously compounded rate of .24 per year. If a 25-year old pearl weighs 1 gram, what did it weigh when it began to form?

- ◯ 0.2478
- ◯ 0.0002478
- ◯ 0.02478
- ⦿ 0.002478

> ✓ Correcto
>
> $$e^{(0.24 \times 25)} = \frac{1}{x}$$
>
> $$x = \frac{1}{\left(e^{0.24 \times 25}\right)}$$

12. $\log_2 z = 6.754$. What is $\log_{10}(z)$?

○ 1.3508

○ 0.82956

◉ 2.03316

○ 0.49185

✓ Correcto

$$\frac{\log_2 z}{\log_2 10} =$$

$$(\log_{10} z) \times (\log_2 10) = 3.321928$$

Therefore, $\log_{10} z = \dfrac{6.754}{3.321928} = 2.03316$

13. Suppose that $g : \mathbb{R} \to \mathbb{R}$ is a function, and that $g(1) = 10$. Suppose that $g'(a)$ is negative for every single value of a. Which of the following could possibly be $g(1.5)$?

○ $g(1.5) = 10.1$

◉ $g(1.5) = 9.7$

○ $g(1.5) = 11$

○ $g(1.5) = 103.4$

✓ Correcto

Since the slope of the tangent line to the graph of g is negative everywhere on the graph, we know that g is *decreasing* function! And therefore we must have $g(1.5) < g(1)$. That is the case here, so this value is at least possible.

Practice quiz on Probability Concepts

PUNTOS TOTALES DE 9

---

1. If $x =$ "It is raining," what is $\sim (\sim x)$?   `1/1 puntos`

   ○ "It is not raining"

   ○ "It is always raining"

   ◉ "It is raining"

   ○ "It is never raining"

   > ✓ Correcto
   >
   > The second negation cancels out the first one.
   >
   > Similarly $\sim (\sim (\sim x)) =\sim x$

2. If the statement "I am 25 years old" is assigned probability 0, what probability is assigned to the statement   `1/1 puntos`
   "I am not 25 years old"?

   ○ -1

   ◉ 1

   ○ 0

   ○ Unknown

   > ✓ Correcto
   >
   > It is always the case that $p(x) + p(\sim x) = 1$.

3. If I assign to the statement x = "it will rain today" a probability of $p(x) = 0.35$, what probability must I assign to the statement "it will not rain today?"

- ○ 0
- ○ .35
- ○ .5
- ◉ .65

✓ **Correcto**

$$p(x) + p(\sim x) = 1$$

4. Is the following collection of statements a probability distribution?

1. I own a Toyota pickup truck

2. I do not own a Toyota pickup truck

3. I own a non-Toyota pickup truck

4. I do not own a non-Toyota pickup truck

- ◉ No
- ○ Yes

✓ **Correcto**

The statements are not *exclusive*:1 and 4 could both be true, 2 and 3 could both be true, 2 and 4 could both be true, and even (1) and (3) could both be true (if I owned more than one pickup truck).

5. I don't know what it means to be "ingenuous." What
   probability would I assign to the statement, "I am ingenuous OR I am not
   ingenuous"?

- ◉ 1
- ○ .5
- ○ -1
- ○ 0

✓ **Correcto**
   It is always the case, regardless of the content of the statement x, that $p(x \text{ or} \sim x) = 1$

6. A friend of mine circumscribes a circle inside a square, so
   that the diameter of the circle and the edge of the square are the same length.
   He asks me to close my eyes and pick a point at random inside the square. He
   says the probability that my point will also be inside the circle is $\frac{\pi}{4}$

   Is this correct?

- ◉ Yes
- ○ No

✓ **Correcto**
   Probabilities can be any real number between 0 and 1. They do not need
   to be rational numbers – a numerator that is a transcendental number like Pi is
   acceptable.

   Note that
   the correct probability does not depend on the length r of the circle's radius.
   For a circle with any radius r to be circumscribed inside a square, the square
   must have sides each of length 2r. The area of the circle is Pi*r^2 and the
   area of the square is (2r)^2 = 4*r^2 = The

7. The probability of drawing a straight flush (including a Royal Flush) in a five-card poker hand is 0.0000153908

What is
the probability of **not** drawing a
straight flush?

- ○ .9967253809
- ○ .9999745688
- ● .9999846092
- ○ .9996582672

✓ **Correcto**
$$p(\sim x) = 1 - p(x)$$

8. What is the probability that a fair, six-sided die will come up with a prime number? (Recall that prime numbers are positive integers other than 1 that are divisible only by themselves and 1)

- ○ $\dfrac{2}{3}$
- ● $\dfrac{1}{2}$
- ○ $\dfrac{1}{6}$
- ○ $\dfrac{1}{3}$

✓ **Correcto**
The faces with 2, 3 and 5 satisfy the condition – which makes 3 relevant outcomes out of the "universe" of 6 outcomes $= \dfrac{3}{6} = \dfrac{1}{2}$

9. The joint probability p (the die will come up 5, the next card will be a heart) Is equal to the joint probability:

○ p (the next card will be a heart, the die will come up 5)

○ p (the die will **not** come up 5, the next card will **not** be a heart)

○ p (the next card will **not** come up 5, the next card will be a heart)

◉ p (the next card will be a heart, the die will **not** come up 5)

> **!** **Incorrecto**
> In joint probabilities, the order does not change the probability: $p(A, B) = p(B, A)$

Practice quiz on Problem Solving

PUNTOS TOTALES DE 9

1. I am given the following 3 joint probabilities:

p(I am leaving work early, there is a football game that I want to watch this afternoon) = $.1$

p(I am leaving work early, there is not a football game that I want to watch this afternoon) = $.05$

p(I am not leaving work early, there is not a football game that I want to watch this afternoon) = $.65$

What is the probability that there is a football game that I want to watch this afternoon?

○ .35

○ .1

◉ .2

○ .3

> **!** **Incorrecto**
> Getting the answer is a two-step process. First, recall that the sum of probabilities for a probability distribution must sum to 1. So the "missing" joint distribution
>
> p(I am not leaving work early, there is a football game I want to watch this afternoon) must be $1-(0.1 + 0.05 + 0.65) = 0.2$
>
> By the sum rule, the marginal probability p(there is a football game that I want to watch this

2. The

Joint probability of my summiting Mt. Baker in the next two years AND publishing a best-selling book in the next two years is .05. If
the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, are these two events dependent or independent?

◉ Dependent

◯ Independent

✓ **Correcto**

We know this because the joint distribution of 5% does not equal the product distribution of $(0.1) \times (0.3) = 3\%$. If I summit Mt. Baker, I am more likely to publish a best-selling book, and vice versa.

3. The

Joint probability of my summiting Mt. Baker in the next two years AND my
publishing a best-selling book in the next two years is .05.

If
the probability of my publishing a best-selling book in the next two years is 10%, and the probability of my summiting Mt. Baker in the next two years is 30%, what is the probability that (sadly) in the next two years I will neither summit Mt. Baker nor publish a best-selling book?

◯ .25

◉ .65

◯ .9

◯ .95

✓ **Correcto**

Set A = I will summit Mt. Baker in the next two years

Set B = I will publish a best-selling book in the next two years.

Since $p(A) = 0.3$ and $p(A, B) = 0.05$, by the SUM RULE we know that $p(A, \sim B) = (0.3 -$

4. I

have two coins. One is fair, and has a probability of coming up heads of .5.
The second is bent, and has a probability of coming up heads of .75. If I toss
each coin once, what is the probability that *at least* one of the coins will
come up heads?

○ .375

○ .625

○ 1.0

◉ .875

✓ **Correcto**

We apply the rule p(A or B or both)

= 1 - (p(~A)p(~B))

= 1 - ((1- .5)(1-.75))

= 1 - .125

=.875

5. What is $\dfrac{11!}{9!}$?

◉ 110

○ 110, 000

○ 4, 435, 200

○ 554, 400

✓ **Correcto**

$$\frac{11!}{9!} = 11 \times 10 = 110$$

What is the probability that, in six throws of a die, there will be exactly one each of "1" "2" "3" "4" "5" and "6" ?

○ .00187220

○ .01176210

○ .01432110

◉ .01543210

✓ **Correcto**

There are $6! = 720$ permutations where each face occurs exactly once.

There are $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 46656$ total permutations of 6 throws.

The probability is therefore $\dfrac{720}{46656} = 0.01543210$

---

7. On 1 day in 1000, there is a fire and the fire alarm rings.

On 1 day in 100, there is no fire and the fire alarm rings (false alarm)

On 1 day in $10,000$, there is a fire and the fire alarm does not ring (defective alarm).

On $9,889$ days out of $10,000$, there is no fire and the fire alarm does not ring.

If the fire alarm rings, what is the (conditional) probability that there is a fire?

Written $p(\text{there is a fire} \mid \text{fire alarm rings})$

○ 1.1%

○ 90.9%

8. On 1 day in 1000, there is a fire and the fire alarm rings.

On 1 day in 100, there is no fire and the fire alarm rings
(false alarm)

On 1 day in $10,000$, there is a fire and the fire alarm does
not ring (defective alarm).

On $9,889$ days out of $10,000$, there is no fire and the fire
alarm does not ring.

If the fire alarm does not ring, what is the (conditional)
probability that there is a fire?

p(there is a fire | fire alarm does not ring)

○ 1.0001%

◉ 0.01011%

○ .01000%

○ .10011%

✓ **Correcto**
On $(1 + 9,889) = 9,890$ days out of every $10,000$ the fire alarm does not ring.

On 1 of those $10,000$ days there is a fire.

$$\frac{1}{9890} = 0.01011\%$$

9. A group of 45 civil servants at the State Department are
newly qualified to serve as Ambassadors to foreign governments. There are 22
countries that currently need Ambassadors. How many distinct groups of 22
people can the President promote to fill these jobs?

○ =1.06*(10^35)

○ $$4.1167 \times (10^12)

◉ =2.429*(10^-13)

○ 8.2334 \times (10^12)

! **Incorrecto**
This
is the probability of drawing any particular permutation.

$$\binom{45}{22}$$

$$= \frac{45!}{23! \times 22!}$$

Practice quiz on Bayes Theorem and the Binomial Theorem

PUNTOS TOTALES DE 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customers.

The store does some research and learns that:

- 10% of the times that a jewelry store is robbed, a customer is in the store.

- A jewelry store has a customer on average 20% of each 24-hour day.

- The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

What is the probability that a robbery will occur while a customer is in the store?

○ $\dfrac{1}{500000}$

○ $\dfrac{1}{2000000}$

◉ $\dfrac{1}{4000000}$

○ $\dfrac{1}{5000000}$

✓ **Correcto**

What is known is:

A: "a customer is in the store," $P(A) = 0.2$

B: "a robbery is occurring," $P(B) = \frac{1}{2,000,000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

2. If I flip a fair coin, with heads
   and tails, ten times in a row, what is the probability that I will get exactly six heads?

- ○ 0.021
- ○ 0.187
- ◉ 0.2051
- ○ 0.305

✓ Correcto
By Binomial Theorem, equals

$$\binom{10}{6}\left(0.5^{10}\right)$$

$$= \left(\frac{10!}{4! \times 6!}\right)\left(\frac{1}{1024}\right)$$
$$= 0.2051$$

3. If a coin is bent so that it has a 40% probability of coming
   up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

- ○ 0.0974
- ○ 0.1045
- ◉ 0.1115
- ○ 0.1219

✓ Correcto
$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

○ 0.0132

◉ 0.0123

○ 0.0312

○ 0.0213

✓ Correcto

The answer is the sum of three binomial probabilities:

$$((\tbinom{10}{8}) \times (0.4^8) \times (.6^2)) + ((\tbinom{10}{9}) \times (0.4^9) \times (0.6^1)) +$$

$$((\tbinom{10}{10})) \times (0.4^{10}) \times (0.6^0))$$

5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

What is the value of the "likelihood" term in Bayes' Theorem -- the conditional probability of the data given the parameter.

○ 0.168835

○ 0.122885

○ 0.043945

◉ 0.120932

✓ Correcto

Bayesian "likelihood" --- the p(observed data | parameter) is

p(8 of 10 heads | coin has p = .6 of coming up heads)

6. We have the following information about a new medical test for diagnosing cancer.

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer. The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of "Negative" for cancer. The other 10% get a false test result of "Positive" for cancer.

What is the conditional probability that I have Cancer, if I get a "Positive" test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

○ 9.5%

○ 67.9%

○ 4.5%

◉ 32.1% probability that I have cancer

✓ **Correcto**

I still have a more than $\frac{2}{3}$ probability of not having cancer

Posterior probability:

p(I actually have cancer | receive a "positive" Test)

By Bayes Theorem:

$$= \frac{\text{(chance of observing a PT if I have cancer)(prior probability of having cancer)}}{\text{(marginal likelihood of the observation of a PT)}}$$

7. We have the following information about a new medical test for diagnosing cancer.

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

The other 5% get a false test result of "Positive" for cancer.

What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- ○ 0.9%
- ◉ $.80\%$
- ○ 88.2%
- ○ 99.1%

! Incorrecto

$.80\%$ is the numerator, but this must be divided by

$$\frac{p(\text{negative test} \mid \text{Cancer})\, p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer})\, p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer})\, p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%) + (95\%)(92\%)}$$

$$\frac{0.8\%}{0.8\% + 87.4\%}$$

$$\frac{0.8\%}{88.2\%}$$

$$= 0.9\%$$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

You are not told whether the draw was done "with replacement" or "without replacement."

What is the probability that the draw was done with replacement?

○ 13.98%

⦿ 12.27%

○ 1

○ 87.73%

✓ **Correcto**

p(40 blue and 10 white | draws without replacement) = 1 [this is the only possible outcome when 50 draws are made without replacement]

p(40 blue and 10 white | draws with replacement)

S = 40

N = 50

P = .8 [for draws with replacement] because 40 blue of 50 total means p(blue) = 40/50 = .8

$$\left(\binom{50}{40}\right)(0.8^{40})(0.2^{10})$$

$$= 13.98\%$$

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers.

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ○ 7.92%
- ○ 92.42%
- ◉ 7.58%
- ○ 8.57%

✓ **Correcto**

By
Bayes' Theorem, the answer is

$$\frac{(.65)(.01)}{((.65)(.01) + (.08)(.99))}$$

$$= 7.58\%$$

Probability (basic and Intermediate) Graded Quiz

CALIFICACIÓN DEL ÚLTIMO ENVÍO

100%

1. What additional statement, added to the three below, forms a probability distribution? **1 / 1 puntos**

   (1) I missed only my first class today

   (2) I missed only my second class today

   (3) I missed both my first and second class today

   ✓ Correcto

2. My friend takes 10 cards at random from a 52-card deck, and places them in a box. Then he puts the other **1 / 1 puntos** 42 cards in a second, identical box. He hands me one of the two boxes and asks me to draw out the top card. What is the probability that the first card I draw will be the Ace of Spades?

   ✓ Correcto

3. I will go sailing today if it does not rain. Are the following two statements Independent or dependent? **1 / 1 puntos**

   (1) "I will go sailing today"

   (2) "It will not rain today"

   ✓ Correcto

3. I will go sailing today if it does not rain. Are the following two statements Independent or dependent?   **1 / 1 puntos**

   (1) "I will go sailing today"

   (2) "It will not rain today"

   ✓ Correcto

4. The probability that I will go sailing today AND the fair six-sided die will come up even on the next roll is .3.   **1 / 1 puntos**

    If these events are independent, what is the probability that I will go sailing today?

   ✓ Correcto

5. I have two coins. One is fair, and has a probability of coming up heads of .5.   **1 / 1 puntos**
   The second is bent, and has a probability of coming up heads of .75.
    If I toss each coin once, what is the probability that at least one of the coins will come up tails?

   ✓ Correcto

6. What is the probability, when drawing 5 cards from a fair 52-card deck, of drawing a "full house" (three of a   **1 / 1 puntos**
   kind and a pair) in the form AAABB?

   ✓ Correcto

7. If it rains, I do not go sailing. It rains 10% of days; I go sailing 3% of days.

   If it does not rain, what is the (conditional) probability that I go sailing?

   Written "p(I go sailing | it does not rain)"?

   ✓ Correcto

8. I am at my office AND not working 2% of the time. I am at my office 10% of the time. What is the conditional probability that I am not working, if I am at my office?

   ✓ Correcto

9. The factory quality control department discovers that the conditional probability of making a manufacturing mistake in its precision ball bearing production is 4% on Tuesday, 4% on Wednesday, 4% on Thursday, 8% on Monday, and 12% on Friday.

   The Company manufactures an equal amount of ball bearings (20%) on each weekday. What is the probability that a defective ball bearing was manufactured on a Friday?

   ✓ Correcto

10. An Urn contains two white marbles and one black marble. A marble is drawn from the Urn without replacement and put aside without my seeing it. Then a second marble is drawn, and it is white.

    What is the probability that the unknown removed marble is white, and what is the probability that it is black?

    ✓ Correcto

11. What is the probability, if I flip a fair coin with heads and tails ten times in a row, that I get at least 8 heads?

    ✓ Correcto

12. Suppose I have either a fair coin or a bent coin, and I don't know which. The bent coin has a 60% probability of coming up heads.

    I throw the coin ten times and it comes up heads 8 times. What is the probability I have the fair coin vs. the probability I have the bent coin?

    Assume at the outset there is an equal $(.5, .5)$ prior probability of either coin.

    *Please note that in order to fit the entire formula in the feedback, probability has been abbreviated to "prob."

    ✓ Correcto