COMP307 Week 7 (Tutorial)

1. Announcements
   - ***Assignment 2 due***
   - Assignment 3 released
   - Fangfang's teaching evaluation

2. Reasoning Under Uncertainty
   - Product rule
   - Sum rule
   - Normalisation rule
   - (Conditional) Independence

Proposition, Variable, Domain

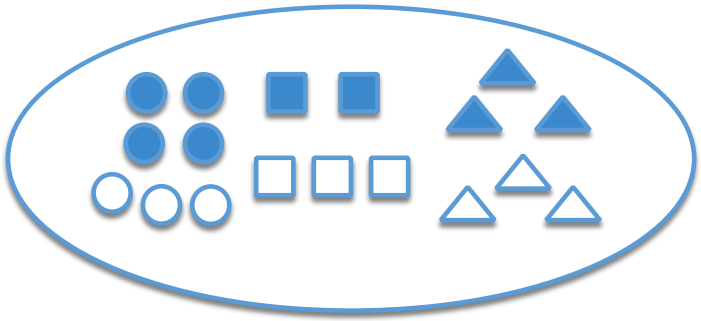
- **Proposition:**
  - Tomorrow will be rainy
  - The die will give 6
  - ...
- **Variable:**
  - Weather tomorrow
  - Outcome of a die
  - ...
- **Domain (all the possible values a variable can take):**
  - {rainy, sunny, cloudy, ...}
  - {1, 2, 3, 4, 5, 6}

Probability notations

- P(A): unconditional/prior probability that A is true
- P(A | B): conditional probability that A is true given B is true
- P(A, B): joint probability that both A and B are true

Picking Objects

- P(Shape = blue, Colour = White)?
- P(Shape = Square)?
- P(Colour = White | Shape = Triangle)?
- …

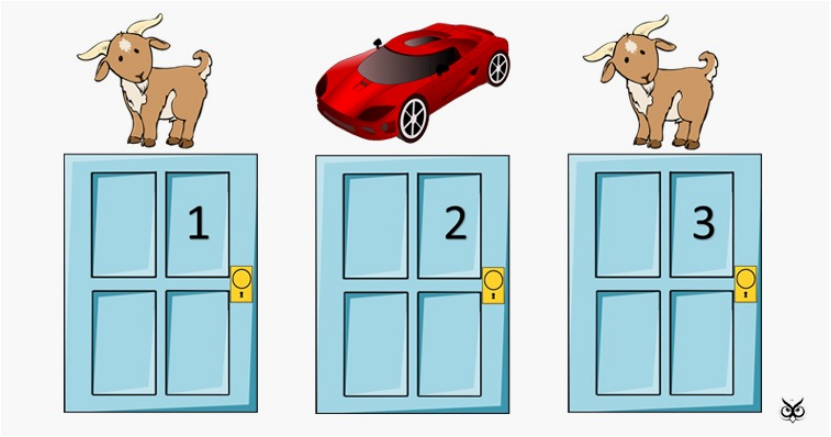


		Shape			
		Circle	Square	Triangle	
Colour	Blue	**4**	**2**	**3**	**9**
	White	**3**	**3**	**3**	**9**
		7	**5**	**6**	**18**

Monty Hall Problem

- Assuming initially picking door 1

Door 1	Door 2	Door 3	Stay	Switch
Car	Goat	Goat	**Win**	Lose
Goat	**Car**	Goat	Lose	**Win**
Goat	Goat	**Car**	Lose	**Win**



Probability Rules

- The product rule:
  - P(A, B) = P(B) * P(A | B) = P(A) * P(B | A)
  - P(A, B, C)?
  - P(X1, X2, X3, …, Xn)?

- The sum rule
  - $P(X = x) = \sum_{y \in \Omega} P(X = x, Y = y)$
  - $P(X_1 = x_1, \dots X_n = x_n) = \sum_{y_1 \in \Omega_1, \dots, y_m = \Omega_m} P(X_1 = x_1, \dots X_n = x_n, Y_1 = y_1, \dots, Y_m = y_m)$

- The normalisation rule
  - $\sum_x P(X = x) = 1$
  - $\sum_x P(X = x \mid Y = y) = 1$

Independence

- If A and B are independent of each other, then
  - $A \perp B$
  - P(A | B) = P(A)
  - P(B | A) = P(B)
  - P(A, B) = P(A) * P(B)

- If A and B are conditionally independent of each other given C, then
  - $A \perp B \mid C$
  - P(A | B, C) = P(A | C)
  - P(B | A, C) = P(B | C)
  - P(A, B | C) = P(A | C) * P(B | C)

Independence

- If $P(a \mid b) = 0.5$, $P(a) = 0.5$, $P(b) = 0.8$, are A and B independent?
- If A and B are **independent**, $P(a) = 0.7$, $P(b) = 0.6$, what is $P(a, b)$?
- If A and B are **independent given C**, $P(a, b \mid c) = 0.3$, $P(a \mid c) = 0.5$, what is $P(b \mid c)$?

Quiz: Weather Forecast

- Random variable: Day_1, Day_2
- Domain: $\{\boldsymbol{W}indy, \boldsymbol{C}alm\}$

- $P(Day_1 = W) = 0.5, P(Day_1 = C) = 0.5$
- $P(Day_2 = W \mid Day_1 = W) = 0.6, P(Day_2 = C \mid Day_1 = W) = 0.4$
- $P(Day_2 = W \mid Day_1 = C) = 0.3, P(Day_2 = C \mid Day_1 = C) = 0.7$

- $P(Day_2 = W) = ?$