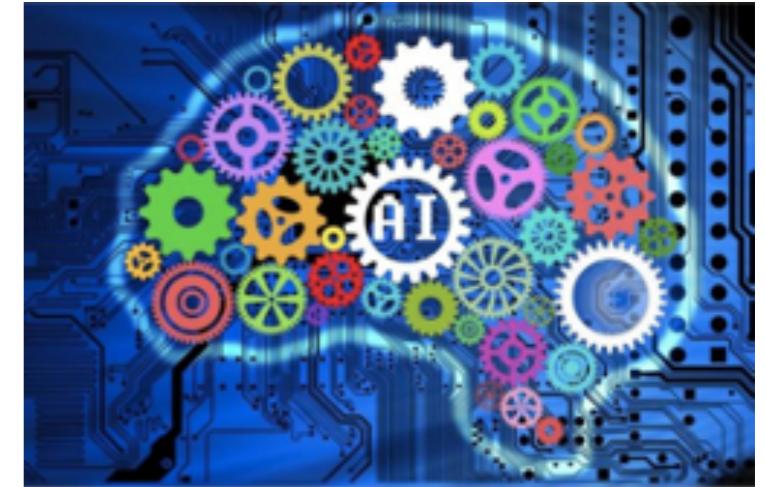


# Content

## 0. Introduction



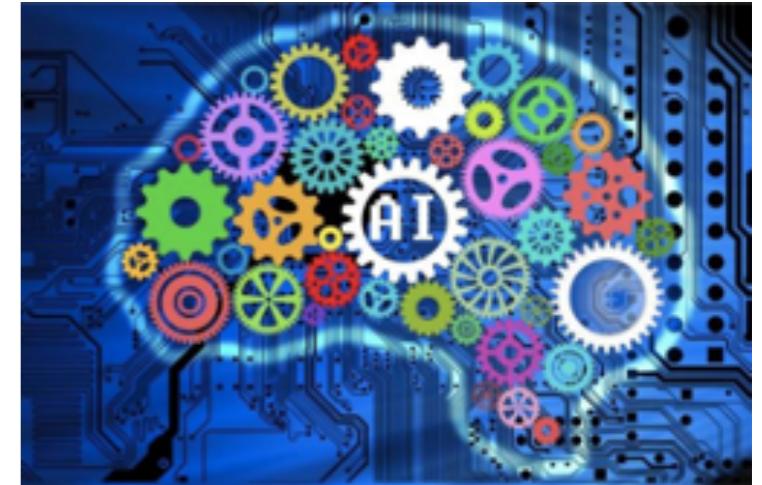
## 1. Regression

- 1.1 Multivariate Linear Regression (curve fitting)
- 1.2 Regularization (Lagrange multiplier)
- 1.3 Logistic Regression (Fermi-Dirac distribution)
- 1.4 Support Vector Machine (high-school geometry)

## 2. Dimensionality Reduction/feature extraction

- 2.1 Principal Component Analysis (order parameters)
- 2.2 Recommender Systems
- 2.3 Clustering (phase transition)

# Content



## **3. Neural Networks**

**3.1 Biological neural networks**

**3.2 Mathematical representation**

**3.3 Factoring biological ingredient**

**3.4 Feed-forward neural networks**

**3.5 Learning algorithm**

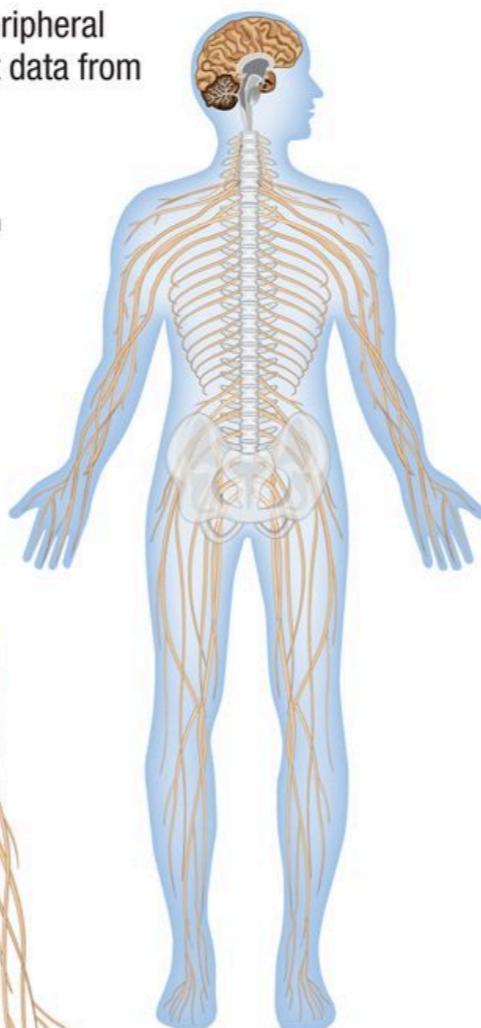
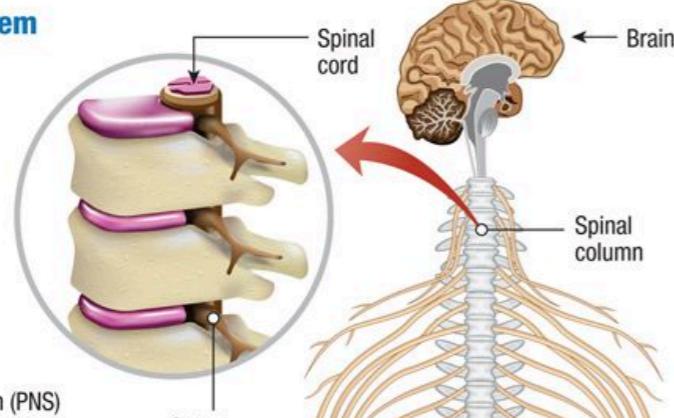
**3.6 Universal Approximation Theorem**

# Human Body: Nervous System

The nervous system is made up of the central nervous system and peripheral nervous system. These systems work together to collect and interpret data from the body's internal and external environment and control responses.

## Central Nervous System

The central nervous system (CNS) manages the body's essential functions. Made up of the brain and spinal cord, the CNS receives sensory information and coordinates an appropriate response.

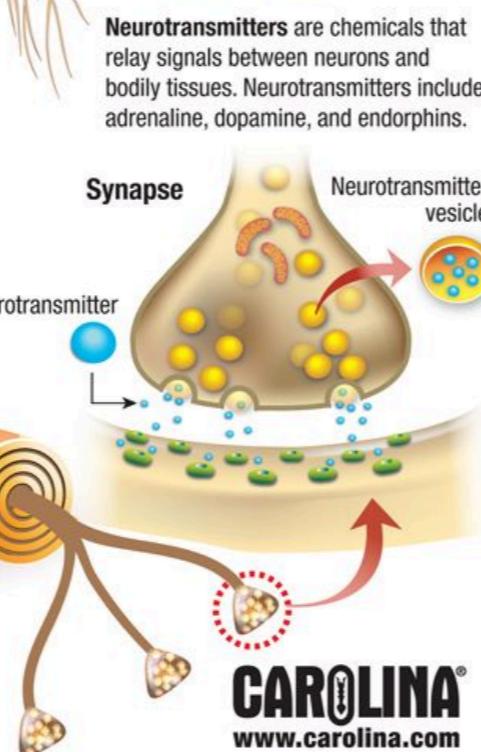
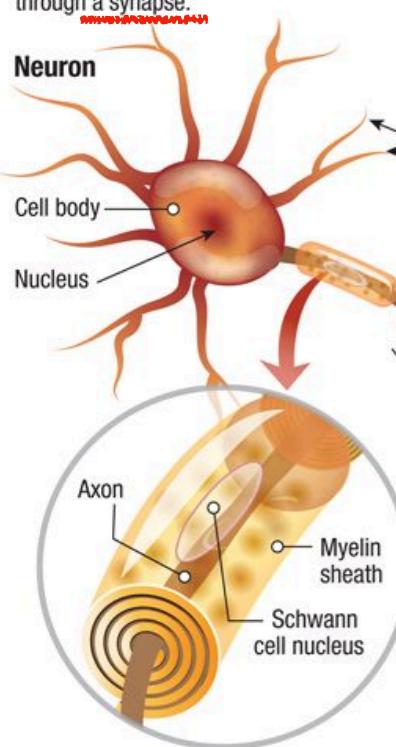


## Peripheral Nervous System

The peripheral nervous system (PNS) connects the CNS to the rest of the body. Nerves branch out from the brain and spinal cord, extending to the organs, muscles, and other parts of the body.

Neurons are highly specialized cells that transmit chemical and electrical information in the body. Neurons use short, branched extensions called dendrites to receive nerve impulses from surrounding cells. These messages then travel through the cell body to the axon, a threadlike structure. The impulse moves through the axon and is transmitted via chemical or electrical signals that pass through a synapse.

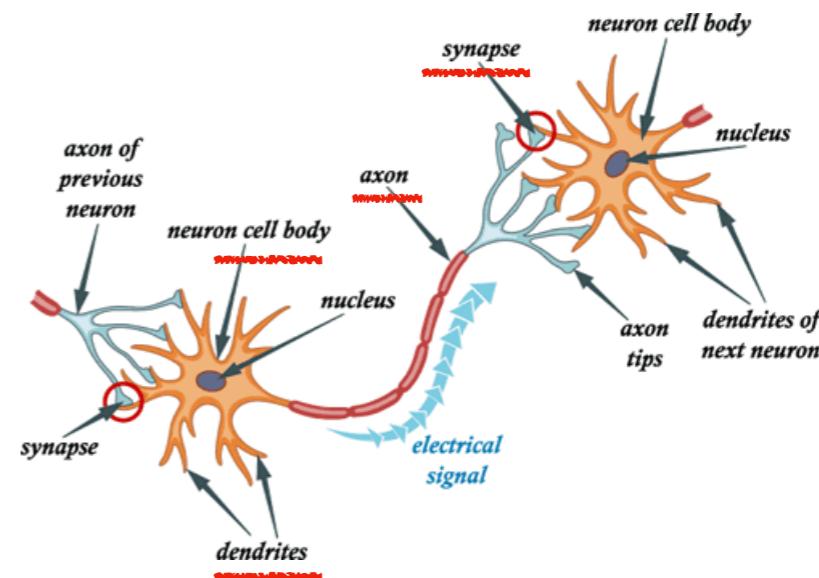
## Neuron



CAROLINA  
[www.carolina.com](http://www.carolina.com)

Brain consists of  $\sim 10^{11}$  neurons, building bricks for the central nervous systems

Neurons are interconnected by synapses, the complexity of the Brain is due to massive highly interconnected neurons working in parallel, One neuron receives inputs from  $\sim 10^4$  others.

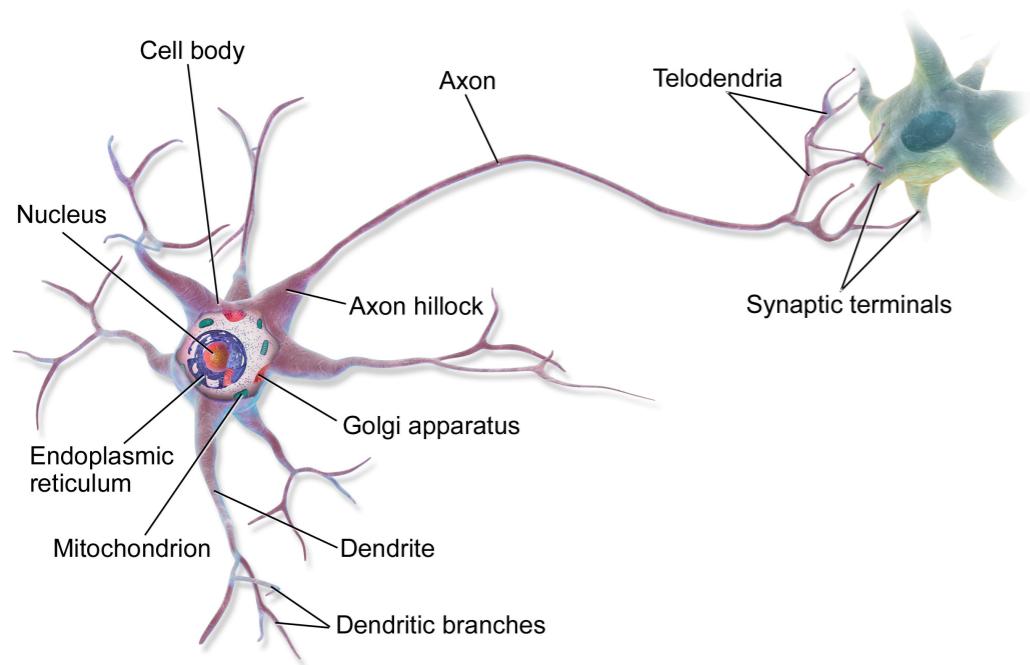


All the inputs to the neuron, are summed up. The input sum is processed by a threshold function and produces an output signal. The processing time  $\sim 1\text{ms}$  per cycle

Brain works in both a parallel and serial way. Picture recognition of human  $\sim 100\text{ ms}$  Around 100 neurons are involved in serial Complexity requires parallel processing

Biological neural systems have high fault tolerance  
People with brain injuries can perform normally

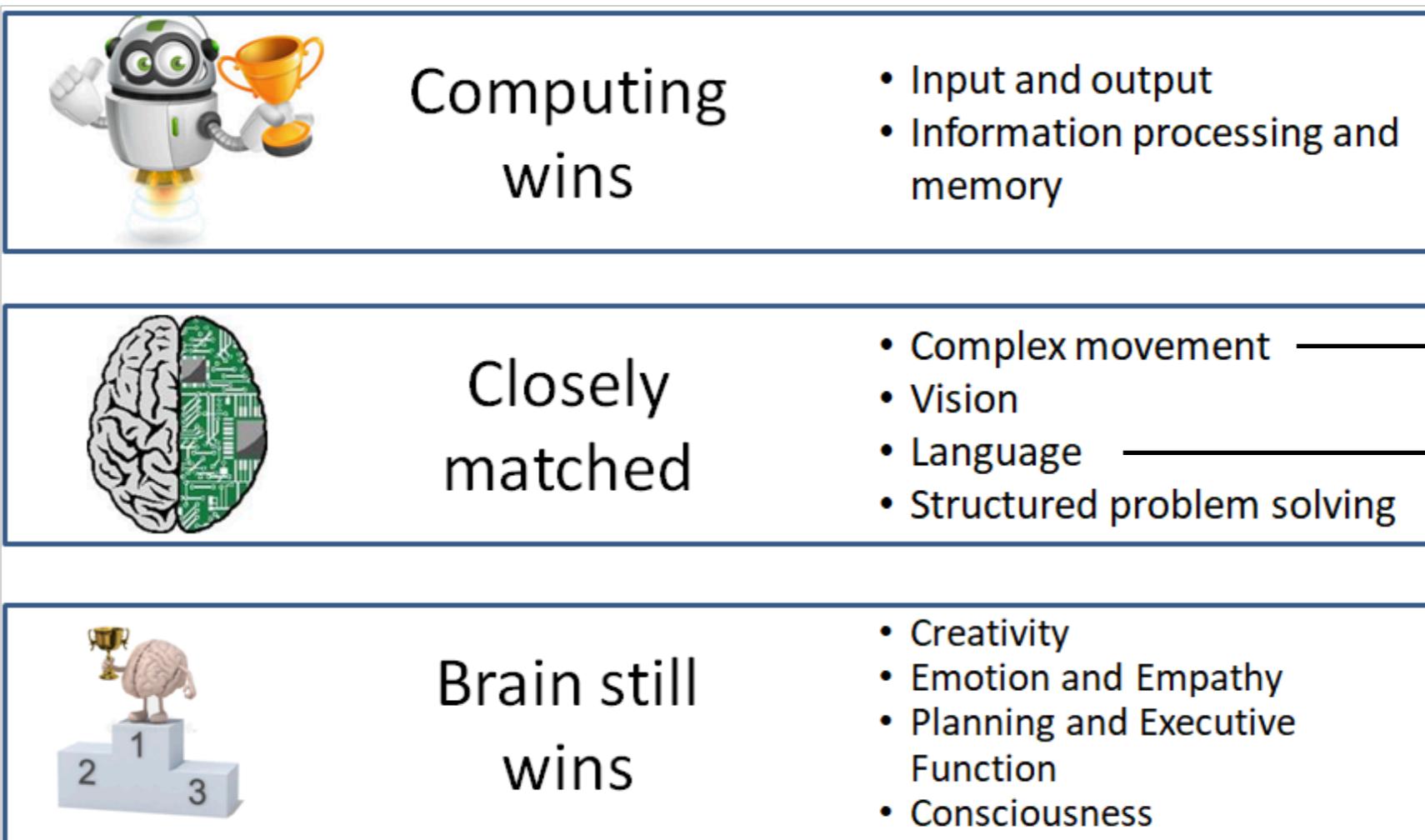
# AI & Machine Learning Basics



	Supercomputer	Personal Computer	Human Brain
Computational Units	4,000 Xeon/AMD CPUs $10^{12}$ transistors	4 CPUs, $10^9$ transistors	$10^{11}$ neurons
Storage units	$10^{14}$ bits RAM $10^{15}$ bits Storage	$10^{11}$ bit RAM $10^{13}$ bit Storage	$10^{11}$ neurons $10^{14}$ synapses
Cycle time	$10^{-9}$ sec	$10^{-9}$ sec	$10^{-3}$ sec
Operations/sec	$10^{15}$	$10^{10}$	$10^{17}$
Memory updates/sec	$10^{14}$	$10^{10}$	$10^{14}$
Weight / Space	150 tons / Basketball court	1 Kg / A4 Paper	1.5 Kg / 1/6 basketball
Power consumption	500 megawatt	100 watt	20 watt

## Peripheral nervous system

- Touching a hot object
- Sensory nerves carry information about the heat to the brain
- Brain, via motor nerves, tells the muscle of the hand to withdraw
- The whole process takes less than a second



<https://becominghuman.ai/brains-vs-computers-f769548010f1>

## ***Neuroscience For Kids***

<http://faculty.washington.edu/chudler/neurok.html>

# Biological neurons (nerve cell)

Functional and structural units of nervous system

$10^{11}$  in human body

80% in human Brain

Input: Dendrites:

neuron receives one or more inputs through dendrites

Hypothesis: Cell Body (Soma):

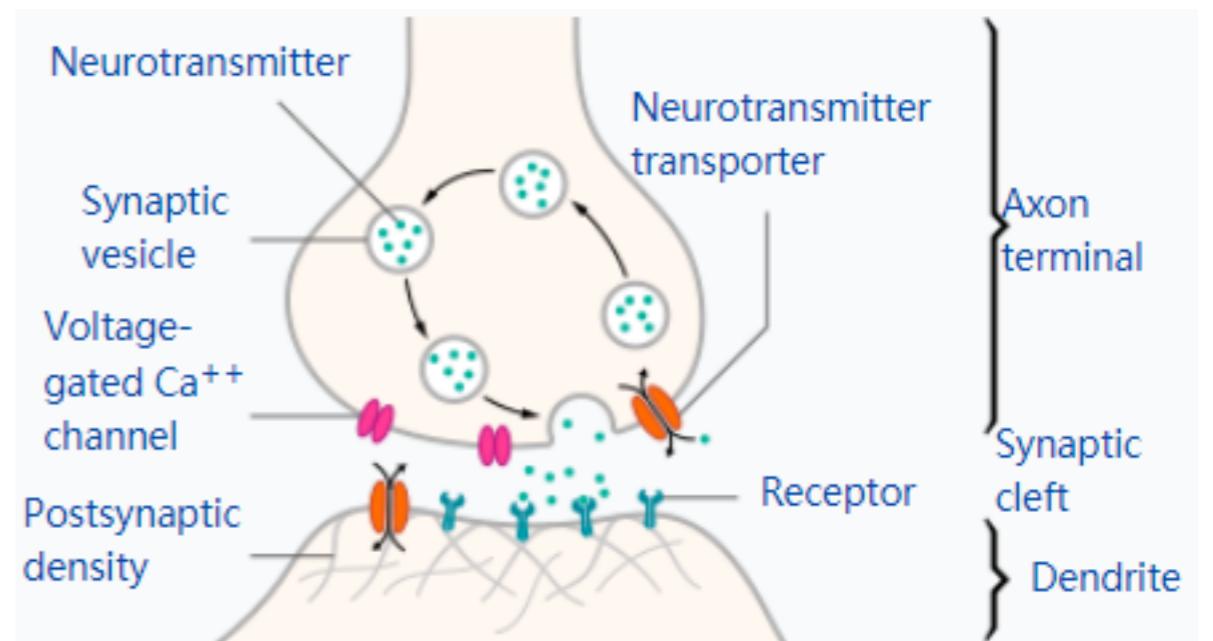
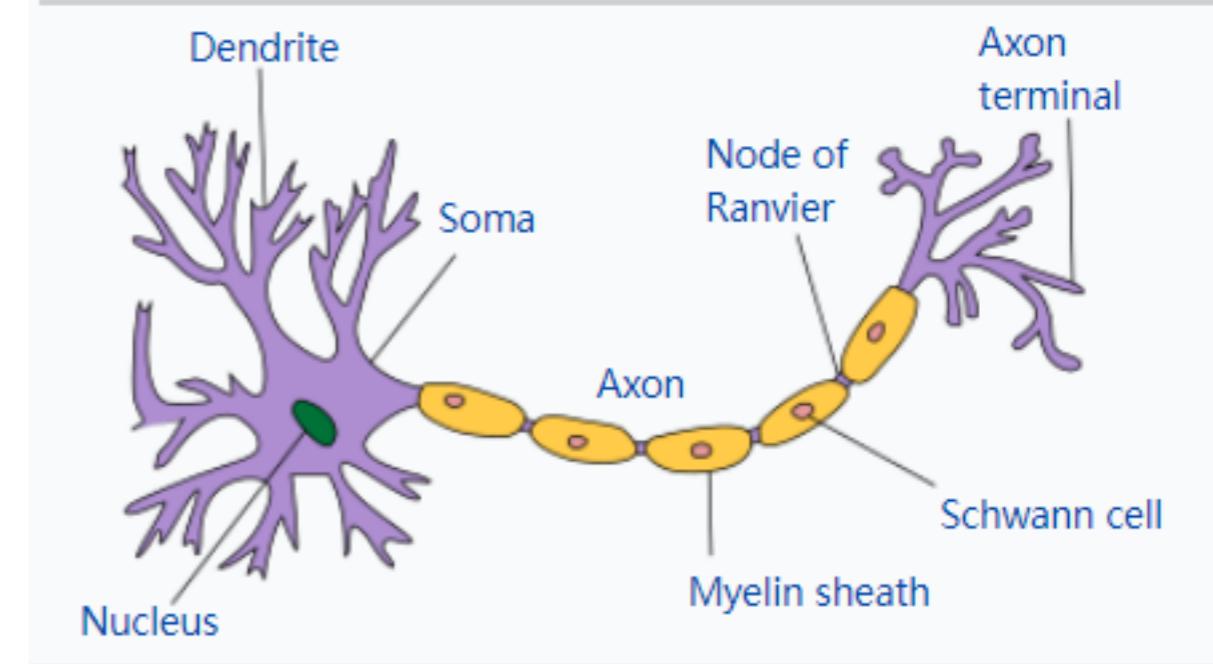
Processing the information

Output: Axon:

Send output through axon

Neurons do not touch each other, they form tiny gaps called synapses,  
pass electrical or chemical signal to another neuron or target cell

<https://en.wikipedia.org/wiki/Synapse>

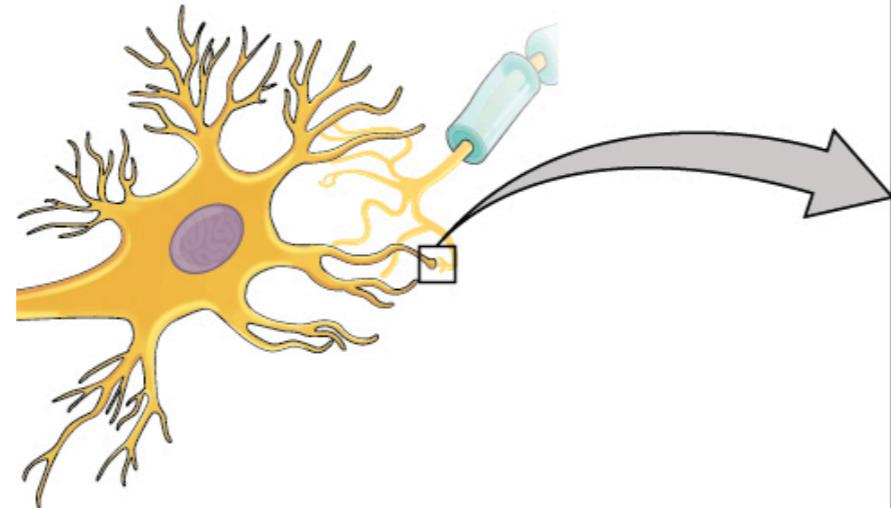


Chemical synapse

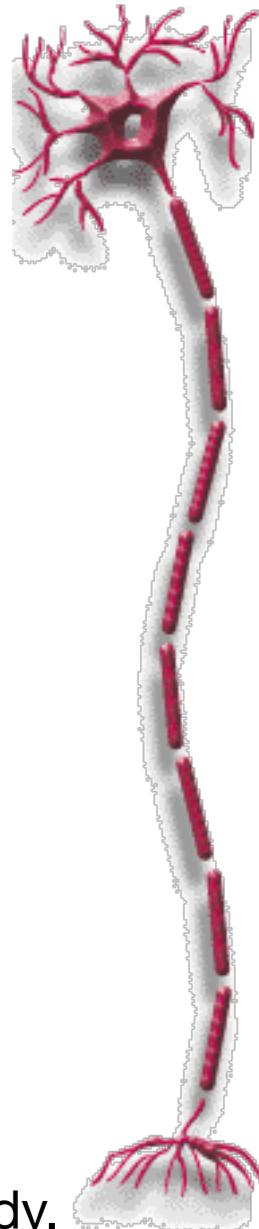
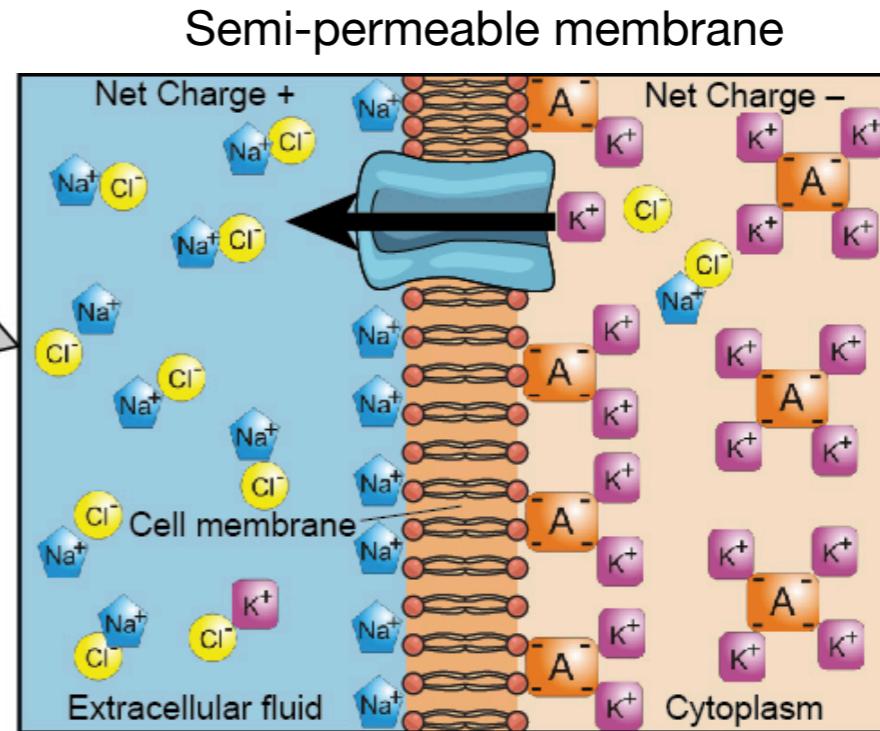
# Biological neurons

Neurons send message **electrochemically**

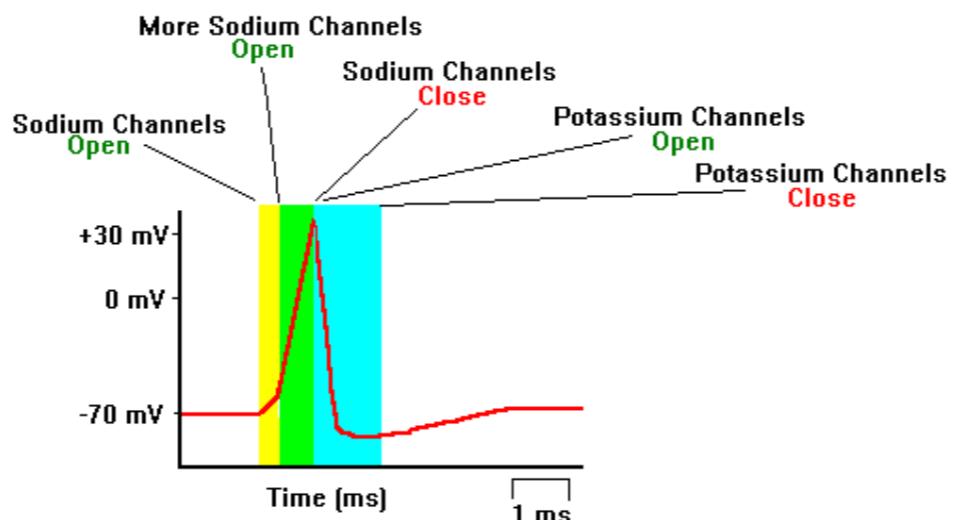
Sodium Na<sup>+</sup> ion  
Potassium K<sup>+</sup> ion  
Calcium Ca<sup>++</sup> ion  
Chloride Cl<sup>-</sup> ion  
Protein molecules A-



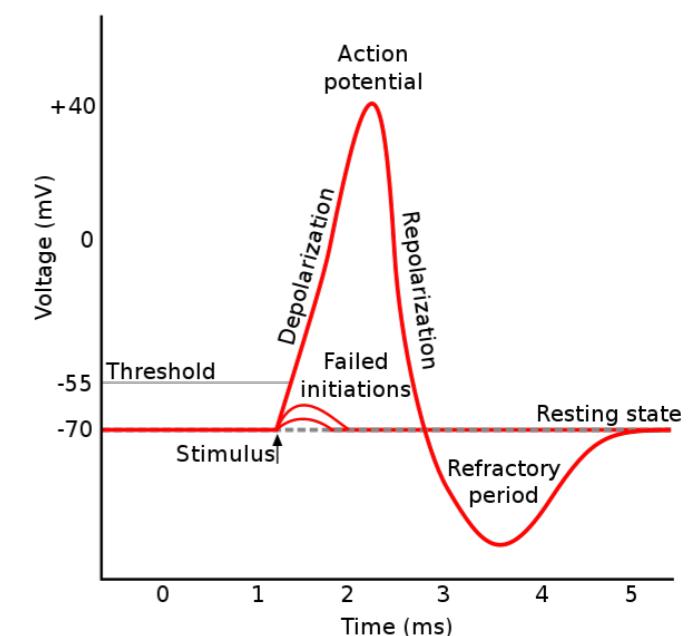
When a neuron at rest (not sending a signal)  
Inside is negative relative to the outside  
**Resting potential** about -70 mV (millivolt)  
More sodium outside and more potassium inside



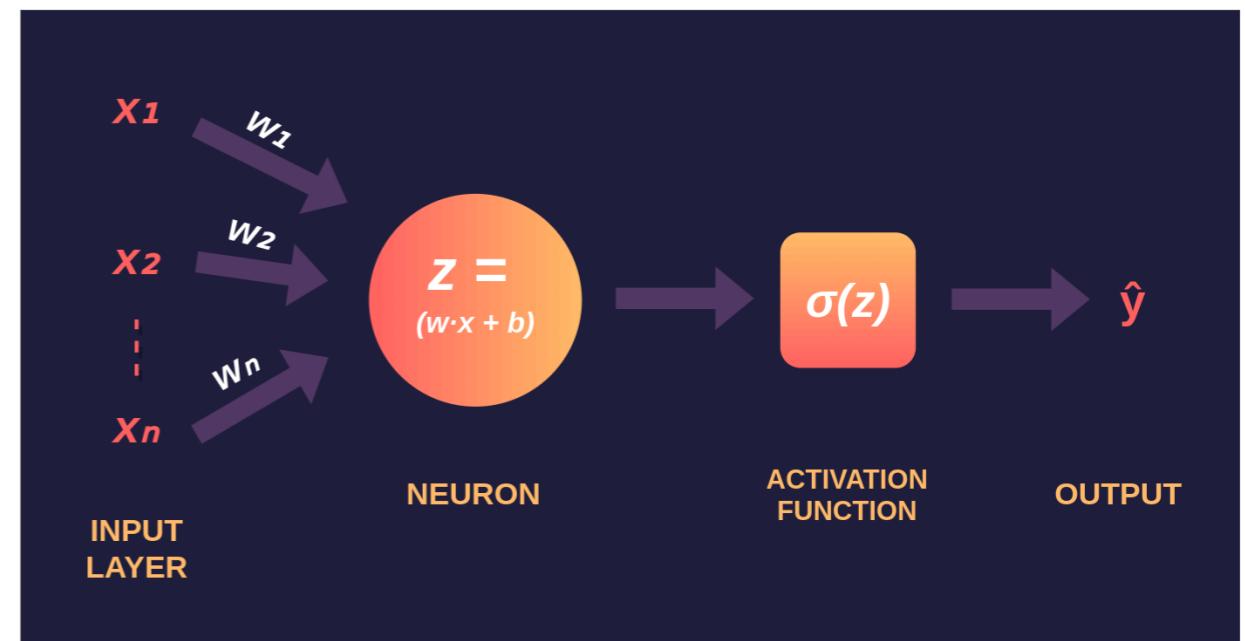
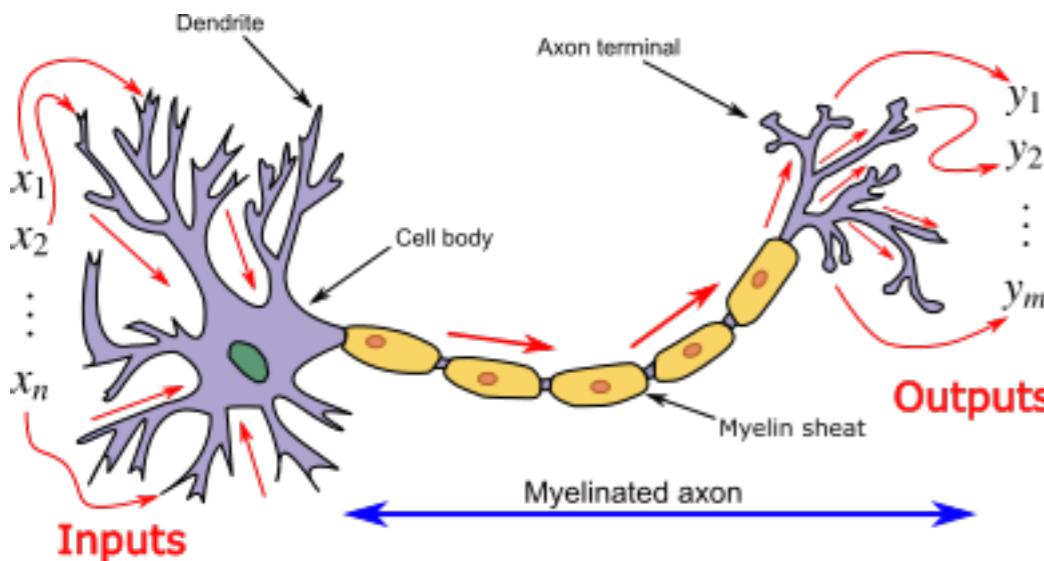
**Action potential** occurs when a neuron sends information down an axon, away from the cell body.  
An explosion of electrical activity created by depolarizing current  
A stimulus causes the resting potential to move towards 0 mV.  
Depolarisation reaches about -55 mV (threshold), neuron will fire an action potential



Spike or impulse  
Depolarization  
Repolarization

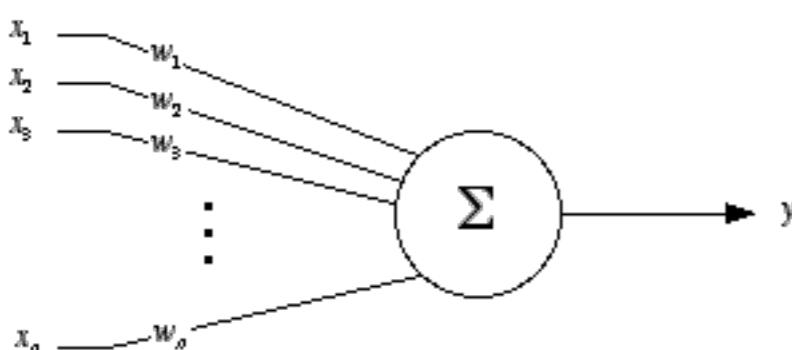


# Mathematical representation for neurons: Linear model



- Brain contains neurons, each neuron can be thought of as a device having inputs and outputs.
- Inputs consist of  $10^3\text{-}10^4$  synapses on the dendritic tree, outputs consist of action potential carried by the axon sent to other neurons.
- The input currents are (roughly) summed together into the cell body, whose voltage rises and decays with the fluctuations in current.
- When the cell body voltage exceeds a certain threshold, an action potential is fired, propagates down the axon.

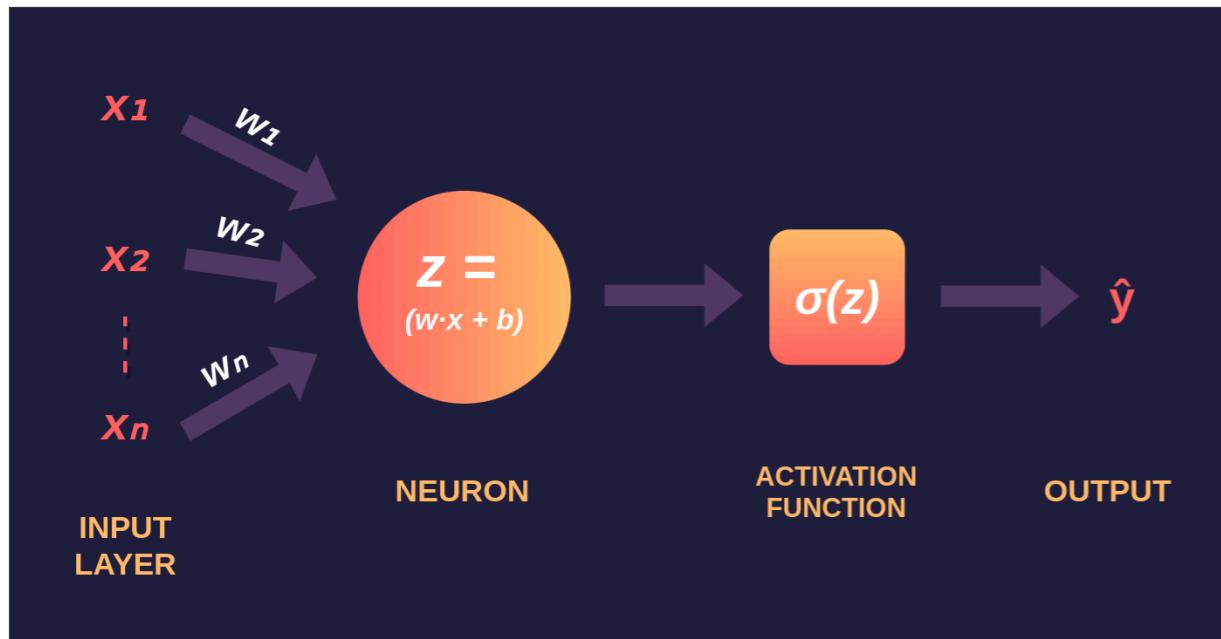
## Linear neuron models



$$y = \sum_{i=1}^n w_i x_i = \theta^T x$$

$w_i$  weight for each input

# Mathematical representation for neutrons: Perceptrons



- Frank Rosenblatt in 1958
- n inputs, n weights, one neuron, one output
- Passing data through via forward propagation

Binary activation function

$$w \cdot x + b = \theta^T x = z$$

$$\begin{aligned} y &= 0 && \text{if } \theta^T x \leq 0 \\ y &= 1 && \text{if } \theta^T x > 0 \end{aligned}$$

$$\begin{bmatrix} 1 & x_1^{(1)} & x_2^{(1)} & \dots & x_N^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} & \dots & x_N^{(2)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_1^{(M)} & x_2^{(M)} & \dots & x_N^{(M)} \end{bmatrix} \cdot \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_N \end{bmatrix} = \underline{\underline{X}} \cdot \underline{\underline{\Theta}}$$

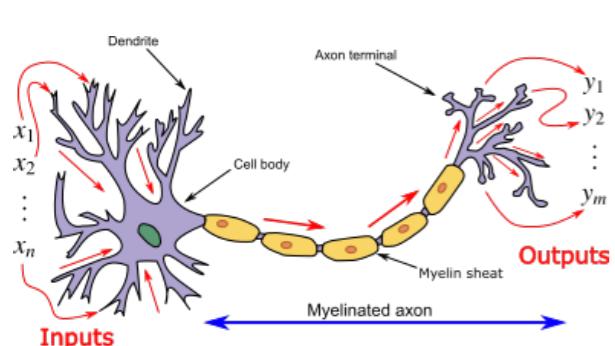
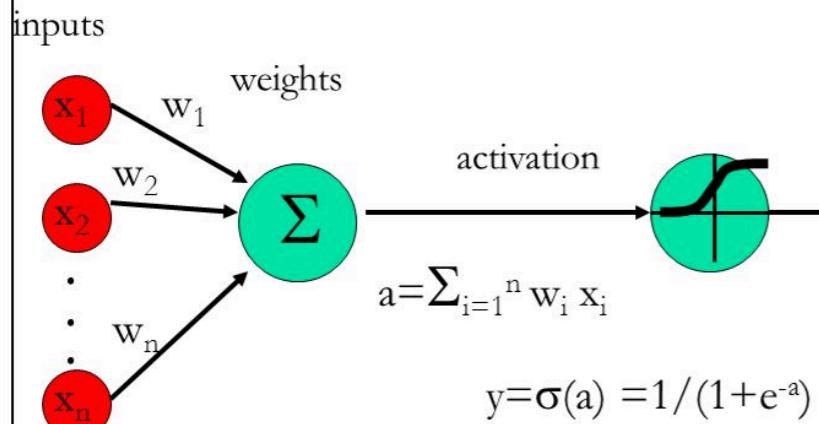
$\mathbb{R}^{M \times 1}$  vector

Logistic/Sigmoid activation function

$$y = \frac{1}{1 + \exp(-\theta^T x)}$$

# Perceptron

## Neuron with Sigmoid-Function

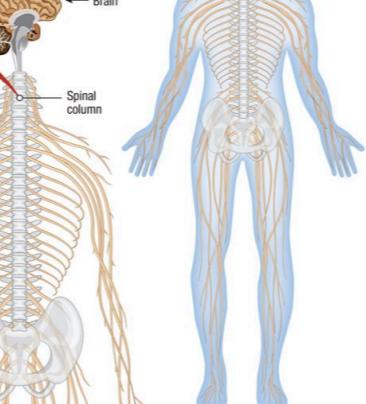


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### Central Nervous System

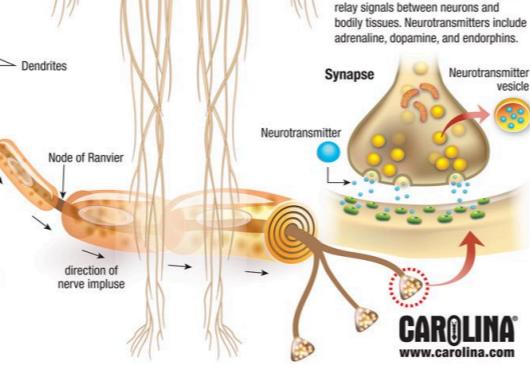
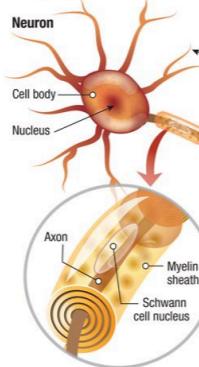
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### Peripheral Nervous System

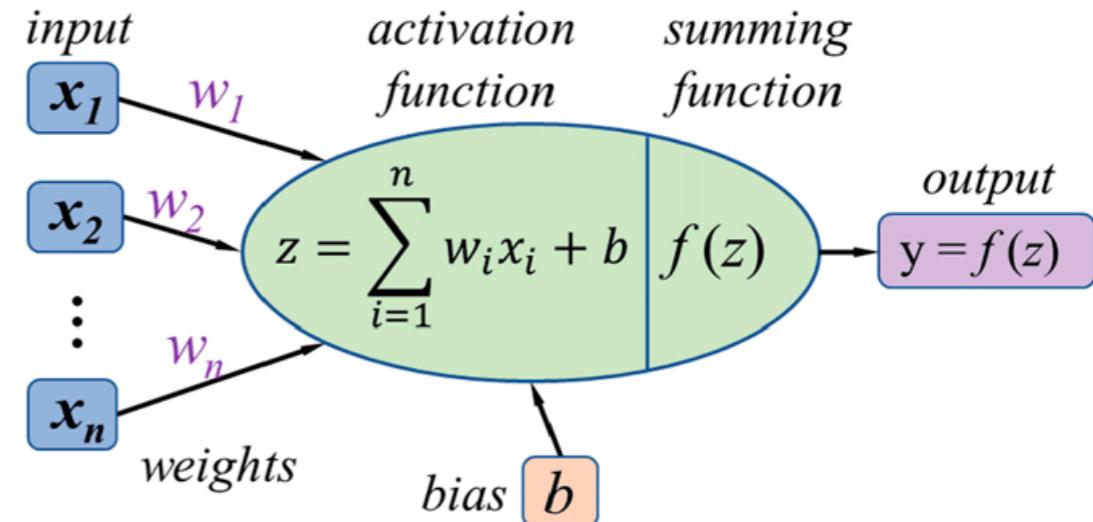
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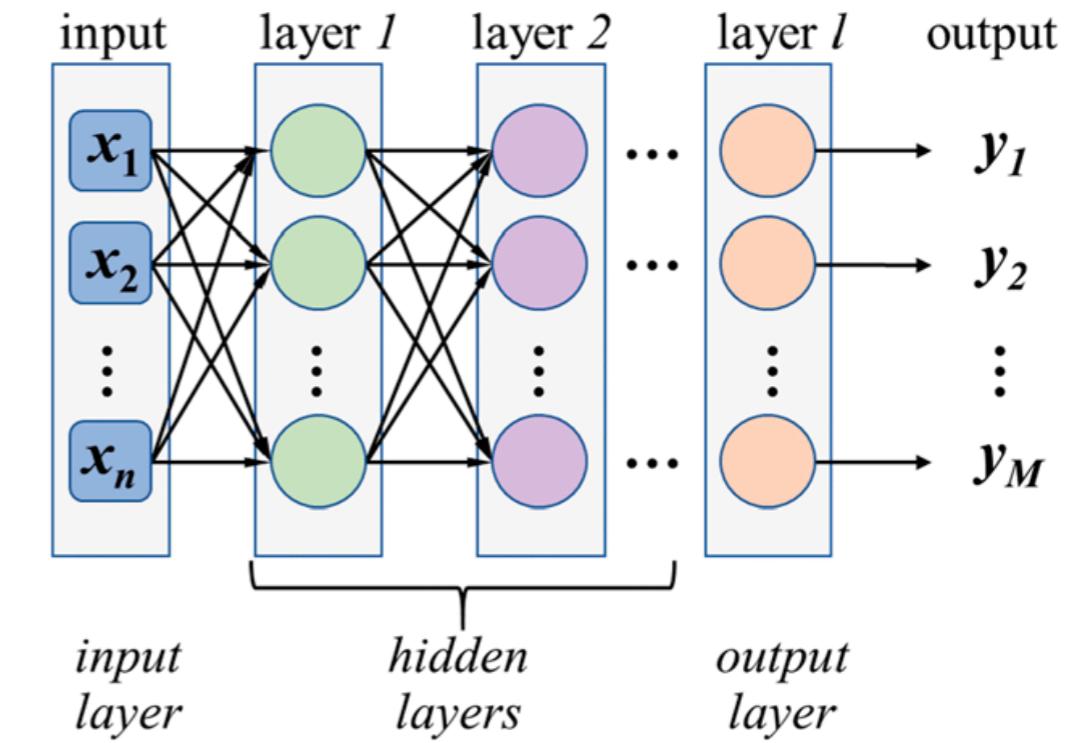


# Neural network

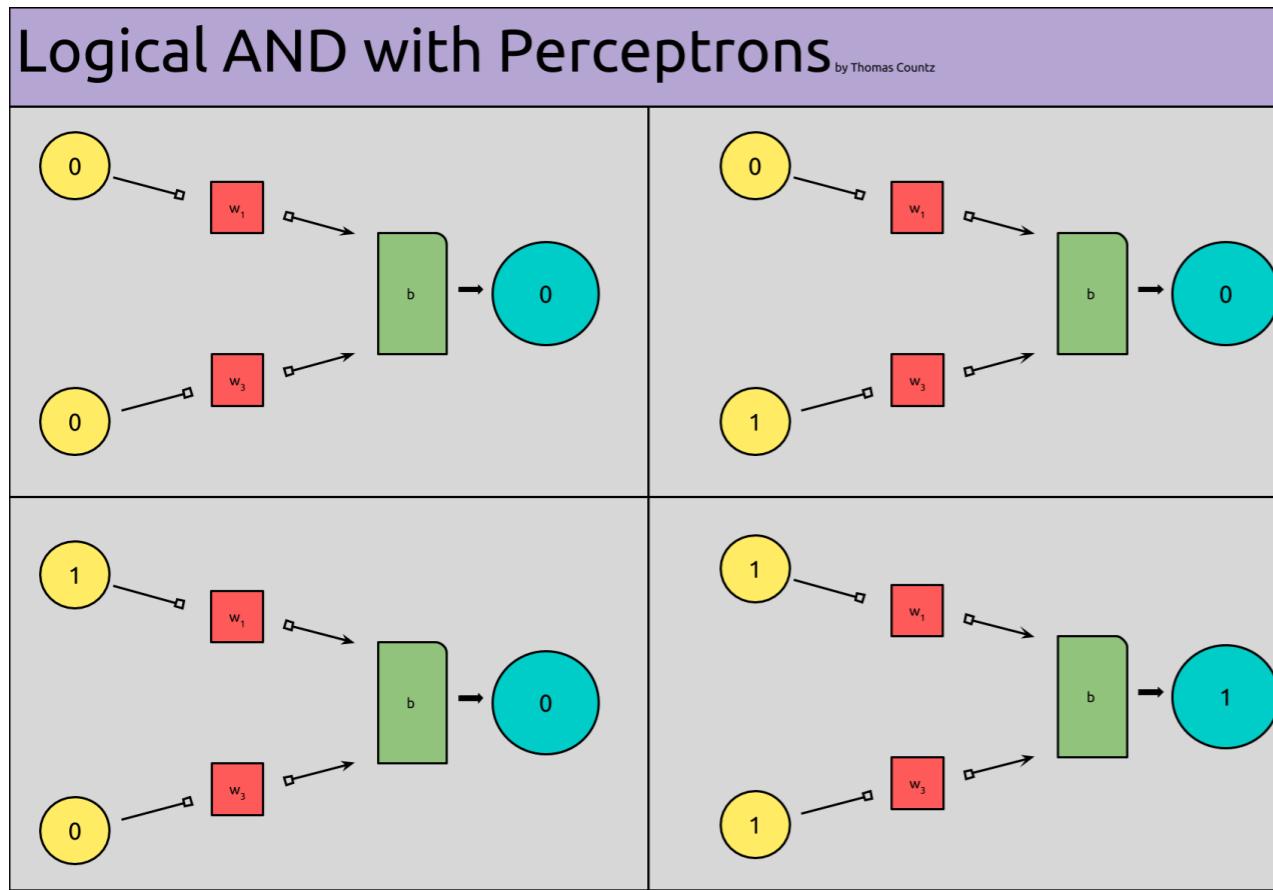
(A) a neuron of an artificial neural network



(B) deep neural network



# A Simple Example of Perceptron AND



A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

$$f(x) = 1 \text{ if } w \cdot x + b > 0$$

$$f(x) = 0 \text{ if otherwise}$$

y	f(x)	y - f(x)
1	1	0
0	0	0
1	0	1
0	1	-1

How to adjust the weights

- If perceptron outputs 0 ( $f(x)=0$ ), when we want 1 ( $y=1$ ), adjust by making  $w \cdot x + b$  larger  
 $y - f(x) == 1$ , then  $w + x \rightarrow w$
- If perceptron outputs 1 ( $f(x)=1$ ), when we want 0 ( $y=0$ ), adjust by making  $w \cdot x + b$  smaller  
 $y - f(x) == -1$ , then  $w - x \rightarrow w$
- If perceptron outputs expected value,  $f(x) == y$ , adjust nothing  
 $y - f(x) == 0$ , then  $w \rightarrow w$

Since  $y-f(x)$  only produces 1, -1, 0, simplify  $w + (y-f(x)) * x \rightarrow w$

<https://medium.com/@thomascountz/perceptron-implementing-and-part-2-84bfb1f46597>

<https://medium.com/@thomascountz/19-line-line-by-line-python-perceptron-b6f113b161f3>