THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 4

Course: *Machine Learning in Physics (PHYS3151)* – Professor: *Dr. Ziyang Meng*
Due date: *Apr. 7th, 2023*

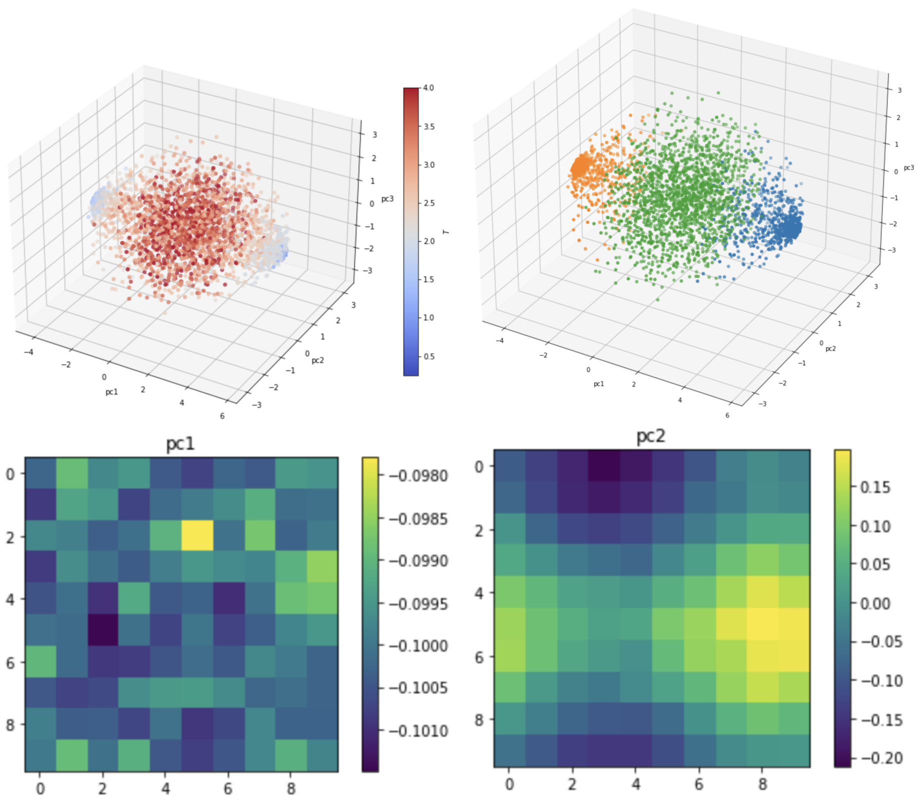
1. PCA and Clustering on 2D ferromagnetic Ising model

For this question, please load the 10×10 Ising data files conf.csv and temp.csv from folder *principal-component-analysis/Ising_model* on Github. You can refer to the PCA Colab example on how to do this.

(a) Perform PCA to the data, plot the sample points against their first **three** principal component. (See the plot below at the top left panel)

(b) Perform K-mean clustering with 3 centroids, plot the resulting clusters in different colors. (Top right panel)

(c) Plot the distribution of eigenvectors for the first two component as heat map on the 10×10 lattice. (Two bottom panels)



2. Understand the theoretical aspect of 2D Ising Model

The 2D Ising model is the simplest example of many-body system that undergo continuous phase transition. In low temperature, the system is in an ordered state, most spins tends to align in the same direction, which leads to non-zero magnetization. When the temperature is higher than the critical temperature $T_c \approx 2.269$, the system will become disordered and magnetization become 0.

However, phase transition occurs only when system size goes to infinity (thermodynamic limit). For finite size, magnetization will decrease after passing the critical temperature, but remains a finite value. In this question, we will see the Ising model in finite size (L=6, 8, 10).

Load the data files F_conf_L6.csv, F_conf_L8.csv, F_conf_L10.csv, and F_temp.csv from folder *principal-component-analysis/Ising_model* on Github. The first three files are spin configurations in different temperatures and system sizes, and the last is the temperature list that the three sizes share (T=[0.25, 0.5, 0.75, . . . , 4]).

(a) The magnetization is defined as $m = \frac{1}{L^2}|\sum_i \sigma_i|$, where $\sigma_i = \pm 1$ is the spin orientation on the i-th site. Find its average values for all system sizes at different temperature. (Top panel)

Hint: measure m at all provided configurations, then take average over all configurations with the corresponding system size and temperature.

(b) Describe the trend of the above plots with system size increase. Compare them to the theoretical prediction in the thermodynamic limit. (Bottom panel)