THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 2

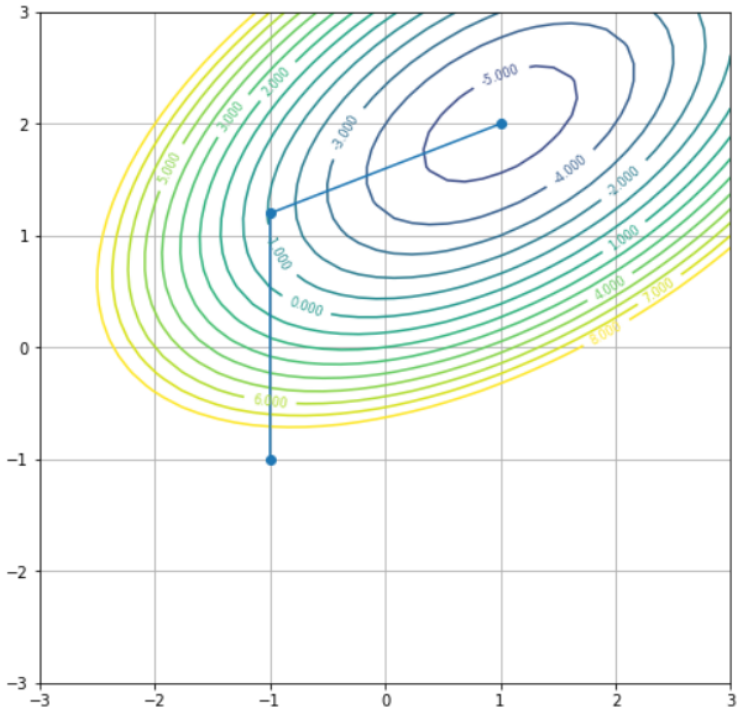
Course: *Machine Learning in Physics (PHYS3151)* – Professor: *Dr. Ziyang Meng*
Due date: *Mar. 3rd, 2023*

1. **Conjugate gradient method**
   Given

$$A = \begin{pmatrix} 3 & -2 \\ -2 & 5 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \qquad c = 2$$

(a) Use conjugate gradient method with initial guess $\mathbf{x}_{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, find the the optimal $\mathbf{x}$ where the quadratic form $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A\mathbf{x} - \mathbf{b}^T\mathbf{x} + c$ attains its minimum. Plot the path of iterations .

(b) Check that:
(i) $\mathbf{x}$ converges to the exact solution to the exact solution in 2 steps. i.e. $A\mathbf{x}_{(2)} = \mathbf{b}$.
(ii) $\mathbf{p}_{(1)}^T A\mathbf{p}_{(2)} = 0$.
(iii) $\mathbf{r}_{(1)}^T\mathbf{r}_{(2)} = 0$.



2. **Sigmoid function**

In logistic regression, we use the sigmoid function $g(z) = \frac{1}{1+e^{-z}}$. Prove the following properties for sigmoid function.

(a) $g(z) + g(-z) = 1$
(b) $g'(z) = g(z)g(-z) = g(z)[1 - g(z)]$
(c) $g'(-z) = g'(z)$
(d) $g''(z) = g(z)g(-z)[g(-z) - g(z)]$