THE UNIVERSITY OF HONGKONG
DEPARTMENT OF PHYSICS

Assignment 1

Course: *Machine Learning in Physics (PHYS3151)* – Professor: *Dr. Ziyang Meng*
Due date: *Feb. 17th, 2023*

1. **Being familiar with Python**

Please read the Python Basic Tutorial Module at Python Tutorial and the colab notebook *Gradient descent & conjugate gradient* to learn the following concepts:

- the useage of different variable types (integer, float, and strings)
- the useage of list
- the useage of if and for statements
- defining a function with return statement, and show the output
- plotting
  - a title of the graph
  - x label and y label
  - the legend of data
  - setting the color manually
  - log-scale in x- and y-axis

(a) Use a for- or while- loop to construct a list containing the first 50 Fibonacci numbers, i.e. the sequence defined as

$$F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$$

Plot them with y-axis in log-scale. Please also add suitable axis-labels and title for the plot.

(b) Define a function func(x), where x is an input of float type, and the output shows the largest Fibonacci number that is **not larger than** x. Print the output in the following format:

The largest Fibonacci number that is not larger than x is y, and it is the N-th element in the sequence.

Show x and y as float numbers and N as an integer. Below is a typical output.

```
[55] func(17.5)

     The largest Fibonacci number that is not larger than  17.5  is  13.0 , and it is the  7 -th element in the sequence.
```

2. **Math practice**
   Given

$$A = \begin{pmatrix} 2 & 1 & -1 \\ -2 & 5 & -1 \\ -2 & 1 & 3 \end{pmatrix}$$

(a) Find all eigenvalues of A. Show that A is positive definite.

(b) Please calculate the Matrix $Q = \frac{1}{M} X^T X$ of $J(\theta)$ in the example 3 of the colab notebook *Example-on-real-life data.ipynb*. Check that Q is positive definite.

3. **Gradient descent method and steepest descent method**
   Given

$$A = \begin{pmatrix} 3 & -2 \\ -2 & 5 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \qquad c = 2$$

(a) Use gradient descent method and steepest descent method with initial guess $\mathbf{x}^{(0)} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, find the the optimal $\mathbf{x}$ where the quadratic form $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T A\mathbf{x} - b\mathbf{x} + c$ attains its minimum. Plot the two paths of iterations.

(b) Compare their performance by comparing the convergence number of iterations, and plotting $J^{(i)} = ||\mathbf{r}^{(i)}||$ against number of iteration, that is the vector norm of the residual $\mathbf{r}^{(i)} = A\mathbf{x}^{(i)} - \mathbf{b}$.

(c) Check that, for steepest descent method, $\mathbf{r}^{(k)} \cdot \mathbf{r}^{(k-1)} = 0$, i.e. Any two consecutive search line are orthogonal.