

# stereotype-simulations-summary

November 28, 2023

## 1 Generational Stereotype Simulations

This notebook: - Replicates results from Bai's prior work - Explores social learning in this context  
- Demonstrates that several hypothesized effects emerge in generational simulations

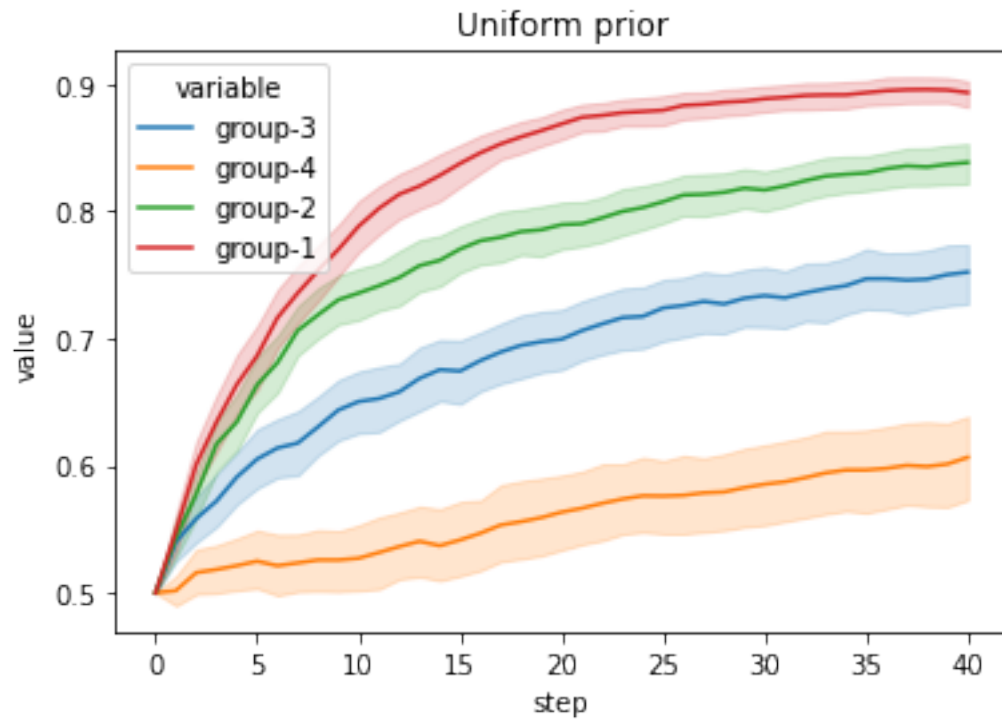
```
[1]: import seaborn as sns
import pandas as pd
import matplotlib.pyplot as plt

from individual_learner import LearnerAgent, FEATURES, BIASED_PRIOR_PARAMS
import simulations
```

```
[2]: %load_ext autoreload
%autoreload 2
```

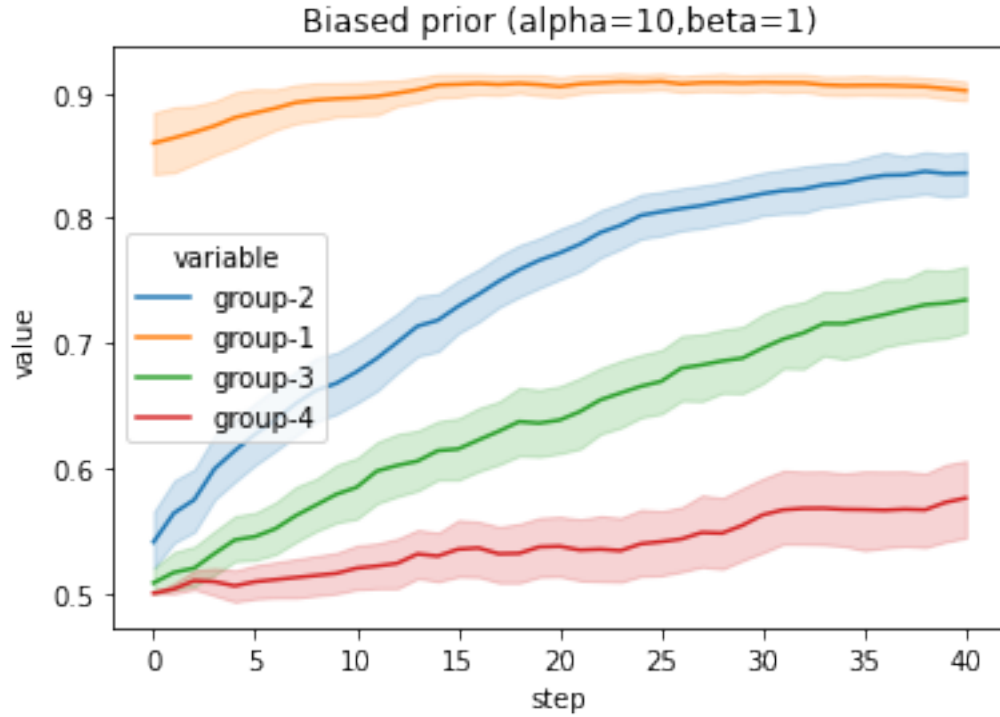
```
[3]: results = simulations.individual_learner_simulations(100,40)
sns.lineplot(data=pd.melt(results, ['step']), x='step', y='value',
             hue='variable')
plt.title("Uniform prior")
```

```
[3]: Text(0.5, 1.0, 'Uniform prior')
```



```
[4]: results = simulations.individual_learner_simulations(100,40,
    ↪prior=BIASED_PRIOR_PARAMS)
sns.lineplot(data=pd.melt(results, ['step']), x='step', y='value',
    ↪hue='variable')
plt.title("Biased prior (alpha=10,beta=1)")
```

```
[4]: Text(0.5, 1.0, 'Biased prior (alpha=10,beta=1)')
```



## 2 Social Learning

### 2.1 Listener agents

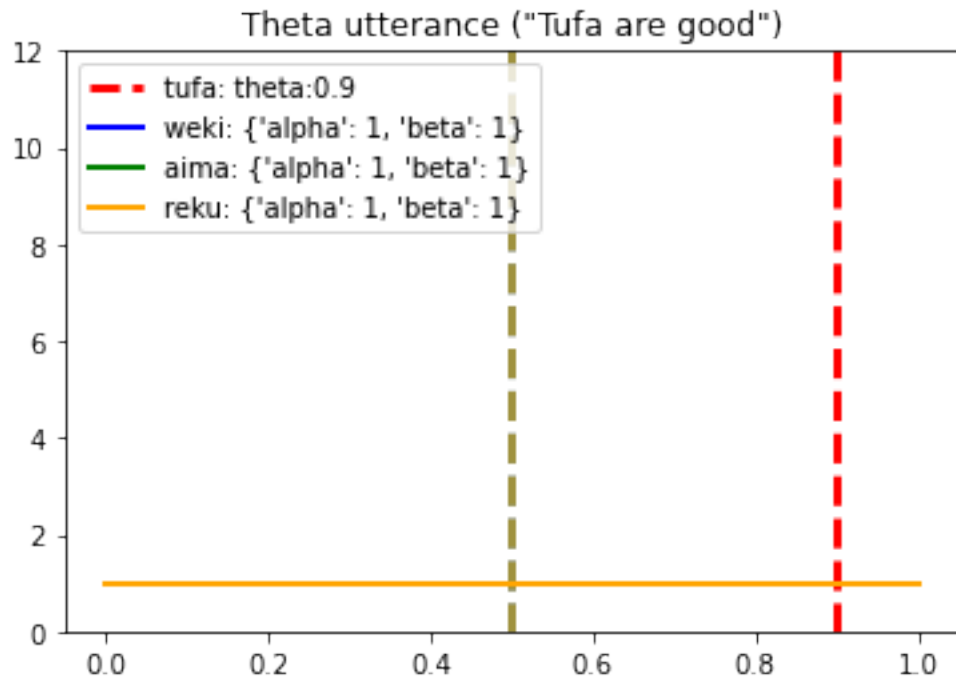
This section tests speakers which choose a fixed utterance instead of passing the full posteriors.

Utterances are either about mean of a population (the  $\theta$  itself) or some underlying observations  $(\alpha, \beta)$ .

$\theta$ -based utterances are of the form  $\langle \text{Tufa}, .9 \rangle$ , while  $\alpha, \beta$  based ones are of the form  $\langle \text{Tufa}, (8, 0) \rangle$ .

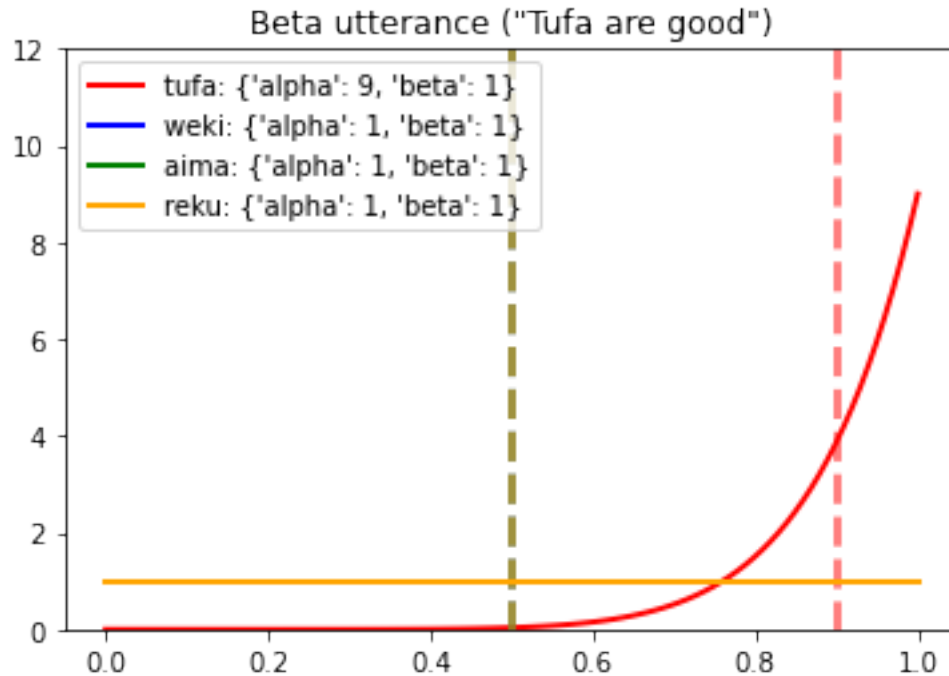
```
[5]: theta_utterance = ('tufa', .90)

LearnerAgent().social_learning_from_utterance(theta_utterance).plot_beliefs()
plt.title('Theta utterance ("Tufa are good")');
```



```
[6]: beta_utterance = ('tufa', (8, 0))

language_conditioned = LearnerAgent().
    ↳social_learning_from_utterance(beta_utterance)
language_conditioned.plot_beliefs()
plt.title('Beta utterance ("Tufa are good")');
```



## 2.2 Speaker Agents

We first have an individual agent gain experience, then evaluate the utility of different utterances conditioned on their posterior.

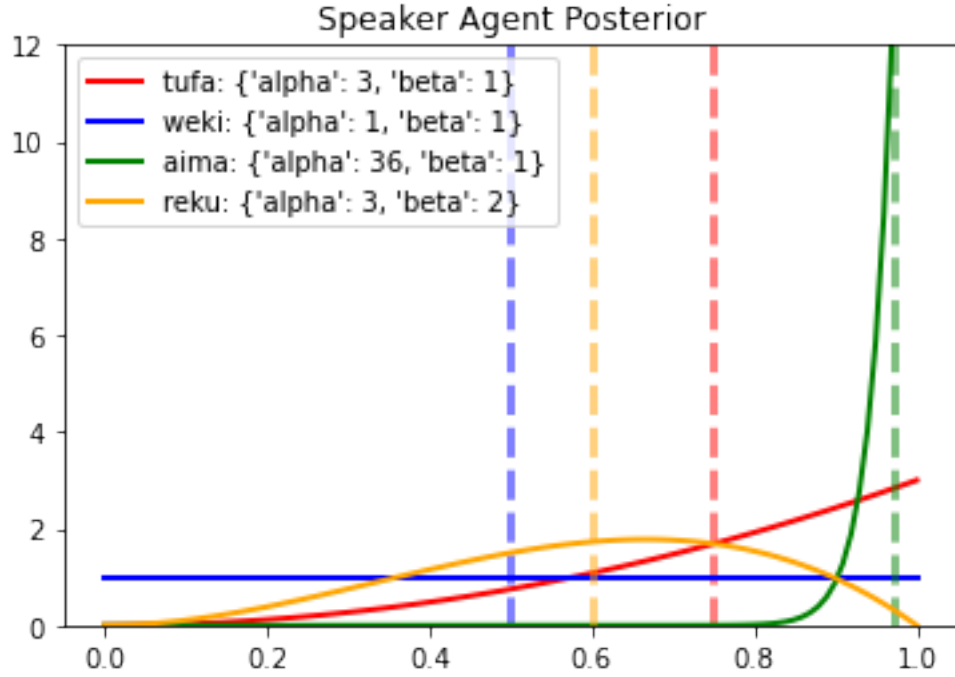
```
[7]: from speakers import ExpectedDecisionTheoreticUtility, BETA_UTTERANCES
import visualizations as viz
```

```
[33]: individual_learner = LearnerAgent()

for i in range(0, 40):
    individual_learner.thompson_sampling()

individual_learner.plot_beliefs()
plt.title('Speaker Agent Posterior')
```

```
[33]: Text(0.5, 1.0, 'Speaker Agent Posterior')
```



### 2.2.1 Decision-Theoretic Utility

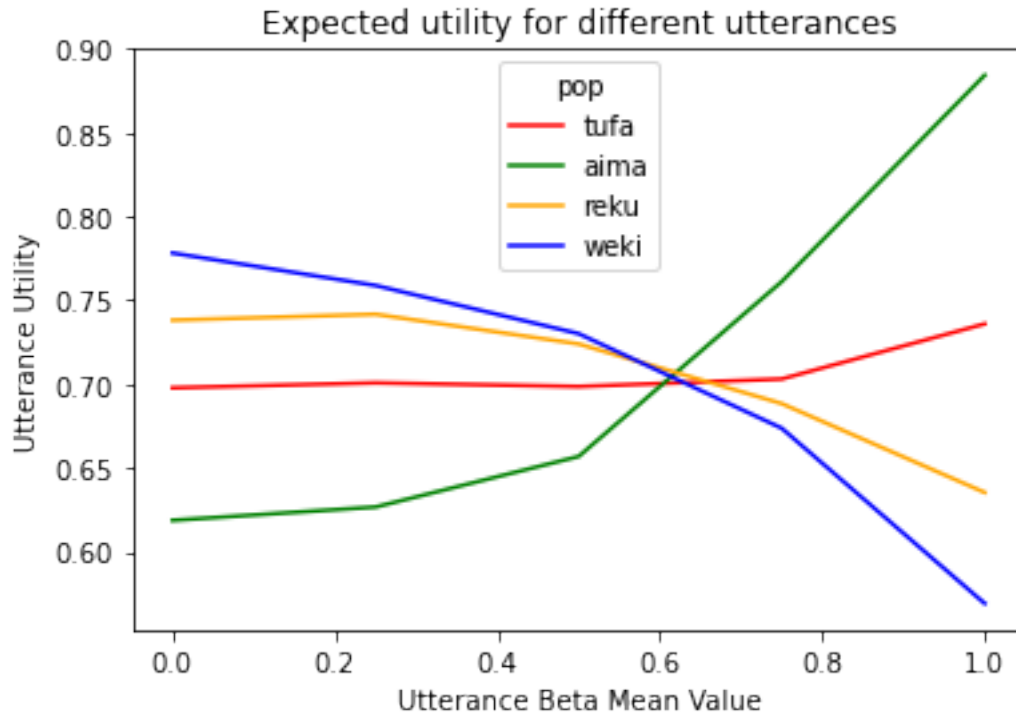
The speaker's utility for an utterance is the expected value of the *listener's* policy, under the *speaker's* posterior:

$$U_u = V_{\pi_L}(\theta_S) = \sum_k \pi_L(a_k | u) * \theta_k.$$

```
[36]: dt_speaker = ExpectedDecisionTheoreticUtility(individual_learner,
↳ BETA_UTTERANCES)

viz.visualize_utterance_utilities(dt_speaker, BETA_UTTERANCES)
plt.title("Expected utility for different utterances");

plt.xlabel('Utterance Beta Mean Value')
plt.ylabel('Utterance Utility');
```



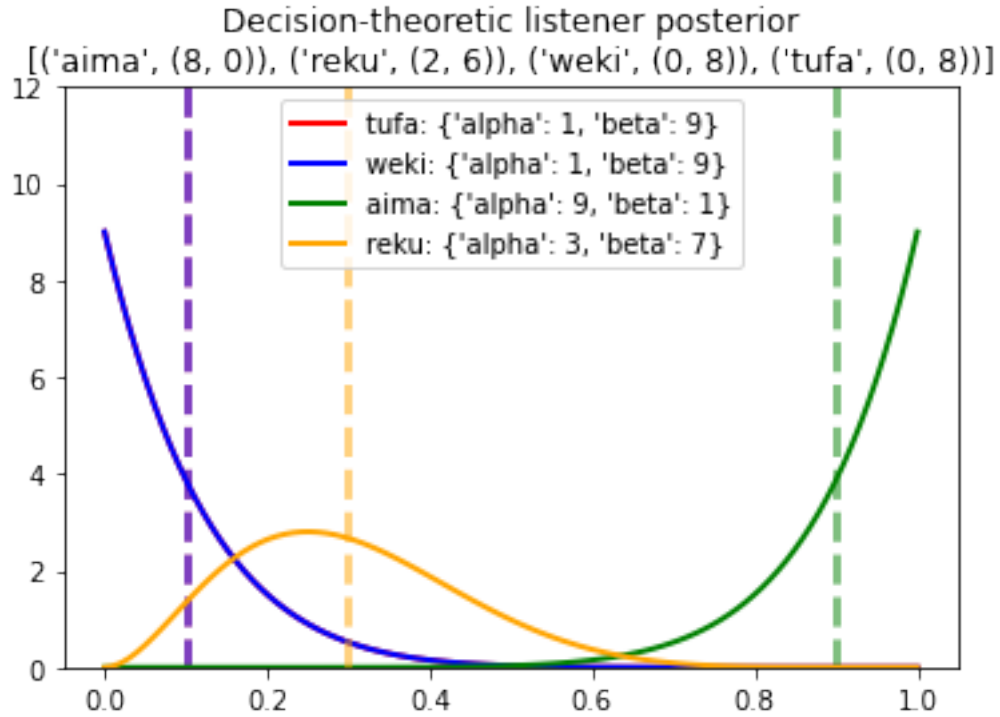
We can use this utility to choose up to  $k$  utterances, which determine the listener's posterior beliefs:

```
[37]: print(dt_speaker.choose_multiple_utterances(1))
      print(dt_speaker.choose_multiple_utterances(3))

[('aima', (8, 0))]
[('aima', (8, 0)), ('tufa', (6, 2)), ('reku', (2, 6))]

[38]: utts = dt_speaker.choose_multiple_utterances(4)
      listener = LearnerAgent()

      for u in utts:
          listener = listener.social_learning_from_utterance(u)
      listener.plot_beliefs()
      plt.title(f'Decision-theoretic listener posterior\n{utts}');
```



### 2.2.2 Epistemic Utility

The epistemic speaker tries to minimize the KL divergence between their beliefs and the listener's.

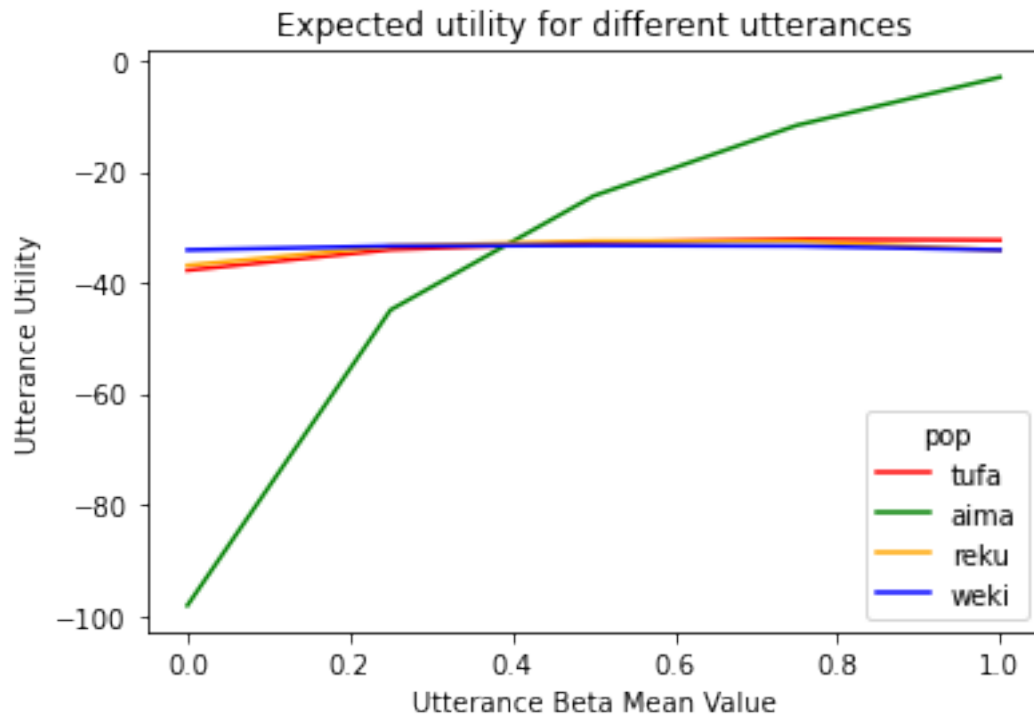
```
[39]: from speakers import EpistemicUtility

epistemic_speaker = EpistemicUtility(individual_learner, alpha=10)

viz.visualize_utterance_utilities(epistemic_speaker, BETA_UTTERANCES)
plt.title("Expected utility for different utterances");

plt.xlabel('Utterance Beta Mean Value')
plt.ylabel('Utterance Utility');
```

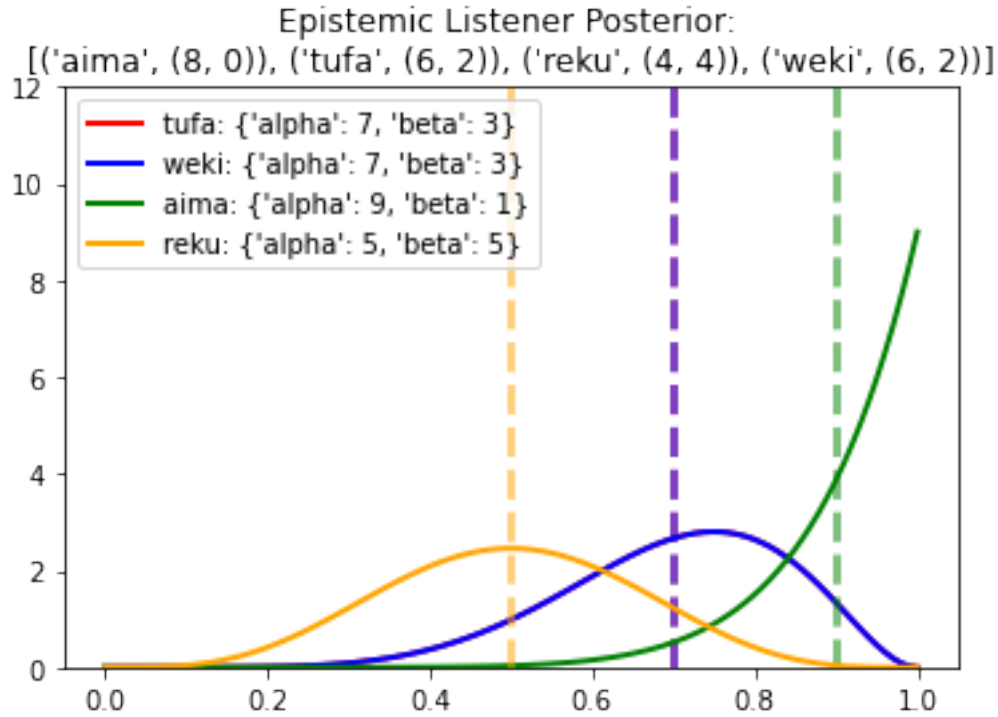




```
[40]: utts = epistemic_speaker.choose_multiple_utterances(4, visualize=False)

listener = LearnerAgent()

for u in utts:
    listener = listener.social_learning_from_utterance(u)
listener.plot_beliefs()
plt.title(f'Epistemic Listener Posterior: \n{utts}');
```



```
[41]: from matplotlib import gridspec

gs = gridspec.GridSpec(2, 2)
fig = plt.figure(figsize=(10, 8))

ax1 = fig.add_subplot(gs[0,0:2])
ax2 = fig.add_subplot(gs[1,0])
ax3 = fig.add_subplot(gs[1,1])

listener = LearnerAgent()
epistemic_speaker.individual_learner.plot_beliefs(ax=ax1)
ax1.set_title("Speaker's Initial Beliefs")

utts = epistemic_speaker.choose_multiple_utterances(4, visualize=False)
for u in utts:
    listener = listener.social_learning_from_utterance(u)
listener.plot_beliefs(ax=ax2)
ax2.set_title(f'Epistemic Listener Posterior');

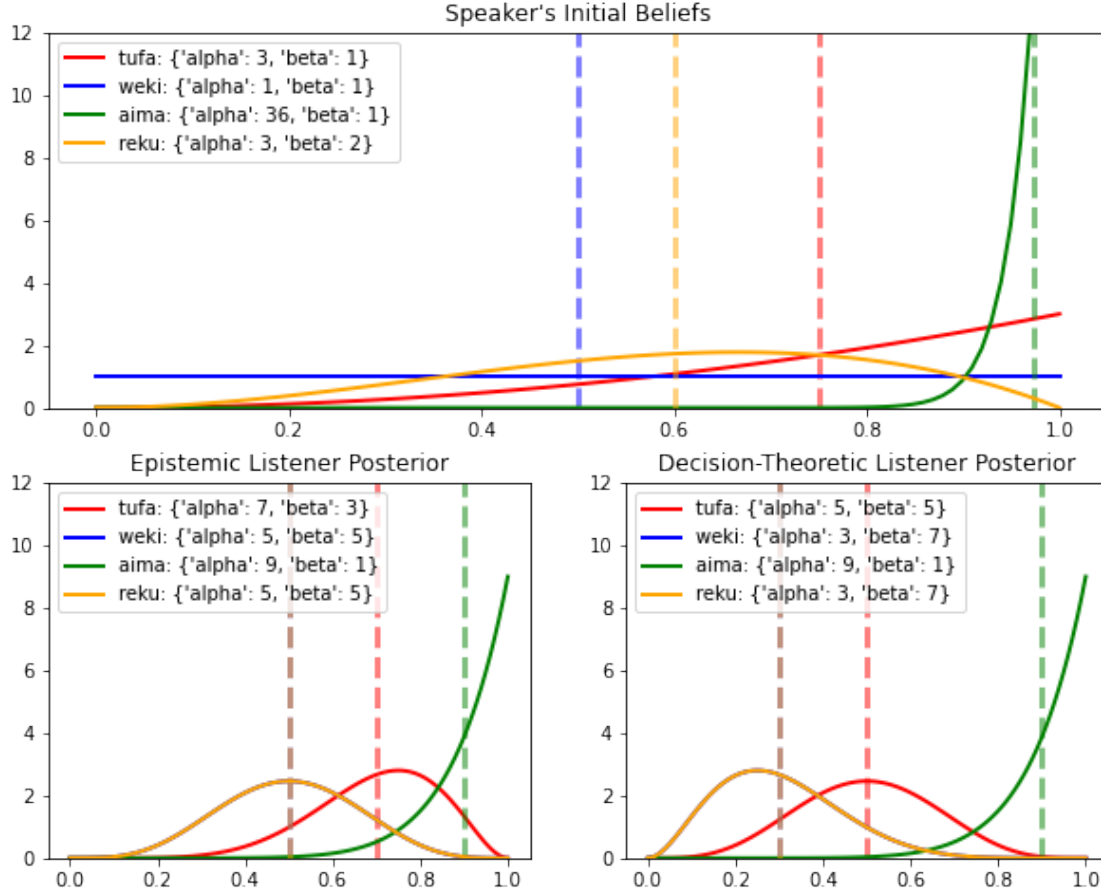
listener = LearnerAgent()
utts = dt_speaker.choose_multiple_utterances(4, visualize=False)
for u in utts:
    listener = listener.social_learning_from_utterance(u)
```

```

listener.plot_beliefs(ax=ax3)
ax3.set_title(f'Decision-Theoretic Listener Posterior');

plt.show()

```



### 3 Generational Simulations

#### 3.0.1 Simulation 1: “Ordering Effect”

The “ordering effect” is the hypothesis that agents which receive social information first (*prior* to individual learning) will display *more bias* than agents which receive social information second.

The “Social first” conditions should show: - **lower** Herfindahl score (less diversity in interaction) - **higher** reward stddev (amplified social stereotypes)

We test two communication variants: “full beta”, in which agents directly transfer their beta distributions, and “dt\_beta” in which agents produce a single beta utterance using decision-theoretic utility.

```
[15]: n_chains = 1000
n_generations = 10
n_rounds = 10

social_first = [True, False]
transmissions = ['full_beta', 'dt_beta', 'epistemic_beta']

results_df_list = []
for social in social_first:
    for t in transmissions:

        res = simulations.generational_simulations(n_chains=n_chains,
                                                    n_generations=n_generations,
                                                    n_rounds=n_rounds,
                                                    social_first=social,
                                                    transmission=t,
                                                    n_utterances=1)

        results_df_list.append(res)
```

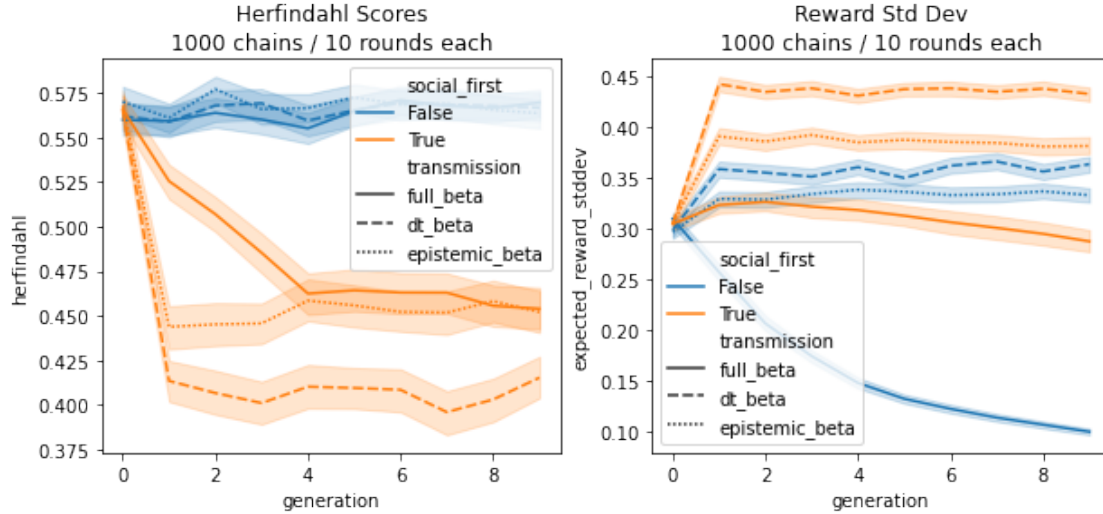
full\_beta, social first, 1 utts, no initial bias --> 1000 chains took 0.26 minutes.  
dt\_beta, social first, 1 utts, no initial bias --> 1000 chains took 1.9 minutes.  
epistemic\_beta, social first, 1 utts, no initial bias --> 1000 chains took 0.46 minutes.  
full\_beta, individual first, 1 utts, no initial bias --> 1000 chains took 0.28 minutes.  
dt\_beta, individual first, 1 utts, no initial bias --> 1000 chains took 1.92 minutes.  
epistemic\_beta, individual first, 1 utts, no initial bias --> 1000 chains took 0.48 minutes.

```
[16]: results = pd.concat(results_df_list)
```

```
[20]: fig, ax = plt.subplots(1, 2, figsize=(10, 4))

sns.lineplot(data=results, x='generation', y='herfindahl', hue='social_first',
             style='transmission', ax=ax[0])
ax[0].set_title(f'Herfindahl Scores\n{n_chains} chains / {n_rounds} rounds_\n\neach');
ax[0].legend(loc=1)

sns.lineplot(data=results, x='generation', y='expected_reward_stddev',
             hue='social_first', style='transmission', ax=ax[1])
ax[1].set_title(f'Reward Std Dev\n{n_chains} chains / {n_rounds} rounds each');
plt.legend(loc=3);
```



**Discussion Social first shows more bias.** When learners *don't* receive social information first, their behaviors are unaffected (left: Herfindahl score remains the same). As a result, the chains accumulate information about different sub-populations. When passing full betas, this reduces bias (right: reward stddev decreases with time). Notably, when producing utterances, this reduces bias *relative to social-first*, but overall still produces bias.

In “social first”, the agents effectively start Thompson sampling with a strong prior. Chains “lock in” to a belief state and then propagate that.

**Utterances show more bias than full parameter passing.** Utterance-based Herfindahl scores drop immediately (left) and bias spikes (right). In contrast, with full betas, the Herfindahl drops gradually; and the bias actually decreases a bit over time. This is particularly interesting given that agents can communicate about only a single population in the “utterance” condition.

### 3.0.2 Simulation 2: “Multiple Utterances”

Multiple utterances allow agents to pass information about >1 population. We expect that **bias will increase** with the number of utterances agents produce.

```
[22]: n_chains = 1000
n_generations = 10
n_rounds = 10

social_first = True
transmission = ['dt_beta', 'epistemic']

n_utterances = [1, 2, 3, 4]

multi_utterance_results_df_list = []
```

```

for n_utt in n_utterances:
    for t in transmission:

        res = simulations.generational_simulations(n_chains=n_chains,
                                                    n_generations=n_generations,
                                                    n_rounds=n_rounds,
                                                    social_first=social_first,
                                                    transmission=t,
                                                    n_utterances=n_utt)

        multi_utterance_results_df_list.append(res)

full_results = pd.concat(multi_utterance_results_df_list)

```

dt\_beta, social first, 1 utts, no initial bias --> 1000 chains took 1.89 minutes.

epistemic, social first, 1 utts, no initial bias --> 1000 chains took 0.46 minutes.

dt\_beta, social first, 2 utts, no initial bias --> 1000 chains took 3.1 minutes.

epistemic, social first, 2 utts, no initial bias --> 1000 chains took 0.59 minutes.

dt\_beta, social first, 3 utts, no initial bias --> 1000 chains took 3.86 minutes.

epistemic, social first, 3 utts, no initial bias --> 1000 chains took 0.69 minutes.

dt\_beta, social first, 4 utts, no initial bias --> 1000 chains took 4.26 minutes.

epistemic, social first, 4 utts, no initial bias --> 1000 chains took 0.75 minutes.

```

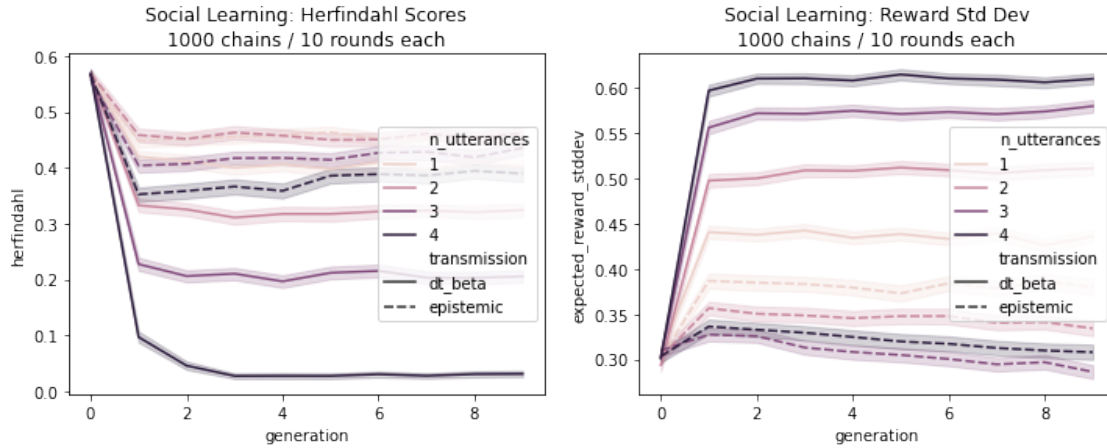
[29]: fig, ax = plt.subplots(1, 2, figsize=(12, 4))

sns.lineplot(data=full_results, x='generation', y='herfindahl',
             hue='n_utterances',
             style='transmission', ax=ax[0])
ax[0].set_title(f'Social Learning: Herfindahl Scores\n{n_chains} chains /\n{n_rounds} rounds each');
ax[0].legend(loc=5);

sns.lineplot(data=full_results, x='generation', y='expected_reward_stddev',
             hue='n_utterances',
             style='transmission', ax=ax[1])

ax[1].set_title(f'Social Learning: Reward Std Dev\n{n_chains} chains /\n{n_rounds} rounds each');
ax[1].legend(loc=5);

```



**Discussion** We confirmed that more utterances results in more bias.

### 3.0.3 Simulation 3: “Initialization Bias”

```
[30]: n_chains = 1000
n_generations = 10
n_rounds = 10

social_first = [True, False]
initial_bias = [True, False]
transmissions = ['full_beta', 'dt_beta']

results_df_list = []
for social in social_first:
    for t in transmissions:
        for b in initial_bias:

            res = simulations.generational_simulations(n_chains=n_chains,
                                                        ↪n_generations=n_generations,
                                                        n_rounds=n_rounds,
                                                        social_first=social,
                                                        transmission=t,
                                                        initial_bias=b)

            results_df_list.append(res)
```

full\_beta, social first, 1 utts, initial bias --> 1000 chains took 0.26 minutes.  
 full\_beta, social first, 1 utts, no initial bias --> 1000 chains took 0.26 minutes.

dt\_beta, social first, 1 utts, initial bias --> 1000 chains took 1.88 minutes.  
 dt\_beta, social first, 1 utts, no initial bias --> 1000 chains took 1.89

minutes.  
full\_beta, individual first, 1 utts, initial bias --> 1000 chains took 0.28 minutes.  
full\_beta, individual first, 1 utts, no initial bias --> 1000 chains took 0.28 minutes.  
dt\_beta, individual first, 1 utts, initial bias --> 1000 chains took 1.91 minutes.  
dt\_beta, individual first, 1 utts, no initial bias --> 1000 chains took 1.92 minutes.

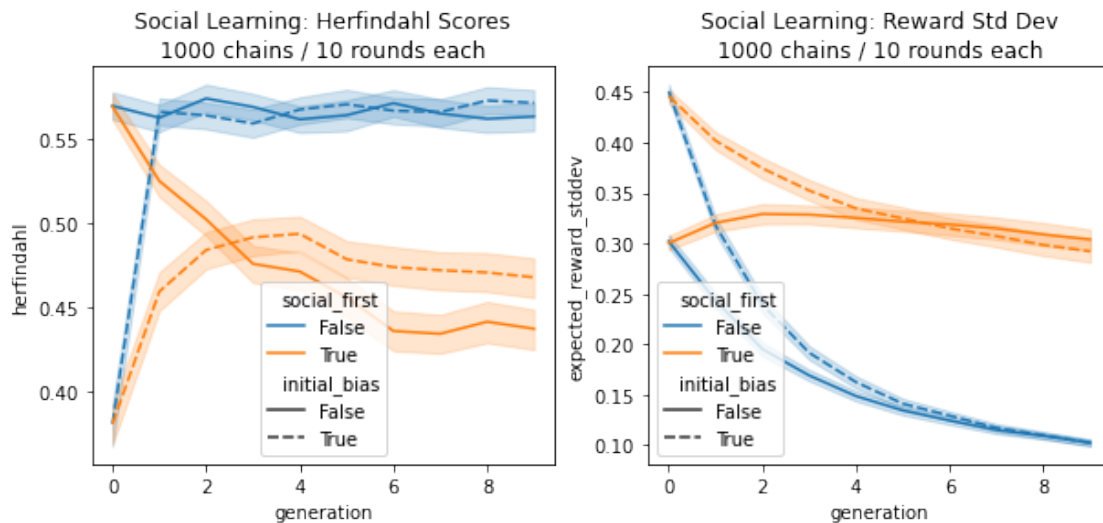
```
[31]: initial_bias_res = pd.concat(results_df_list)
```

```
[32]: fig, ax = plt.subplots(1, 2, figsize=(10, 4))

to_plot = initial_bias_res[initial_bias_res.transmission == 'full_beta']

sns.lineplot(data=to_plot, x='generation', y='herfindahl', hue='social_first',
             style='initial_bias', ax=ax[0])
ax[0].set_title(f'Social Learning: Herfindahl Scores\n{n_chains} chains /\n{n_rounds} rounds each');

sns.lineplot(data=to_plot, x='generation', y='expected_reward_stddev',
             hue='social_first', style='initial_bias', ax=ax[1])
ax[1].set_title(f'Social Learning: Reward Std Dev\n{n_chains} chains /\n{n_rounds} rounds each');
```



```
[35]: fig, ax = plt.subplots(1, 2, figsize=(10, 4))

to_plot = initial_bias_res[initial_bias_res.transmission == 'dt_beta']
```

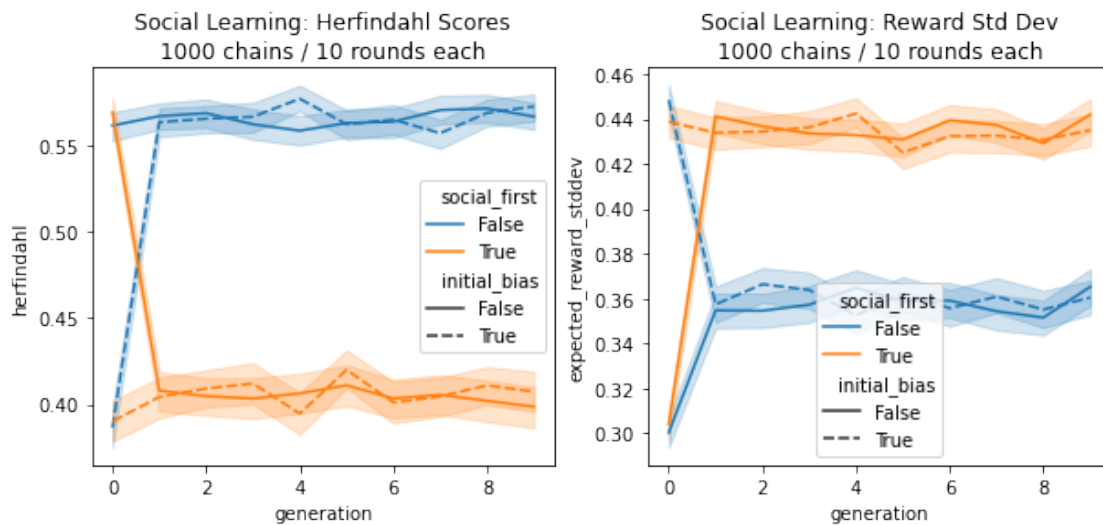


```

sns.lineplot(data=to_plot, x='generation', y='herfindahl', hue='social_first',
             style='initial_bias', ax=ax[0])
ax[0].set_title(f'Social Learning: Herfindahl Scores\n{n_chains} chains /\n{n_rounds} rounds each');

sns.lineplot(data=to_plot, x='generation', y='expected_reward_stddev',
             hue='social_first', style='initial_bias', ax=ax[1])
ax[1].set_title(f'Social Learning: Reward Std Dev\n{n_chains} chains /\n{n_rounds} rounds each');

```



[ ]: