Polynomial Regression

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Polynomial regression is a tool that allows us to understand and predict the behavior of complex data. While linear regression assumes a linear relationship between the independent variables and the dependent variable, polynomial regression allows for modeling nonlinear relationships by incorporating polynomial terms.

First, we consider the univariate model of the form:

$$y_i = f(x_i) + \varepsilon_i \quad i = 1, \ldots, n, \tag{1}$$

According to polynomial regression, f is a polynomial function with degree q. i.e.,

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_q x^q$$

for certain coefficients β_j where $j = 0, \ldots, q$. Now, the model above can be written as

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_q x_i^q + \varepsilon_i \quad i = 1, \ldots, n.$$

It is possible to estimate the coefficients β using linear regression considering the model $y = X\beta + \varepsilon$ where the model matrix is $X = [1, x, x^2, \ldots, x^q]$ and the coefficient vector is $\beta = [\beta_0, \beta_1, \beta_2, \ldots \beta_q]^T$.