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Determination of optimal gas forming conditions from free bulging tests at constant pressure

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ABSTRACT

This study proposes a method for determination of material characteristics by inverse analysis of free bulging tests results. The blow-forming tests were carried out at the temperature of 415 °C using aluminum alloy (AMg-6) sheets of a 0.92 mm thickness. Each test was performed at constant pressure. For each fixed value of pressure, a series of experiments was carried out with different forming times to obtain evolutions of dome height *H* and thickness *s*. Two different constitutive equations were used to describe the dependence of flow stress on the effective strain rate: the Backofen power equation and the Smirnov one taking into account an s-shape of stress–strain rate curve in the logarithmic scale. The constants of these equations were obtained by least squares minimization of deviations between the experimental variations of *H* and *s* and ones predicted by a simplified engineering model formulated for this purpose. Using the Smirnov constitutive model to describe the dependence of flow stress on strain rate, unlike the classical power law, makes it possible to analyze the variation of strain rate sensitivity index *m* with strain rate. On the basis of the obtained data, the optimum strain rate for AMg-6 processing was estimated as one corresponding to the maximum of strain rate sensitivity index. The validity of the proposed method was examined by finite element simulation of free bulging process.

1. Introduction

Gas forming is an advanced method of complex thin-walled parts production which is used mainly in aerospace industry. To increase the hot forming ability and improve the mechanical properties of the final products, the utilization of superplasticity effect is desired. A computer simulation is necessary to optimize the pressure cycles during the development of superplastic forming technologies. Adequacy of the simulation results depends strongly on the accuracy of initial and boundary conditions and constitutive equations of the material.

In isothermal condition, neglecting the effects of strain hardening and grain growth, the mechanical behavior of a material during hot forming is described as a relation between the equivalent stress σ_e, and equivalent strain rate $\dot{\varepsilon}_e$:

$$\sigma_e = f(\dot{\varepsilon}_e) \tag{1}$$

where *f*(. . .) is a single-valued function which should be determined experimentally.

The mechanical behavior of superplastic materials is generally described by standard power relation proposed by Backofen et al. (1964):

$$\sigma_e = K\dot{\varepsilon}_e^m \tag{2}$$

where *K* and *m* are characteristics of the material. The most important characteristic in Eq. (2) is the strain rate sensitivity *m*. Higher values of *m* are responsible for lower rates of flow localization and better plasticity what was first described by Hart (1967). For superplastic materials the value of *m* is greater or equal to 0.3 (Vasin et al., 2000).

The Eq. (2) is still the most commonly used constitutive model for simulation of superplastic forming processes. It is a classic power function which forms a straight line with a slope *m* if it is plotted in logarithmic scale. At the same time it is well known that a sigmoidal variation of the flow stress with strain rate takes place in the logarithmic scale (Vasin et al., 2000). Thus, Eq. (2) may be applied only as a local approximation describing the sigmoidal

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curve within a rather narrow range of the strain rates (Enikeev and Kruglov, 1995).

Smirnov (1979) proposed the rheological model of visco-plastic medium which describes the behavior of superplastic materials in a wide range of strain rates. The constitutive equation corresponding to this model takes the following form:

$$\sigma_e = \sigma_s \frac{\sigma_0 + k_v \dot{\varepsilon}_e^{m_v}}{\sigma_s + k_v \dot{\varepsilon}_e^{m_v}} \tag{3}$$

where σ_0 is the threshold stress which corresponds to the small strain rates, σ_s is the yield stress at large strain rates, k_v and m_v are material parameters.

The main advantage of the Smirnov model is its invariance in a wide range of strain rates. At the same time, once the constants σ_0, σ_s, k_v and m_v are found for a material at a given temperature, the optimal forming conditions can be easily evaluated as a strain rate range corresponding to the required strain rate sensitivity (Aksenov et al., 2013).

The strain rate sensitivity index m can be obtained from Eq. (3) as follows:

$$m(\dot{\varepsilon}_e) = \frac{dln(\sigma_e)}{dln(\dot{\varepsilon}_e)} = \frac{m_v k_v \dot{\varepsilon}_e^{m_v}(\sigma_s - \sigma_0)}{(\sigma_0 + k_v \dot{\varepsilon}_e^{m_v})(\sigma_s + k_v \dot{\varepsilon}_e^{m_v})} \tag{4}$$

This function has a peak at the strain rate:

$$\dot{\varepsilon}_{\text{opt}} = \left(\frac{\sqrt{\sigma_0 \sigma_s}}{k_v} \right)^{1/m_v} \tag{5}$$

Flow behavior of superplastic materials can be described in more details by application of complex microstructure mechanisms based constitutive equations. Khraisheh et al. (2006) described constitutive model of AZ31 alloy taking into account microstructural features including grain growth and cavitation. To calibrate the model they used microstructural investigations as well as results of extensive experimental study proposed by Abu-Farha and Khraisheh (2007). Albakri et al. (2013) recently used this constitutive model to construct forming limit diagrams. Constant pressure free bulging tests were used in their work to correct the constitutive model based on uniaxial tensile testing. The same material was investigated by Taleff et al. (2010), they demonstrated that the initial material model based upon uniaxial tensile data is not accurate enough to describe biaxial forming and the corrections should be made by considering constant pressure bulging tests.

This comprehensive approach allows one to achieve accurate constitutive model of investigated alloys but it needs a large amount of experimental work including tensile tests in wide strain rate range, microstructure investigations and bulging tests for calibration of the model. The purpose of this work is to provide a method determining the main characteristics of the material on a base of free bulging tests only.

The determination of material constants on the basis of bulging tests is in the focus of many studies. Enikeev and Kruglov (1995) proposed a method for calculation of constants in Eq. (2) on the basis of free bulging tests carried out to a predetermined dome height. Li et al. (2004) simulated SPF processes by finite element method (FEM) and applied an inverse analysis to obtain the true material constants in Eq. (2). Giuliano and Franchitti (2007) proposed a method of superplastic material characterization which is able to predict a strain hardening index as well as the constants K and m. Giuliano then used this method to determine the Backofen constitutive equation constants for Ti-6Al-6V alloy (Giuliano, 2008) and most recently to AA-5083 alloy (Giuliano, 2012). All of these methods use a Backofen constitutive equation what makes it difficult to determine the optimal forming conditions.

Inverse analysis applied by Li et al. (2004) appears to be the most universal way to determine the material constants. This approach is based on multiple simulations of bulging process and searching the material constants which produce the closest results to the experimental ones by minimization of error function. Using of FEM for simulation of the process provides appropriate accuracy, but calculations are very time consuming. In this regard using of simplified engineering models which provides the solution in analytical or semianalytical form could be more suitable.

Hill (1950) provided one of the earliest formulations of the free bulging process model and general solution for cold-worked metals. Woo (1964) formulated a general method of analysis for axisymmetric forming processes and applied it to cold hydrostatic bulging process using numerical iterative procedure for the solution. Similar numerical procedure was used by Holt (1970) for simulation of superplastic free bulging and bulging in conical grooves. Holt's formulation is based on the assumption of a balanced biaxial stress state, while Guo and Ridley (1989) assume that the ratio of the circumferential and tangential stresses varies with fractional height in a logarithmic manner. Yang and Mukherjee (1992) consider a bulging contour to be a part of an ellipsoid instead of most commonly used sphere to increase an accuracy of the model for a low m materials.

All the approaches of superplastic bulging simulation mentioned above results in equation systems supposed to be solved numerically. Yu-Quan and Jun (1986) proposed a method which provides simple differential equation describing the dependence of dome height in respect to forming time. This equation can be obtained for any constitutive model allowing the construction of inverse function for the $f(\ldots)$ from Eq. (1). The week point of this approach is that it uses the expression of dome apex thickness versus dome height in a form which does not depend on material properties.

Since the main goal of this investigation is the interpretation of bulging forming experiments, the dependence of apex thickness on dome height was obtained as an approximation of the experimental results. This approximation was then used to produce the model of dome height evolution using an approach which is similar to one described by Yu-Quan and Jun (1986). The model of dome height evolution produced by this way is then used in inverse analysis to determine the material constants.

The proposed method was applied to determine the constants σ_0, σ_s, k_v and m_v of Smirnov constitutive equation (Eq. (3)) as well as K and m of Backofen one (Eq. (2)) for AMg6 aluminum alloy. In order to validate the obtained material constants, they were used in finite element simulation of the bulging experiments. Another validation procedure was performed by comparison of obtained strain rate sensitivity values with ones calculated by the method based on Enikeev and Kruglov (1995) equations.

2. Mathematical model of a bulging process

The scheme of a free bulging process is illustrated at Fig. 1. A metal sheet of initial thickness s is formed by pressure P in a die

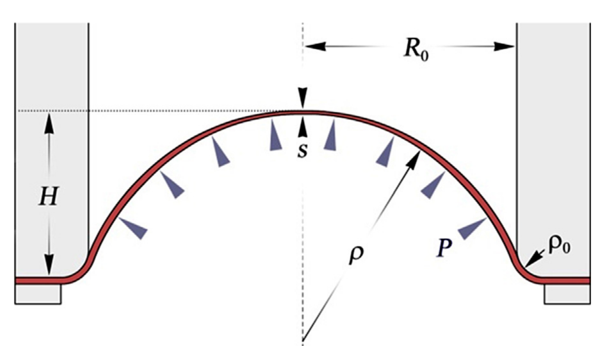

Fig. 1. Scheme of the free bulging process.

with an aperture radius R_0 and entry radius ρ_0. At an instant t, the free part of the dome is assumed to be a spherical surface with radius ρ. H is the height of the dome and s is the current thickness at the dome apex.

By considering the equilibrium conditions of a small element in the dome apex, it is possible to obtain:

$$\sigma_e = \frac{P\rho}{2s}. \tag{6}$$

Equivalent strain and equivalent strain rate in the dome apex can be obtained as:

$$\varepsilon_e = \ln\left(\frac{s_0}{s}\right), \tag{7}$$

$$\dot{\varepsilon}_e = -\frac{1}{s}\frac{ds}{dt} = -\frac{1}{s}\frac{ds}{dH}\frac{dH}{dt}. \tag{8}$$

The value of ρ can be expressed as follows:

$$\rho = \frac{(R_0+\rho_0)^2 + H^2}{2H} - \rho_0. \tag{9}$$

The important point of the mathematical simulation of free bulging process is a relation between the thickness at dome apex s and its height H. There are several approaches to express this relation. Jovane (1968) used the assumption of uniform thickness distribution to obtain the equation which was later used by Cheng (1996) for determination of material constants:

$$s = \frac{s_0 R_0^2}{R_0^2 + H^2}. \tag{10}$$

Enikeev and Kruglov (1995) proposed a formula:

$$s = s_0\left(\frac{\sin\alpha}{\alpha}\right)^2. \tag{11}$$

where

$$\alpha = \arcsin\left(\frac{2HR_0}{R_0^2 + H^2}\right). \tag{12}$$

Yu-Quan and Jun (1986) obtained the geometric relation between the bulging contour and particle path from the viewpoint of quantitative analysis. It was shown by them that a particle path is perpendicular to the bulging contour at any time of the process. The dependence between s and H can be expressed in this case as:

$$s = \frac{s_0 R_0^4}{(R_0^2 + H^2)^2}. \tag{13}$$

All of the Eqs. (10)–(13) are obtained for a case of zero die entry radius ($\rho_0 = 0$). To describe the case with nonzero ρ_0, assume following by Yu-Quan and Jun (1986) that every particle of the dome is moving perpendicular to a dome surface. Using this hypothesis, it can be shown, that:

$$\frac{ds}{dH} = -\frac{2s}{\rho}. \tag{14}$$

Substituting Eq. (10) into Eq. (14) and solving the differential equation with an initial condition $s(0) = s_0$, the relation between s and H which takes into account a nonzero value of ρ_0, can be obtained:

$$s = \frac{s_0(R_0+\rho_0)^4}{((R_0+\rho_0)^2 + H^2 - 2H\rho_0)^2} e^{-4C_1\rho_0\{\mathrm{atan}(C_1\rho_0) - \mathrm{atan}(C_1(\rho_0 - H))\}} \tag{15}$$

where

$$C_1 = (R_0^2 + 2R_0\rho_0)^{-1/2}. \tag{16}$$

It can be seen that if $\rho_0 = 0$, Eq. (15) becomes identical to Eq. (13).

If experimental data of the dome thickness at different heights are available then the relation $s(H)$ can be obtained by fitting the experimental data. Aoura et al. (2004) used polynomial fitting of measured apex thickness of domes with different height obtained by tests with constant stress.

In this paper, the approximation of $s(H)$ dependence was made by integration of following differential equation:

$$\frac{ds}{dH} = -\frac{As^\alpha}{\rho} \tag{17}$$

where A and α were constants to be defined. For the combination $A = 1$, $\alpha = 1$ and $\rho_0 = 0$, the solution of Eq. (17) is identical to Eq. (10). For the combination $A = 2$, $\alpha = 1$, the solution of Eq. (17) is identical to Eq. (15).

After the relation $s(H)$ is obtained, the differential equation describing the dependence of dome height on time can be constructed by substitution of Eqs. (6) and (8) into Eq. (1):

$$\frac{dH}{dt} = -\frac{s}{ds/dH} f^{-1}\left(\frac{P\rho}{2s}\right) \tag{18}$$

where $f^{-1}(\ldots)$ is the inverse function of the one in Eq. (1), $s = s(H)$ is the relation obtained by solution of Eq. (17).

Substituting Eqs. (2) and (17) into Eq. (18), the relation between H and t is obtained for Backofen constitutive equation:

$$\frac{dH}{dt} = \frac{\rho}{A}\left(\frac{P\rho}{2K}\right)^{1/m} s^{1-\alpha-(1/m)} \tag{19}$$

Substituting Eqs. (3) and (17) into Eq. (18), the relation between H and t is obtained for Smirnov constitutive equation:

$$\frac{dH}{dt} = \frac{s^{1-\alpha}\rho}{A}\left(\frac{\sigma_s(P\rho - 2s\sigma_0)}{K_v(2s\sigma_s - P\rho)}\right)^{1/m_v} \tag{20}$$

Effective strain rate $\dot{\varepsilon}_e$ can be determined using Eq. (3) only for effective stress $\sigma_0 < \sigma_e < \sigma_s$. Therefore, the Eq. (20) should be restricted as follows:

$$\frac{dH}{dt} = \frac{s^{1-\alpha}\rho}{A}\left(\frac{\sigma_s \max(P\rho - 2s\sigma_0, 0)}{K_v \max(2s\sigma_s - P\rho, \delta)}\right)^{1/m_v} \tag{21}$$

where δ is a small value which characterizes a rapid growth of H at the beginning of the bulging process.

Zero value of H according to Eq. (9) leads to infinite value of ρ. Therefore, reasonable initial conditions $H(0) = H_0$ should be applied for both differential Eqs. (19) and (21) to avoid dividing by zero. Where H_0 is a small positive value (in this work the value of $H_0 = 0.001$ mm was used).

3. Experiment

A series of bulge forming tests was performed on the aluminum alloy (AMg6) sheets at constant pressures (P_j) of $P_1 = 0.3$, $P_2 = 0.35$, $P_3 = 0.4$, $P_4 = 0.5$ and $P_5 = 0.6$ MPa. The tests were carried out at the temperature of 415 °C with different forming times t_i. The upper die has cylindrical geometry with the aperture radius $R_0 = 50$ mm, die entry radius $\rho_0 = 5$ mm and 50 mm depth. Square sheet specimens of 160 mm side and 0.92 mm mean initial thickness were fixed by cylindrical tool with 110 mm aperture diameter. For each fixed value of the pressure, a series of experiments was carried out with different forming times. After the forming, each specimen was measured to obtain the dome height ($H_i^{\exp}$) and thickness at the apex ($s_i^{\exp}$). Ten tests with different forming times were performed for the pressure $P_1 = 0.3$ MPa. For the rest of pressures P_2–P_5 four tests at each pressure were performed. The picture of a specimen after the forming at the pressure of $P_3 = 0.4$ MPa for a duration of 1250 s is presented in Fig. 2.

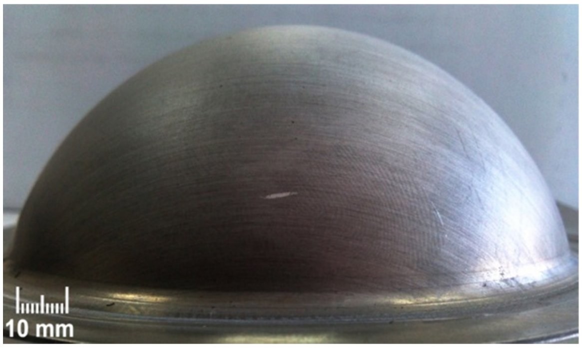

Fig. 2. Specimen after the forming during 1250 s at the pressure of $P = 0.4$ MPa.

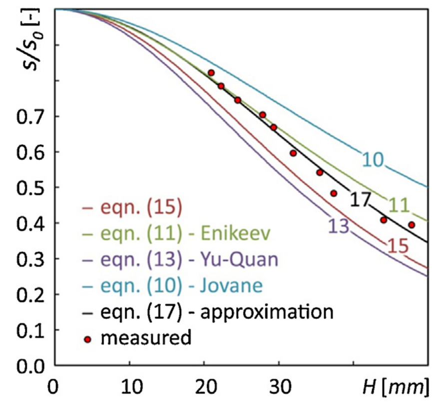

Fig. 3. Comparison between measured and predicted thickness at the dome apex depending on the dome height for forming at constant pressure $P = 0.3$ MPa.

The obtained data were used to determine the constants of Smirnov and Backofen constitutive equations for AMg6 alloy. The analysis of experimental data was carried out in two stages.

At the first stage, the values of constants $A(P_j)$ and $\alpha(P_j)$ of Eq. (17) were found for every pressure P_j by nonlinear regression analysis. The Eq. (17) was numerically integrated with the initial condition $s(0) = s_0$ and initial values of constants $A(P_j) = 2$ and $\alpha(P_j) = 1$. Then, the constants $A(P_j)$ and $\alpha(P_j)$ were corrected in order to minimize the objective function $F_s(P_j)$:

$$F_s(P_j) = \sum_{i=N_0(P_j)}^{N_1(P_j)} \left(\frac{s_i^{\exp} - s(H_i^{\exp})}{s_i^{\exp}} \right)^2 \tag{22}$$

where $N_0(P_j)$ is the number of the first test at pressure P_j and $N_1(P_j)$ is the number of the last test at pressure P_j.

At the second stage, the determination of rheological characteristics of the material was performed. The Eq. (19) was used for the Backofen constitutive model and Eq. (21) for the Smirnov one. The material constants (K and m for the first model and σ_0, σ_s, K_v and m_v for the second one) were found by minimization of objective function F_H :

$$F_{H,s} = \sum_{i=1}^{N} \left\{ \left(\frac{H_i^{\exp} - H(t_i)}{H_i^{\exp}} \right)^2 + \left(\frac{s_i^{\exp} - s(t_i)}{s_i^{\exp}} \right)^2 \right\} \tag{23}$$

where $N = 26$ is a total number of the experiments.

The minimization of the objective functions was performed using the simplex method proposed by Nelder and Mead (1965).

4. Results and discussion

4.1. Determination of material constants

The comparison between dependences of thickness in dome apex on dome height as measured and predicted by different models is shown at Fig. 3. It can be seen that all of the models produces different $s(H)$ curves, the Jovane and Yu-Quane curves have the largest discrepancy and all of the experimental data lies between them.

The values of constants A and α obtained for different pressures are presented in Table 1.

After the $s(H)$ relation was obtained the rheological constants of Backofen and Smirnov models were determined. The Backofen constitutive model constants were found as: $K = 155.7$ and $m = 0.265$. The Smirnov constitutive model constants were found as: $\sigma_0 = 12.07$, $\sigma_s = 45.74$, $k_v = 18{,}196$ and $m_v = 0.929$. The obtained constitutive equations plotted in logarithmic scale are shown in Fig. 4. Experimental and predicted dome height evolutions are illustrated in Fig. 5.

Table 1
Approximation constants A and α, obtained for different pressures.

Pressure (MPa)	A	α	Mean deviation (%)	Number of experiments
0.3	1.52	0.865	2.99	10
0.35	1.19	0.060	4.38	4
0.4	1.26	0.161	2.12	4
0.5	1.46	0.549	0.74	4
0.6	1.63	0.850	1.30	4

4.2. Finite element simulation

The obtained constitutive equations were verified by finite element simulation. Two series of FE simulations were performed with the material behavior described by Eq. (2) for the first series and by Eq. (3) for the second one. The material parameters used in FE simulation were obtained by the proposed method, their values are presented in Section 4.1. The simulations were performed in finite element software developed by the authors. Its basic formulations and experimental verification process are described in Chumachenko et al. (2005). Axisymmetric visco-plastic three-node elements with mean initial side of 0.3 mm were used for the simulations. The kinematic boundary conditions were set for the boundary nodes so that they remain fixed after the contact with the dies. On the free part of the bottom boundary a pressure condition was specified. The initial mesh and the evolution of specimen geometry as well as effective strain distributions obtained by simulation are shown in Fig. 6. It can be seen that the bulge profile has a close to circular form and the thickness distribution is not uniform. Thinning is higher at the dome apex as well as accumulated effective strain.

The $H(t)$ and $s(t)$ relations obtained by FE simulation are compared with the experimental ones in Figs. 7 and 8. In Fig. 7 the

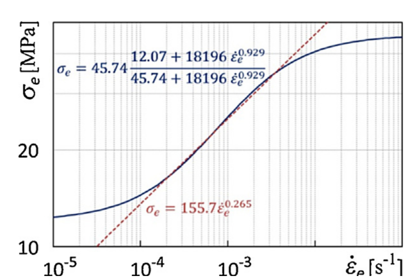

Fig. 4. Constitutive equations obtained for AMg6 alloy according to Backofen (dashed line) and Smirnov (solid line) models.

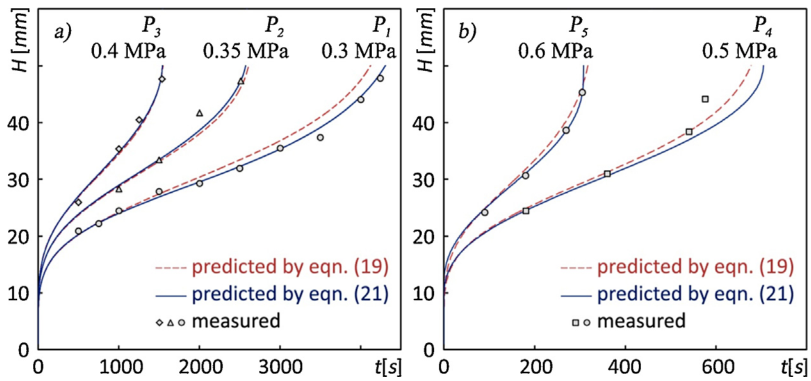

Fig. 5. Evolutions of dome height predicted by the engineering models based on Backofen (dashed line) and Smirnov (solid line) constitutive equations compared with measured values.

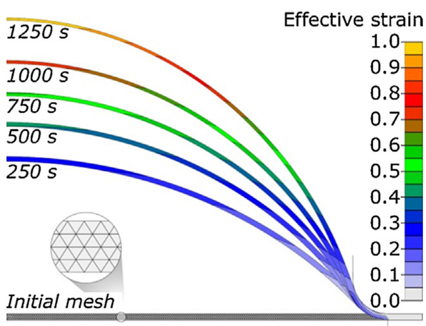

Fig. 6. Initial mesh and evolution of dome geometry obtained by FEM using Smirnov constitutive model for $P_3 = 0.4$ MPa.

experimental values $H(t)$ and ones predicted by FEM are illustrated. Fig. 8 illustrates the $s(t)$ relations for the same pressures. It can be seen that the greatest differences between the results obtained for Backofen constitutive equation and Smirnov one occur at the pressures $P_1 = 0.3$ and $P_5 = 0.6$ MPa. At the other pressures the simulation results are almost identical. The simulation results are in good correspondence with the experimental ones concerning the prediction of $H(t)$. The $s(t)$ obtained by simulation are lower than experimental ones. So the thickness distribution in the real process appears to be more uniform that in the model. This may be partially explained by the fact that FE simulation was performed with neglecting of slip between the die and the material which is apparently occurred in real forming process.

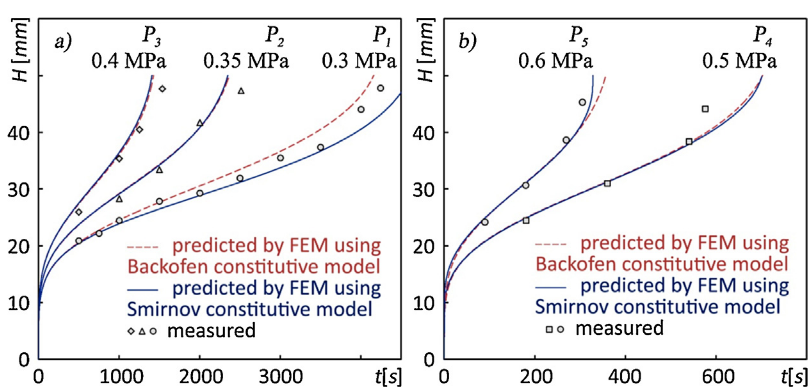

Fig. 7. Evolutions of dome height predicted by FE simulations based on Backofen (dashed line) and Smirnov (solid line) constitutive equations compared with measured values.

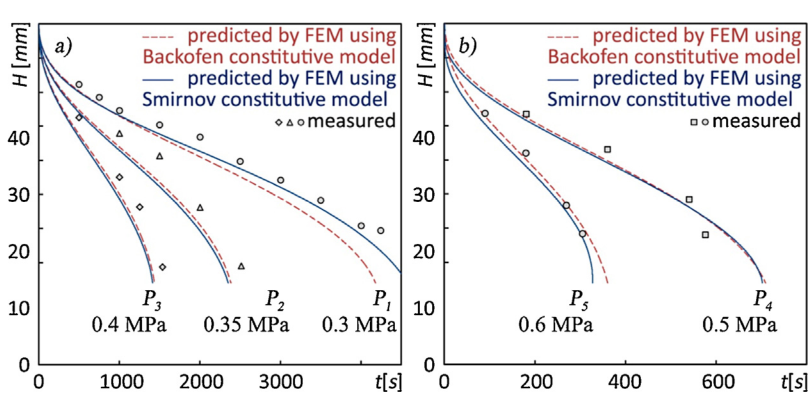

Fig. 8. Evolutions of thickness at the dome apex predicted by FE simulations based on Backofen (dashed line) and Smirnov (solid line) constitutive equations compared with measured values.

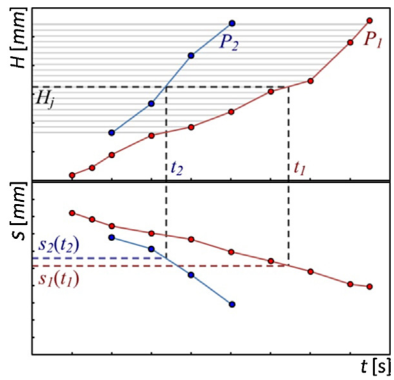

Fig. 9. Linear interpolation of the experimental data and calculation of t_1, t_2, $s_1(t_1)$ and $s_2(t_2)$ for given P_1, P_2 and H_j.

4.3. Variation of m with strain rate

The essential feature of Smirnov constitutive model is the ability to predict the variation of strain rate sensitivity index m with strain rate. Once the material constants σ_0, σ_s, K_v and m_v are determined, the $m(\dot{\varepsilon}_e)$ variation and the optimal strain rate can be evaluated using the Eqs. (4) and (5).

In order to verify the constitutive equations obtained in this work, a method of a strain rate sensitivity evaluation proposed by Enikeev and Kruglov (1995) was applied. This method was developed for the experimental equipment which is capable to determine the moment of contact between the dome apex and the die. Thus, for two different pressures P_1 and P_2 the forming times t_1 and t_2 are known which lead to the same dome height H. Assuming that m is a constant, its value can be found as:

$$m = \frac{\ln(P_1/P_2)}{\ln(t_2(H)/t_1(H))}. \tag{24}$$

Corresponding strain rate value can be estimated as follows:

$$\dot{\varepsilon}_e = \frac{1}{2}\left(\frac{\ln(s_0/(s_1(t_1)))}{t_1} + \frac{\ln(s_0/(s_2(t_2)))}{t_2}\right). \tag{25}$$

In order to apply this method, the experimental values $H_i^{\exp}$ and $s_i^{\exp}$ obtained for the forming time $t_i^{\exp}$ were interpolated by piecewise linear functions. Then, for every pair of successive pressures the values of m and $\dot{\varepsilon}_e$ were calculated by Eqs. (24) and (25) for a number of heights H_j as shown in Fig. 9. In Fig. 10 the comparison between obtained data is presented.

It can be seen from Fig. 10 that the data obtained by proposed technique are in good correspondence with the one obtained by Eq. (24). The dark blue line in Fig. 10 corresponds to variation of strain rate sensitivity m obtained by Smirnov constitutive model

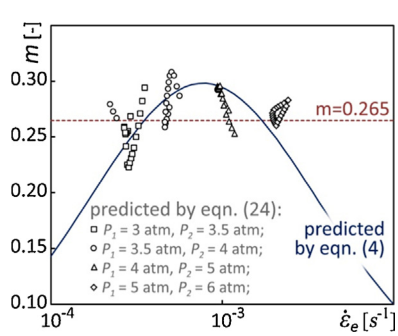

Fig. 10. Variations of strain rate sensitivity m with strain rate $\dot{\varepsilon}_e$ obtained by proposed method and by Enikeev equation.

with the rheological parameters given in Section 4.1. The maximum of this curve corresponds to the optimal strain rate which value can be found using Eq. (5) as $\dot{\varepsilon}_{\text{opt}} = 7.74 \times 10^{-4}\ \text{s}^{-1}$. The maximum strain rate sensitivity value can be determined by Eq. (4) as $m(\dot{\varepsilon}_{opt}) = 0.298$.

5. Conclusions

A free bulging process was considered. Several approaches of semi-analytical simulation of the bulging process were discussed. A special engineering model oriented to utilization in inverse analysis of free bulging tests is constructed. The model is capable to predict the evolution of dome height (H) on forming time (t) taking into account experimentally determined apex thickness (s) related to dome height and a nonzero die entry radius.

A new technique which enables one to determine the Smirnov constitutive equation constants as well as Backofen ones on the basis of free bulging tests is proposed. The proposed technique is based on inverse analysis and consists of two steps. First, the proper approximation is constructed to determine $s(H)$ relation for each pressure. Then, the material constants are obtained by least square minimization of the deviations between the experimental relations of $H(t)$ and $s(t)$ and ones predicted by the engineering model.

The proposed technique was used for characterization of AMg6 aluminum alloy. The obtained material constants were verified by FE simulation and by comparison with estimations made by the equation proposed in Enikeev and Kruglov (1995) work. The optimum strain rate was estimated for the investigated material to be $7.74 \times 10^{-4}\ \text{s}^{-1}$. At this strain rate it almost reaches the superplastic state, having the strain rate sensitivity index $m_{\max} = 0.298$.

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References

Abu-Farha, F., Khraisheh, M., 2007. Mechanical characteristics of superplastic deformation of AZ31 magnesium alloy. J. Mater. Eng. Perform. 16 (2), 192–199.
Aksenov, S.A., Chumachenko, E.N., Logashina, I.V., 2013. Tensile testing of Ti-6Al-4V alloy superplasticity. Mater. Sci. Forum 762, 392–397.
Albakri, M., Abu-Farha, F., Khraisheh, M., 2013. A new combined experimental–numerical approach to evaluate formability of rate dependent materials. Int. J. Mech. Sci. 66, 55–66.
Aoura, Y., Ollivier, D., Ambari, A., Dal Santo, P., 2004. Determination of material parameters for 7475 Al alloy from bulge forming tests at constant stress. J. Mater. Process. Technol. 145, 352–359.
Backofen, W.A., Turner, I.R., Avery, D.H., 1964. Superplasticity in an Al–Zn alloy. Trans. ASM 57 (4), 980–990.
Cheng, J.H., 1996. The determination of material parameters from superplastic inflation tests. J. Mater. Process. Technol. 58, 233–246.
Chumachenko, E.N., Portnoi, V.K., Paris, L., Billaudeau, T., 2005. Analysis of the SPF of a titanium alloy at lower temperatures. J. Mater. Process. Technol. 170, 448–456.
Enikeev, F.U., Kruglov, A.A., 1995. An analysis of the superplastic forming of a thin circular diaphragm. Int. J. Mech. Sci. 37 (5), 473–483.
Giuliano, G., 2008. Constitutive equation for superplastic Ti–6Al–4V alloy. Mater. Des. 29, 1330–1333.
Giuliano, G., 2012. Constitutive modelling of superplastic AA-5083. Tech. Mech. 32 (2–5), 221–226.
Giuliano, G., Franchitti, S., 2007. On the evaluation of superplastic characteristics using the finite element method. Int. J. Mach. Tools Manuf. 47, 471–476.
Guo, Z.X., Ridley, N., 1989. Modelling of superplastic bulge forming of domes. Mater. Sci. Eng. A 114, 97–104.
Hart, E.W., 1967. Theory of tensile test. Acta Metall. 15, 351–353.
Hill, R., 1950. A theory of the plastic bulging of a metal diaphragm by lateral pressure. Philos. Mag. 41, 1133–1143.
Holt, D.L., 1970. An analysis of the bulging of a superplastic sheet by lateral pressure. Int. J. Mech. Sci. 12, 491–497.

Jovane, F., 1968. An approximate analysis of the superplastic forming of a thin circular diaphragm: theory and experiments. Int. J. Mech. Sci. 10, 403–424.
Khraisheh, M.K., Abu-Farha, F.K., Nazzal, M.A., Weinmann, K.J., 2006. Combined mechanics-materials based optimization of superplastic forming of magnesium AZ31 alloy. CIRP Ann.: Manuf. Technol. 55 (1), 233–236.
Li, G.Y., Tan, M.J., Liew, K.M., 2004. Three-dimensional modeling and simulation of superplastic forming. J. Mater. Process. Technol. 150, 76–83.
Nelder, J.A., Mead, R., 1965. A simplex method for function minimization. Comput. J. 7, 308–313.
Smirnov, O.M., 1979. Metal Working in a Superplastic State. Mashinostroyenie, Moscow (in Russian).
Taleff, E.M., Hector Jr., L.G., Verma, R., Krajewski, P.E., Chang, J.K., 2010. Material models for simulation of superplastic Mg alloy sheet forming. J. Mater. Eng. Perform. 19 (4), 488–494.
Vasin, R.A., Enikeef, F.U., Mazurski, M.I., Munirova, O.S., 2000. Mechanical modelling of the universal superplastic curve. J. Mater. Sci. 35, 2455–2466.
Woo, D.M., 1964. The analysis of axisymmetric forming of sheet metal and the hydrostatic bulging process. Int. J. Mech. Sci. 6, 303–317.
Yang, H.S., Mukherjee, A.K., 1992. An analysis of the superplastic forming of a circular sheet diaphragm. Int. J. Mech. Sci. 34 (4), 283–297.
Yu-Quan, S., Jun, Z., 1986. A mechanical analysis of the superplastic free bulging of metal sheet. Mater. Sci. Eng. 84, 111–125.