Guidance and control of vehicle

Assignment 2

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1 Open - Loop Analysis

1.1 Some features without wind

In the absence of wind, we have :

- $V_g = V_a = 552km/h$
- $\beta = -\psi$
- $\beta = arcsin(\frac{v}{V_a})$

$$\beta = arcsin(\frac{v}{V_a}) \tag{1}$$
$$= arcsin(\frac{M * A}{V_a}) \tag{2}$$
$$= arcsin(\frac{0.45 * 340}{552/3.6}) \tag{3}$$
$$= 0.99782 \tag{4}$$

1.2 Dutch roll mode:

We use the model in the book.

$$\begin{pmatrix} \dot{\beta} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} -0.322 & -0.9917 \\ 8.5396 & 0.6646 \end{pmatrix} \begin{pmatrix} \beta \\ r \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha_a^c \\ \alpha_r^c \end{pmatrix} \tag{5}$$

$$\lambda_{dutchroll} = \frac{Y_v + N_r}{2} \pm \sqrt{\frac{Y_v + N_r}{2}^2 - Y_v N_r + N_v Y_r} \tag{6}$$

With the function damp on matlab we can obtain this table:

Pole	Damping	Frequency	Time Constant
1.71e-01 + 2.87i	-5.96e-02	2.87	-5.84
1.71e-01 - 2.87i	-5.96e-02	2.87	-5.84

Dutch roll is a series of out-of-phase turns, when the aircraft rolls in one direction and yaws in the other. In fact, when the aircraft starts to move on the right, a sideslip motion appears. This sideslip generates a roll motion on the right. To correct this, a roll motion and a yaw motion are generated on the opposite side to cancel sideslip. Then, because the directional stability lags significantly behind the lateral stability, when the roll motion is corrected, the yaw motion continues. So the aircraft has over-banked to the left, causing a left sideslip. Then, the process is reversed and starts again. It is an oscillatory motion.

1.3 Spiral-divergence mode:

$$\lambda_{spiral} = \frac{L_v N_r - N_v L_r}{L_v} \tag{7}$$

We know $\dot{\overline{r}} = \lambda_{spiral} * \overline{r} + ... * \overline{\lambda_a}$
We can deduce of the matrix A, this result: $\lambda_{spiral} = -0.4764$

The root is a negative real. Our system with spiral mode is stable.

1.4 Rolling mode:

$$\lambda_{rolling} = L_p = -3,6784 \tag{8}$$

The root is a negative real. Our system with rolling mode is stable but it is also faster than with the spiral mode because his root is closest of 0.

1.5 Open loop mode / close loop mode

The open loop mode is useful to study features like margins, stability and root locus method. The close loop mode is useful to study the response time and the bode. The open loop allow make a first study and have an idea of range value of control gains. Without this, we are blind to choose this value with only the response time.

2 Autopilot for course hold using aileron and successive loop closure

2.1 Compute a_{ϕ_1} and a_{ϕ_2}

$$\frac{p}{\lambda_a} = \frac{a_{\phi_2}}{s + a_{\phi_1}} \tag{9}$$

The matrix A gives us the second equation. After, we just need make an identification.

$$\dot{p} = -a_{\phi_1} * p + a_{\phi_2} * \lambda_a \tag{10}$$

$$\dot{p} = -30.6492 * \beta - 3.6784 * p + 0.6646 * r - 0.7333 * \lambda_a + 0.1315 * \lambda_r \tag{11}$$

We can conclude that $a_{\phi_1} = 3.6784$ and $a_{\phi_2} = -0.7333$

$$\frac{p}{\lambda_a} = \frac{-0.7333}{s + 3.6784} \tag{12}$$

2.2 Compute gains of control structure

2.2.1 Inner loop: H_{ϕ/ϕ_c}

We find this in the book. It misses only the k_{i_ϕ}. We compute this gain with the root locus method on Matlab.

$$k_{p_\phi} = \frac{\delta_a^{max}}{e_\phi^{max}} * sign(a_{\phi_2}) \tag{13}$$

$$w_{n_\phi} = \sqrt{\frac{|a_{\phi_2}|\delta_a^{max}}{e_\phi^{max}}} \tag{14}$$

$$k_{d_\phi} = 2 * \frac{\zeta_\phi w_{n_\phi} - a_{\phi_1}}{a_{\phi_2}} \tag{15}$$

$$\tag{16}$$

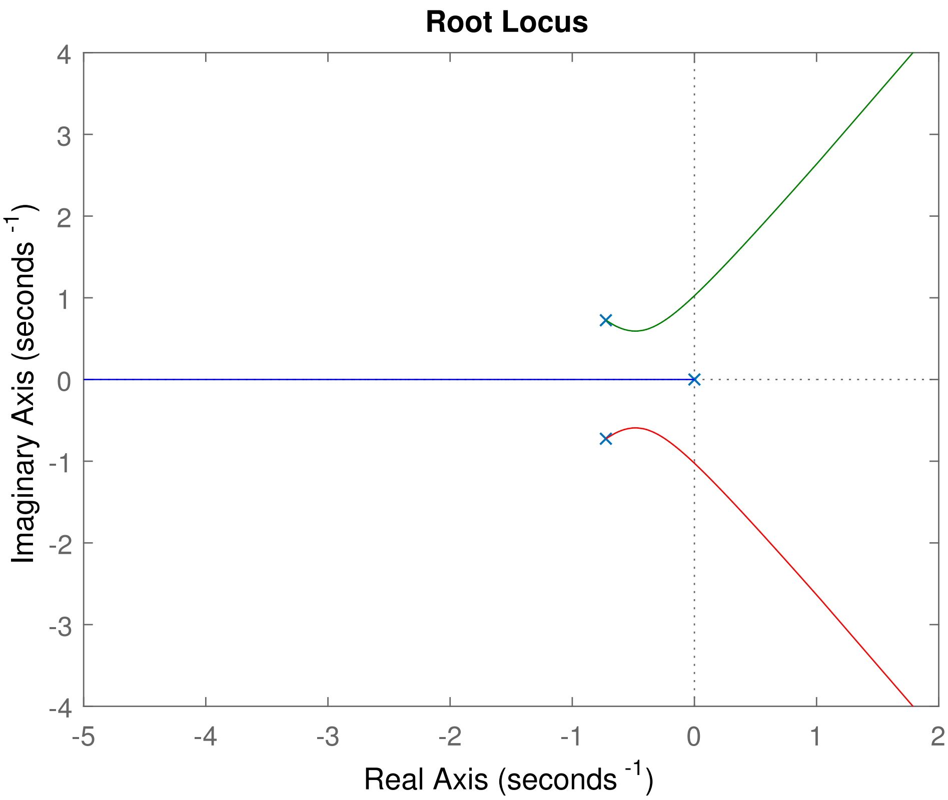

Figure 1: Root locus method on the open loop H_{ϕ/ϕ_c}

We see on this figure, k_{i_ϕ} need to be between -2 and 2.(we take gains writen for each points on the negative part of roots)

In the book, we find the equation of closed loop H_{ϕ/ϕ_c}

$$H_{\phi/\phi_c} = \frac{a_{\phi_2} k_{p_\phi}(s + \dfrac{k_{i_\phi}}{k_{p_\phi}})}{s^3 + (a_{\phi_1} + a_{\phi_2} k_{d_\phi})s^2 + a_{\phi_2} k_{p_\phi} s + a_{\phi_2} k_{i_\phi}} \tag{17}$$

$$= \frac{\dfrac{a_{\phi_2} k_{p_\phi}(s + \dfrac{k_{i_\phi}}{k_{p_\phi}})}{a_{\phi_2} k_{i_\phi}}}{\dfrac{s^3}{a_{\phi_2} k_{i_\phi}} + \dfrac{(a_{\phi_1} + a_{\phi_2} k_{d_\phi})}{a_{\phi_2} k_{i_\phi}} s^2 + s + 1} \tag{18}$$

We evaluate the step of the close loop on matlab to find the good value of k_{i_ϕ}

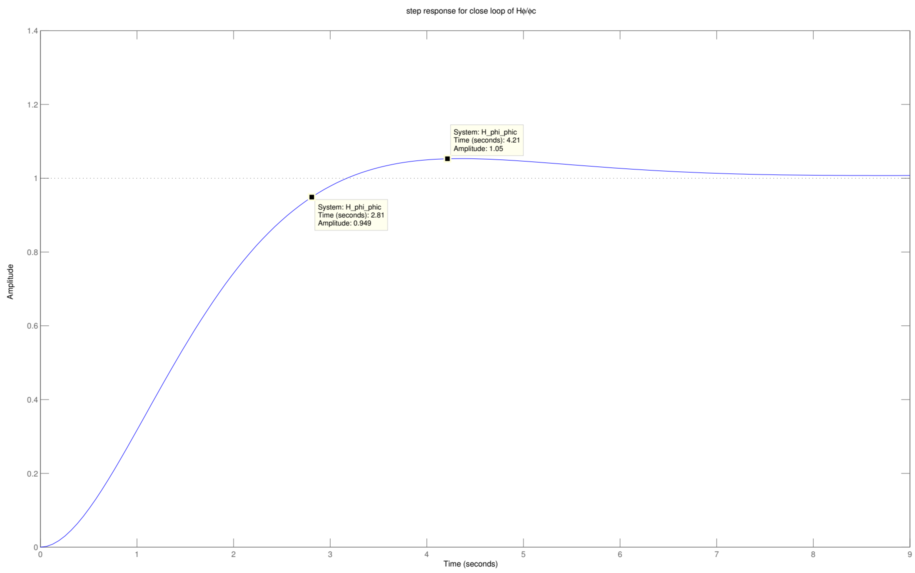

Figure 2: Step response of the feedback inner loop H_{ϕ/ϕ_c}

Finally, we choose $K_{i_\phi} = 0.01$. Our system has a response time of 2.81s and a overshoot of 5% . If we remove the integration effect, the overshoot is remove but the response time became more important. In our case, the overshoot isn't too important so we prefer keep the integration.

2.2.2 Full system: H_{χ/χ_c}

Now, we want find coefficient of the control structure H_{χ/χ_c}

In first, we compute the open loop of system (without the disturbance)

$$H_{\chi/\chi_{c open}} = (k_{p_\chi} + \frac{k_{i_\chi}}{s})H_{\phi/\phi_c} * \frac{V_g}{g * s} \tag{19}$$

$$= \frac{a_{\phi_2} k_{p_\chi} k_{p_\phi}(s + \frac{k_{i_\phi}}{k_{p_\phi}})(s + \frac{k_{i_\chi}}{K_{p_\chi}}) * \frac{V_g}{g}}{s^3 + (a_{\phi_1} + a_{\phi_2} k_{d_\phi})s^2 + a_{\phi_2} k_{p_\phi} s + a_{\phi_2} k_{i_\phi}} \tag{20}$$

Now, we can compute the close loop of system

$$
H_{\chi/\chi_c} = \frac{H_{\chi/\chi_{c open}}}{1 + H_{\chi/\chi_{c open}}} \tag{21}
$$

$$
= \frac{\dfrac{(s + \dfrac{k_{i_\phi}}{k_{p_\phi}})(s + \dfrac{k_{i_\chi}}{k_{p_\chi}})}{\dfrac{1}{k_{p_\chi}} + \dfrac{k_{i_\chi}}{k_{p_\chi}} + \dfrac{k_{i_\phi}}{k_{p_\phi}}}}{\dfrac{g}{V_g * a_{\phi 2} k_{p_\phi}(1 + k_{i_\chi} + k_{p_\chi}\dfrac{k_{i_\phi}}{k_{p_\phi}})} s^3 + (a_{\phi_1} + \dfrac{k_{d_\phi} + k_{p_\chi} k_{p_\phi}}{k_{p_\phi}(1 + k_{i_\chi} + k_{p_\chi}\dfrac{k_{i_\phi}}{k_{p_\phi}})}) s^2 + s + 1} \tag{22}
$$

$$
w_{n_\chi} = \frac{w_{n_\phi}}{W_\chi} \tag{23}
$$

$$
k_{i_\chi} = \frac{w_{n_\chi}^2 V_g}{g} \tag{24}
$$

$$
k_{p_\chi} = 2 * \frac{\zeta_\chi w_{n_\chi} V_g}{g} \tag{25}
$$

We need fix W_χ and ζ_χ. For this, we use the step response of H_{χ/χ_c}

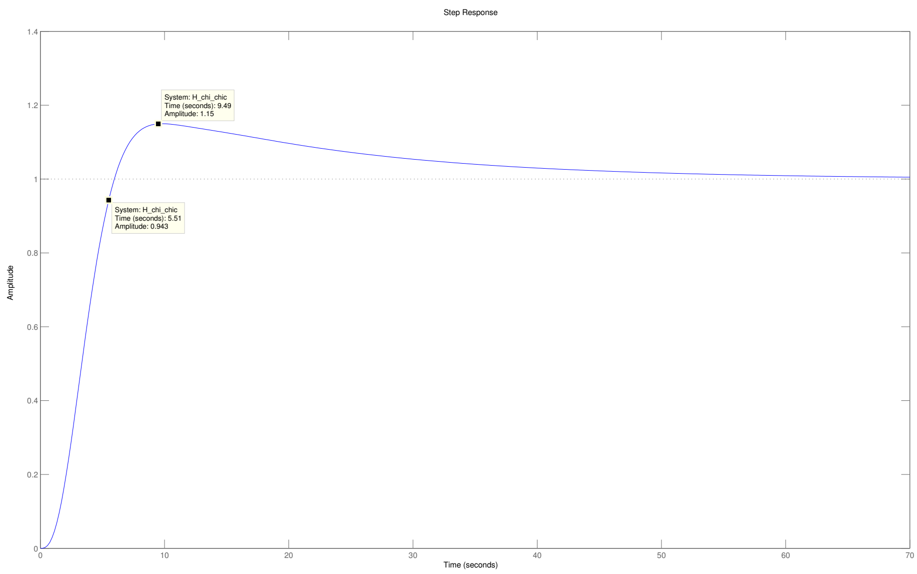

Figure 3: Step response of the feedback loop of system H_{χ/χ_c}

Finally, we choose

$$W_\chi = 9 \tag{26}$$
$$\zeta_\chi = 1.2 \tag{27}$$

Our global system has a response time of 5.51and a overshoot of 15%.

To ensure a sufficient bandwidth separation between the inner and outer feedback loops, with have to respect this condition: $W_\chi >> 1$ We see this problem very well on this figure. When we choose $W_\chi = 9$, our system is stable with

- $phase > -180$ at w_c
- $amplitude < 0dB$ at w_{180}

But if we choose $W_\chi = 9$, our natural pulsation are equal and the system is unstable.

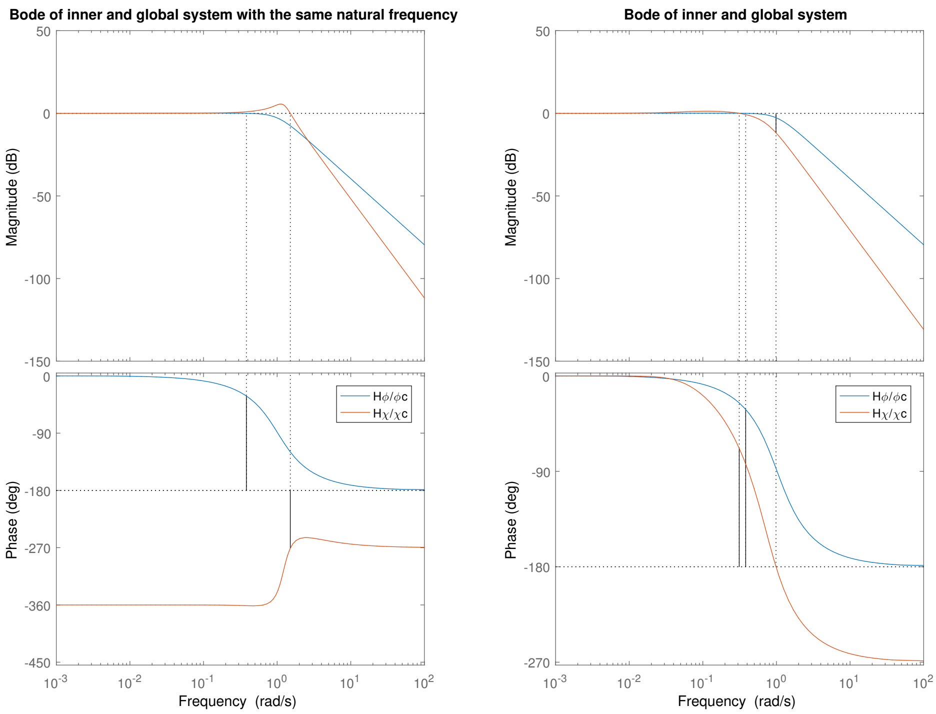

Figure 4: Bode of feedbacks loops

2.3 Simulation of simplified model

We choose two kinds of inputs to simulate the system. (Be careful: we need have $amplitudeofstep < 15$ degrees)

- series of normal step
- series of smoothing step

With the first series, we can study the response time and the overshoot easily but it isn't representative of the reality. It's why we use the second series.

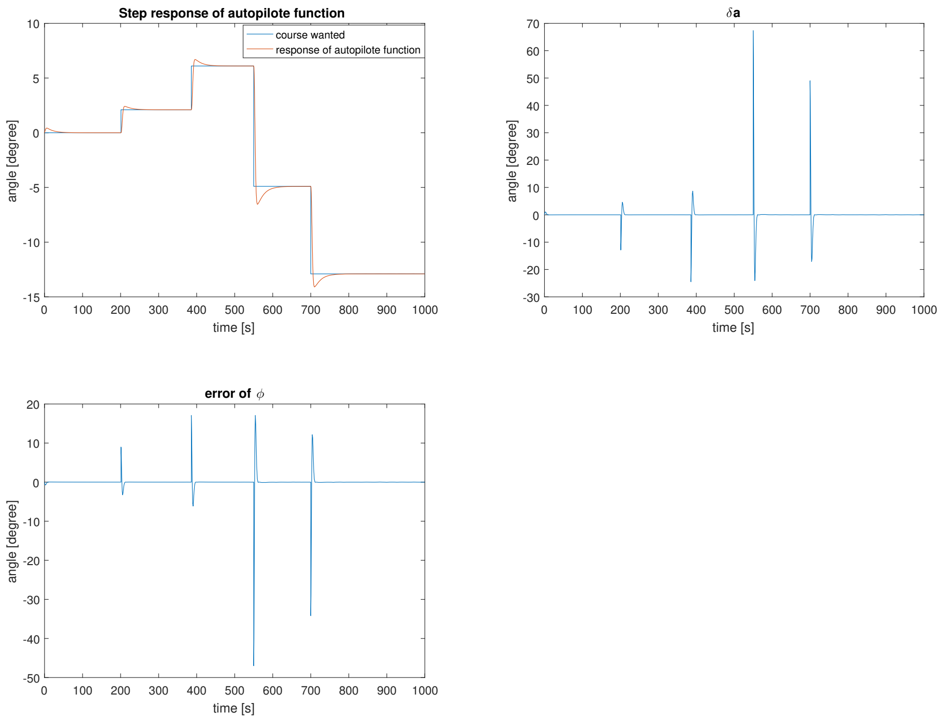

Figure 5: Simulation with the step serie

We can observe a good response time but there are important overshoots ($\approx 25\%$). For the angle of aileron δ_a, we are two times an wrong values upper at 21.5 degrees. It the same for the error of ϕ, we are two times an wrong values lower than -15 degrees. I think, the problem is due to steps which not representative of the reality because when we use the second serie the problem disappeared.

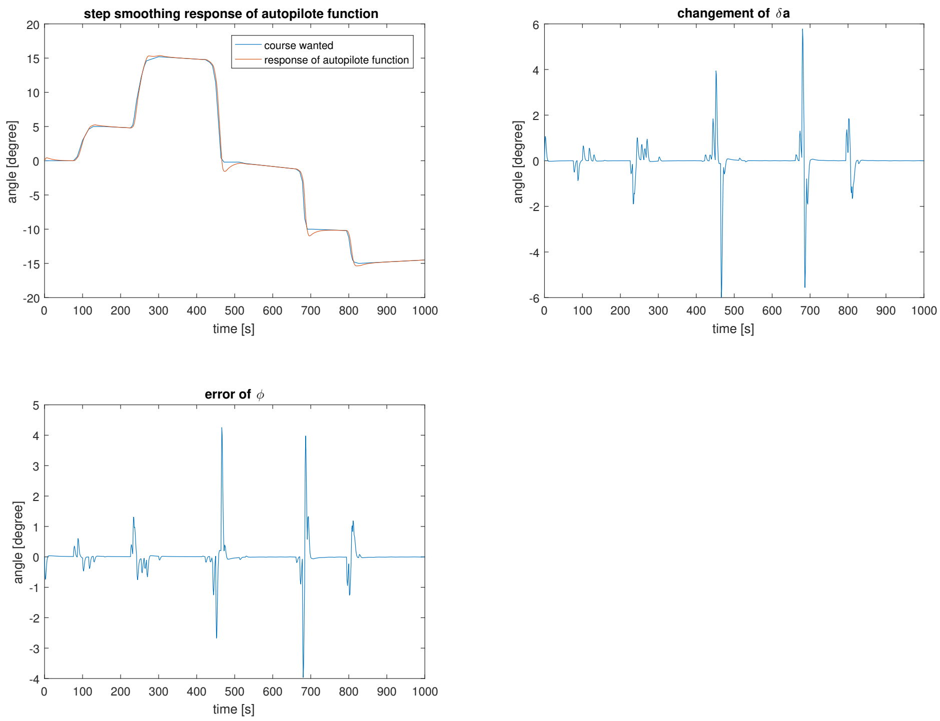

Figure 6: Simulation with the step smoothing serie

This time, we observe the same behavior for the response. The response of angle aileron and the error of ϕ are good values. For conclude this simulation, we have good result which can improve to make disappear the overshoot.

2.4 State space model

It's why, now we want observe results with the complete state space model. We add too the real coordinated turn equation (just $tan(\phi)$ instead of ϕ). We simulate with the same inputs using in the previous question.

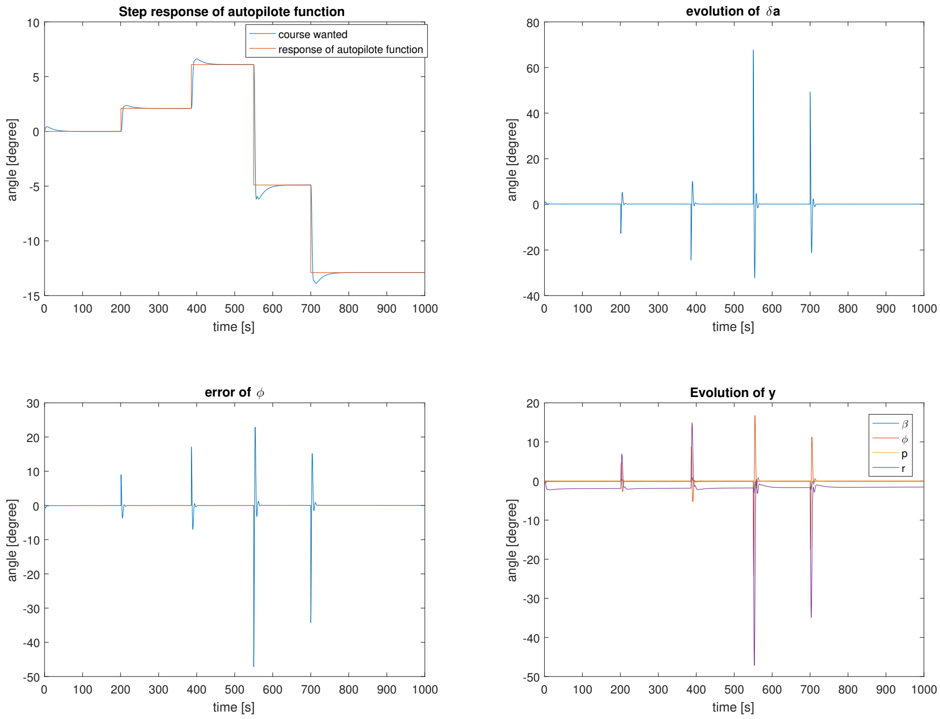

Figure 7: Simulation with the step serie

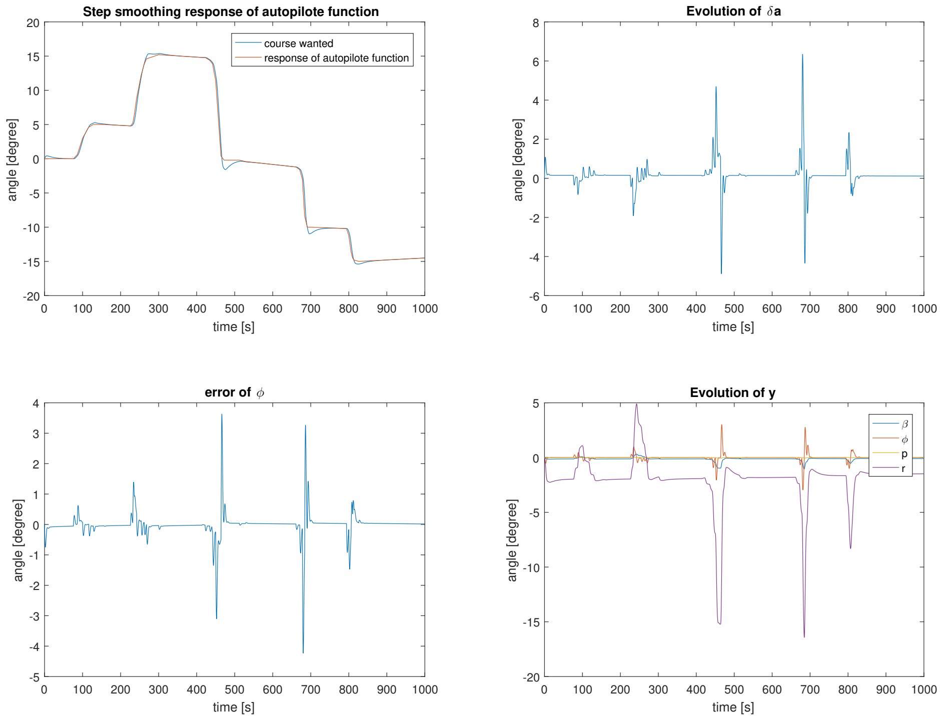

Figure 8: Simulation with the step smoothing serie

The response time and overshoots are improved with this model. The angle aileron and the error of ϕ aren't change. We can see the evolution of y. The output r (angle rudder) has an important variation. It seems interesting to work on this regulation. p and ϕ are an equivalent behavior because $\phi = \frac{p}{s}$

3 MIMO linear autopilot for course hold

In this part, we want implement the regulation of the angle rudder.

3.1 Compute the gain k_r

Like for the loop of ϕ, we use the root locus method on the open loop to obtain an range of value to compute k_r

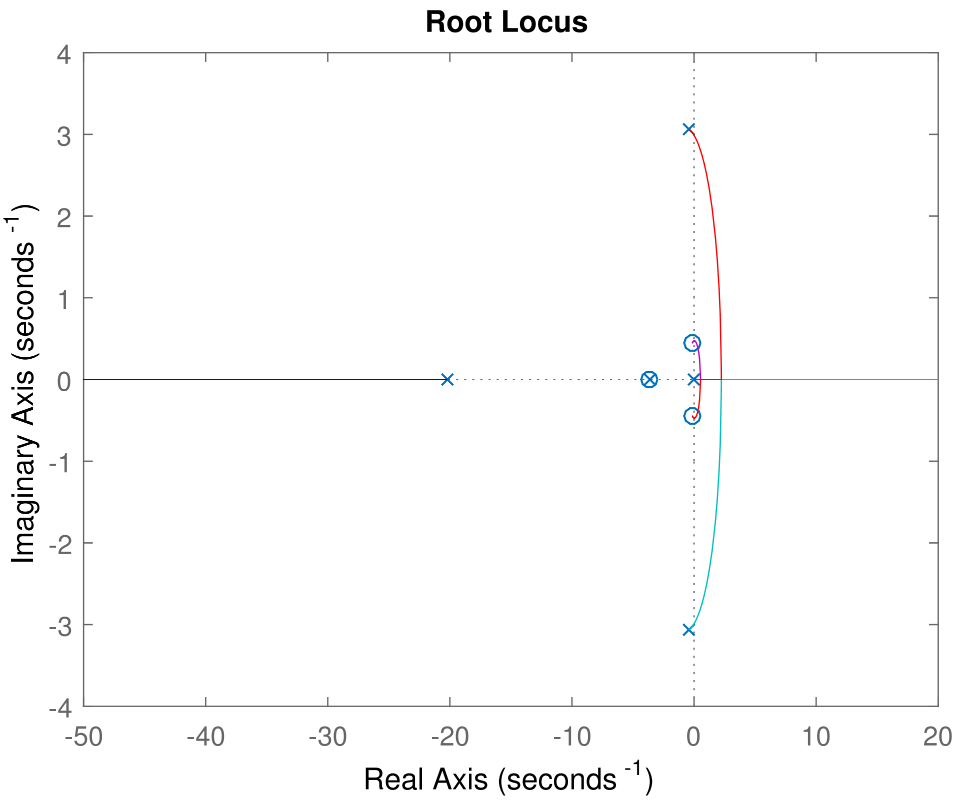

Figure 9: Root locus method on the open loop H_r

We see that k_r need to choose between -12 and 12. For the stability, we know that $k_r < 0$. The range value is $[-12; 0[$.

Finally, we take $k_r = -7$.

Like previous questions, we use same inputs to simulate the new model.

3.2 Step response of system with control of rupper

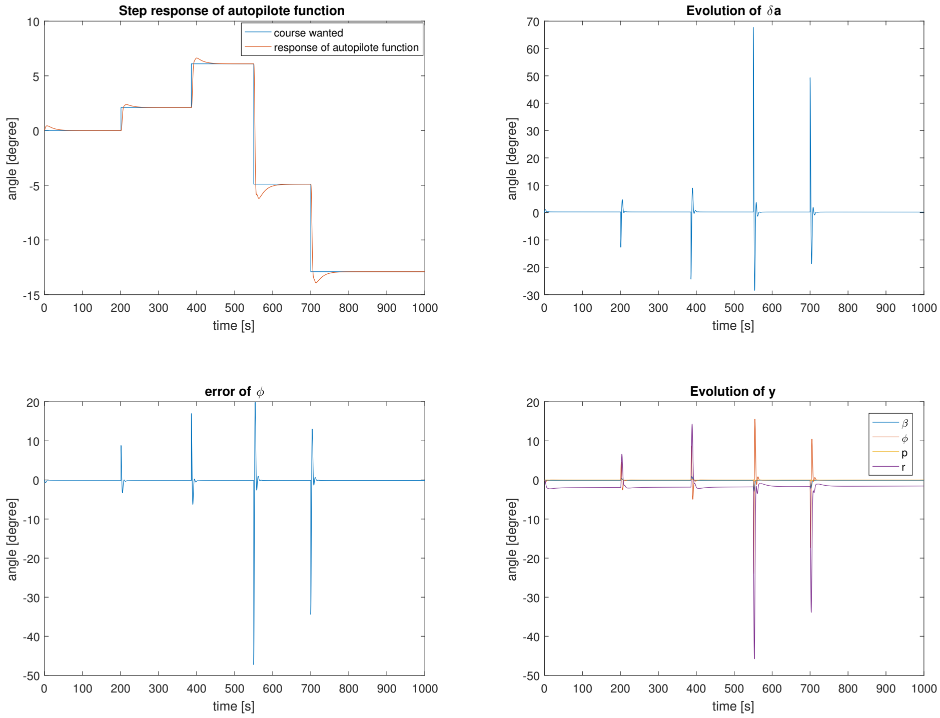

Figure 10: Simulation with the step serie

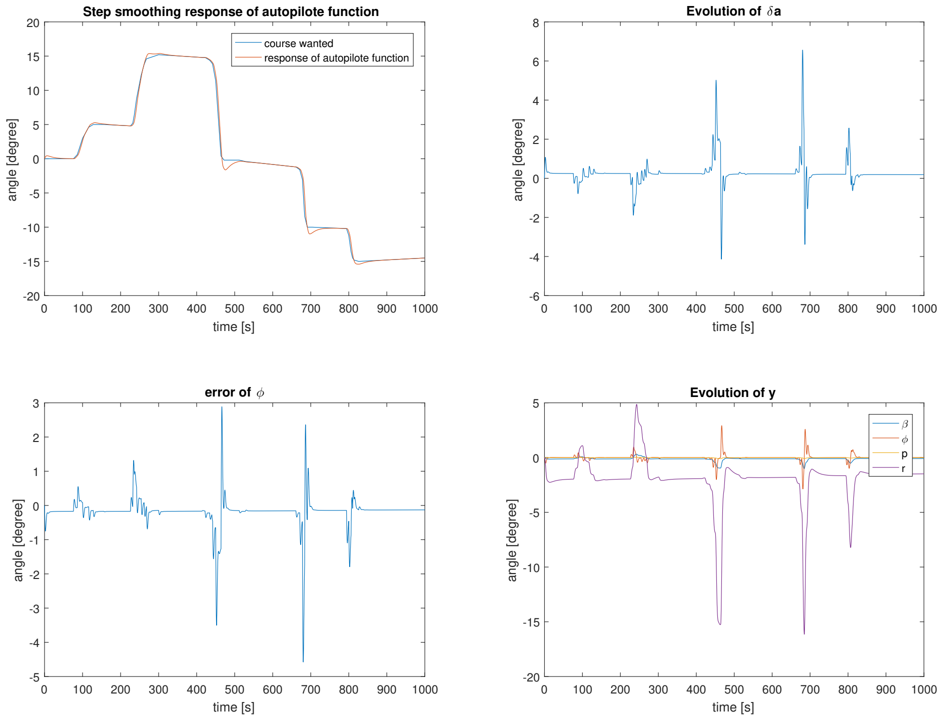

Figure 11: Simulation with the steps smoothing series

The response time and overshoots are improved with this model. The overshoot has almost disappeared on the second serie. We thinks, we can't remove the full overshoot on the first serie because the changement of angle are too suddendly and not representative of the reality. The angle aileron and the error of ϕ aren't change. But this time, we can see that the angle rudder is between the range value allowed $[-30; 30]degrees$. We can see the evolution of y.

3.3 Step response of system with washout filter

We add in this part a washout filter on the feedback of H_r. It modify the dynamic of our loop. So, we take now $k_r = -10$.

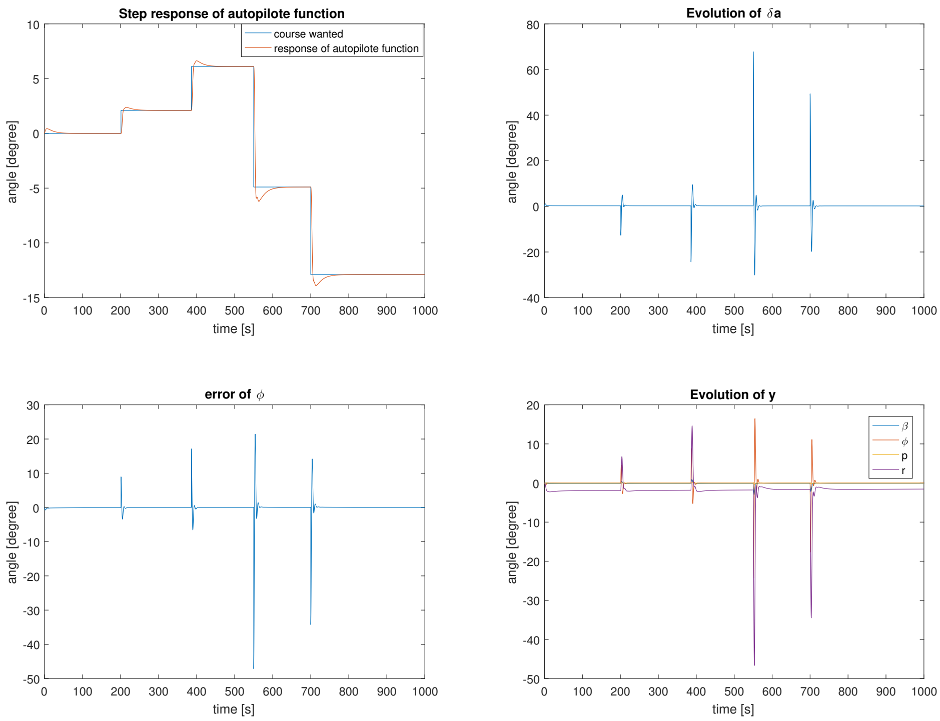

Figure 12: Simulation with the step serie

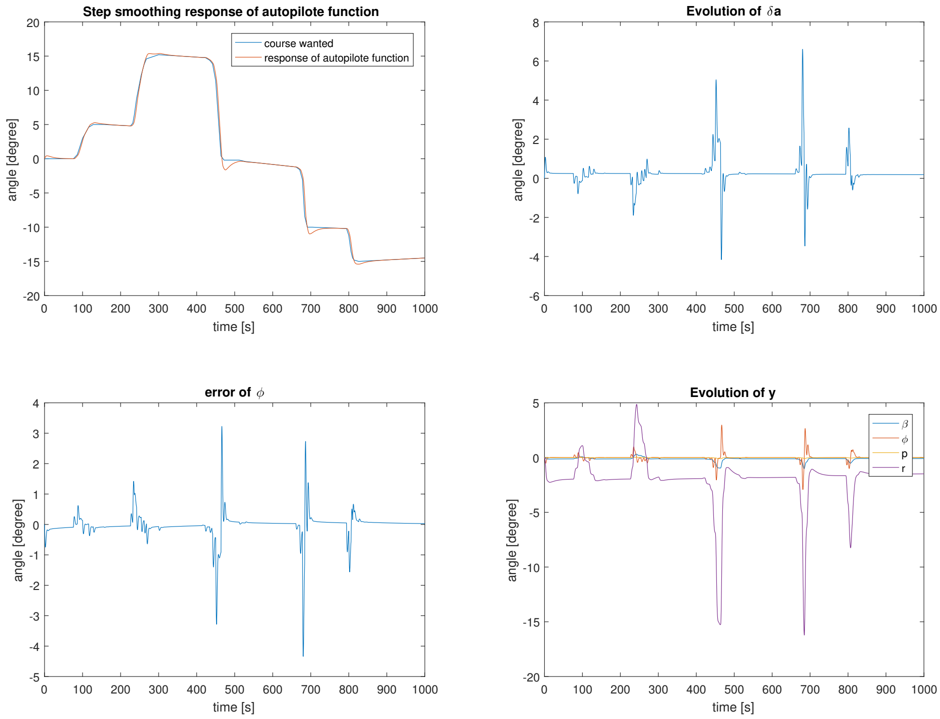

Figure 13: Simulation with the step smoothing serie

In theory, the washout filter remove the overshoot but not for our simulation. So, there are may be an error in the file simulation. But the most important, add the washoot filter for the first serie allow have finally good values for the error of ϕ and for the angle aileron.

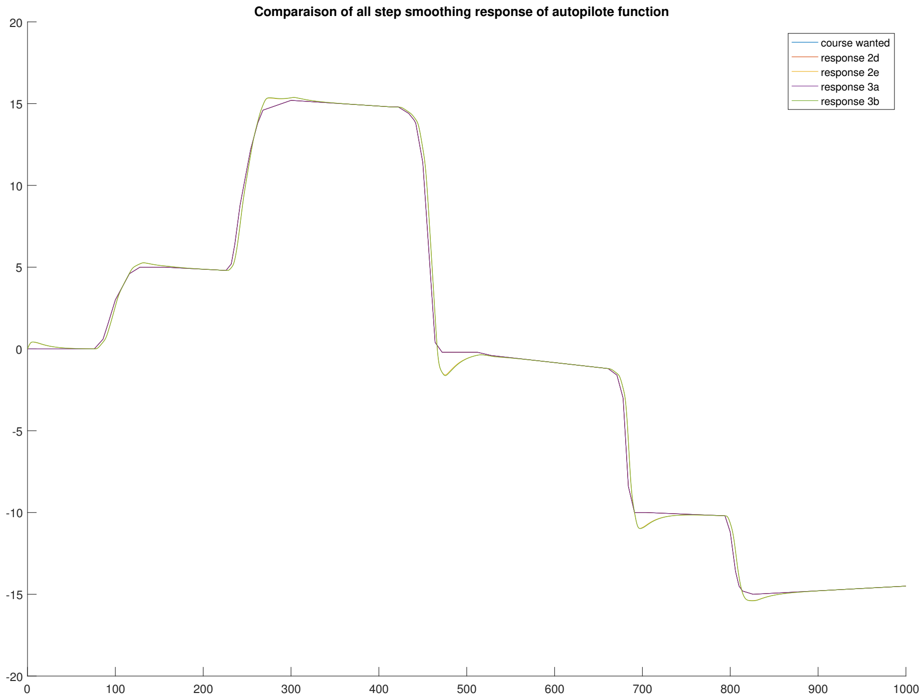

Figure 14: Comparison of all simulation with the second serie

We can see on this, each adding improve the result of our regulation (not for the last part but it seems there are a mistake in my model). At the final, we have a response system with a little overshoot, a fast response time and angle rudder, angle aileron and aileron wich respect features imposed.

We don't have problem of integration. So, we don't need make modification. But if we have this kind of problem due in the integration of too important value, we use a saturateur to keep our value in the logical range.