Biomedical data processing
Assignment 1—Time-domain filtering and notch filtering

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1 Introduction

In this document, the studied signal is a noisy ECG with a sample rate of 1000 Hz. The power spectrum of the ECG allows us to detect the presence of the power-line noise frequency 50 Hz and his harmonics in 150, 250, 350 and 450 Hz. Furthermore, as we can see in the time domain, the signal is also disturbed by a general high-frequency noise and a baseline drift due to a low-frequency noise.

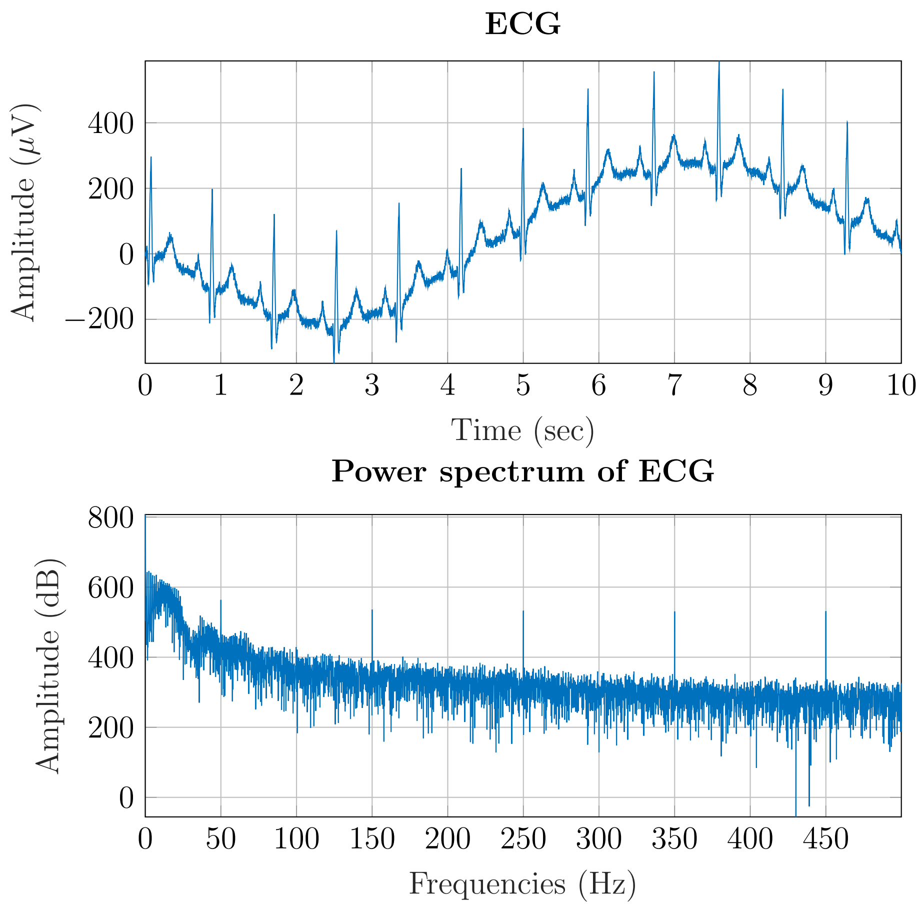

Figure 1: Representation of a noisy ECG

To remove the high/low/power-line frequency in an ECG, the assignment [1] asked to use 3 filters in serial:

- Hanning filter
- Derivative-based filter
- Comb filter

We will begin to study these filters one by one before combining them. We are aware that in the scientific literature, it exists other methods more efficient (with adaptive filters for example). But it is a good beginning to apply in real these filters seen during the lecture. Equations are all from the textbook [3].

2 Hanning Filter

The Hanning filter is the easiest form of moving average (MA) filter. We assume here that the signal is ergodic (which can be a huge assumption).

$$y(n) = \frac{1}{4}(x(n) + 2x(n-1) + x(n-2))$$

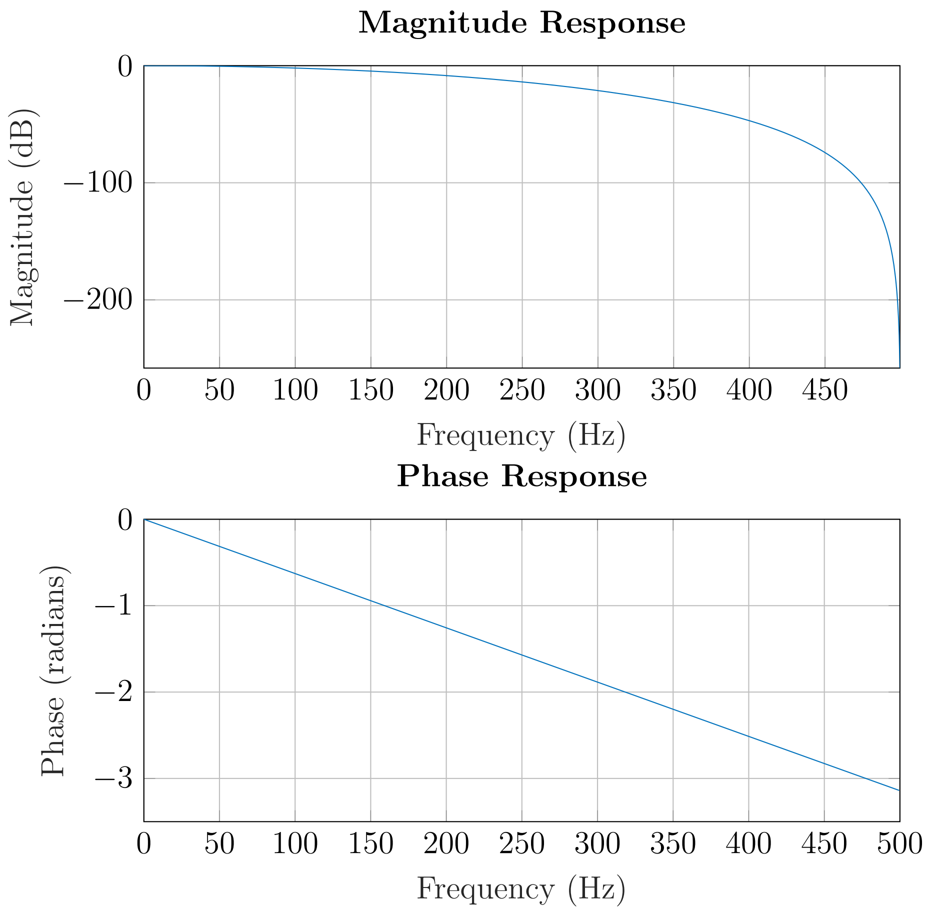

Figure 2: Transfer Function of the Hanning filter

The -3 dB cut-off is in 250 Hz. The signal at all frequencies after that are reduced. We can see this attenuation when we compare the power spectrum of the ECG filtered and the noisy ECG in the next figure.

Another point about this graph is, because of the linear phase, the filtered signal has a very low distortion (or delay). It is the simple way to make a low-pass filter.

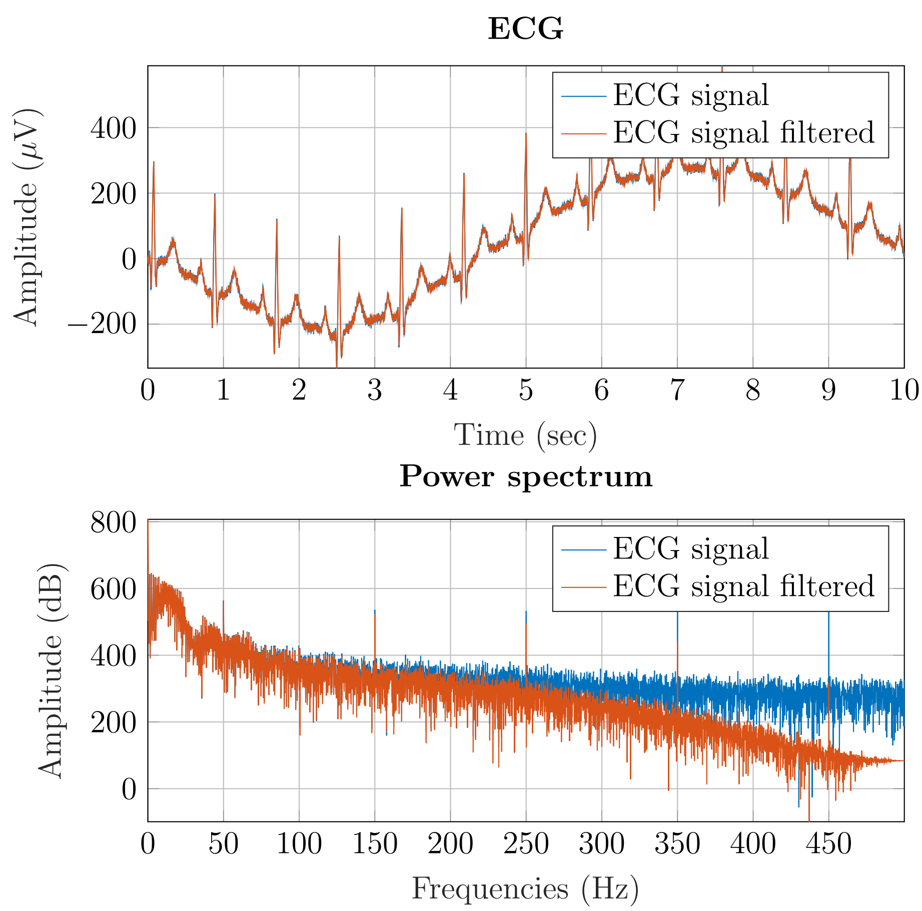

Figure 3: Comparative between noisy ECG and ECG filtered by the Hanning filter

3 Derivative-based filter

At the opposite, the derivative-based filter is a high pass filter.

$$H(z) = f_s(1 - z^{-1})$$

As the filter has a zero at $z = 1$, the DC component is removing which stopped the problem of drift baseline in the time domain.
But the signal is often smoothed and the slow P and the T wave are removed because of the bandwidth of the rejected band is too wide. It is why a pole is often placed at a radius r from the center of the unit circle. It will design a sharper derivative-based filter and improve amplitude response on the right side (because it is a high-pass filter) of the cut-off frequency.

$$H(z) = f_s \frac{(1 - z^{-1})}{1 - r * z^{-1}}$$

$$r = 1 - \frac{\Delta f}{f}$$

With Δf : the ideal bandwidth of the rejected band.

On the ECG's frequency graph, the low-frequency noise is included between 0 and 8 Hz. So, we want to have a 16 Hz bandwidth.
To confirm this, we tested it with 3 different radius :

- 0.984 which means a bandwidth equal to 16 Hz
- 0.95 which means a bandwidth equal to 50 Hz
- 0.90 which means a bandwidth equal to 100 Hz

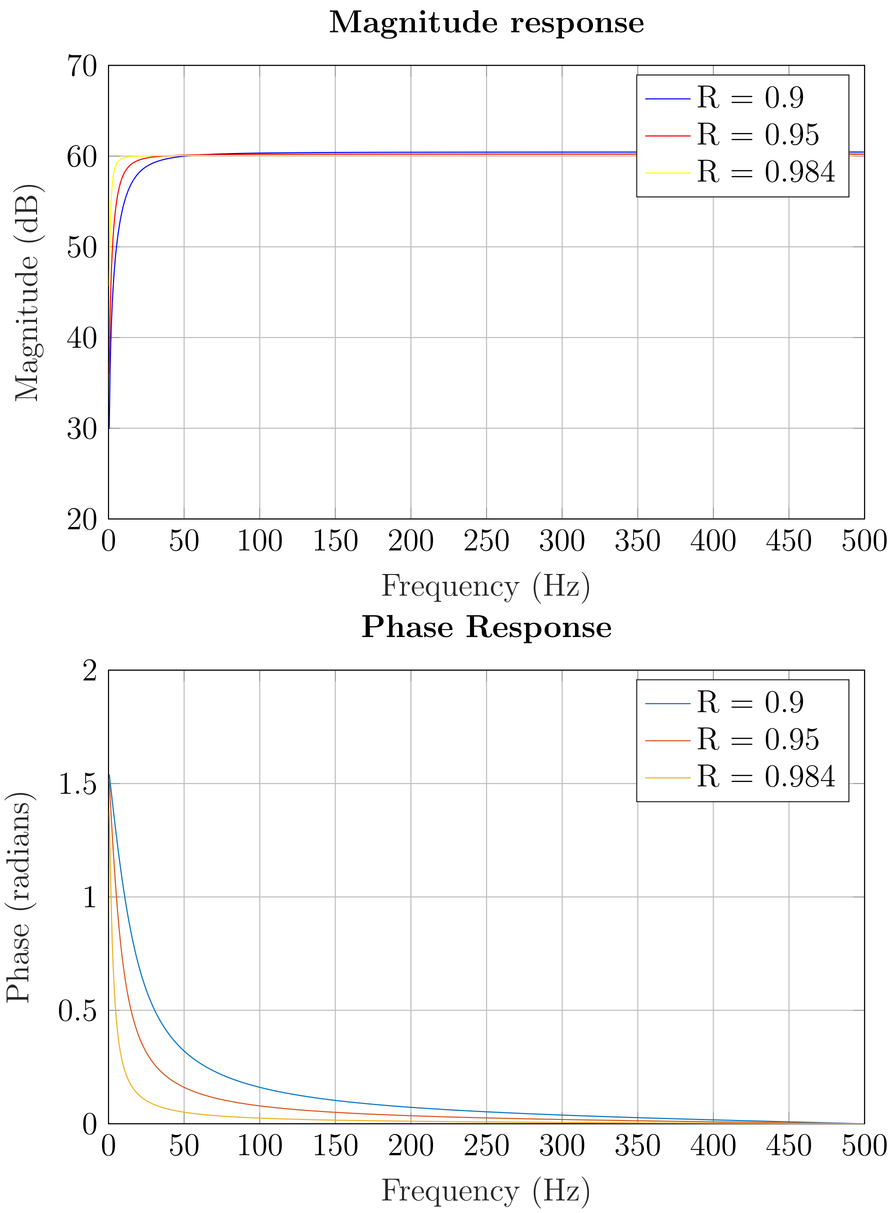

Figure 4: Transfer function of the Derivative-Based filter

By increasing the radius, the filter is sharper. If we take a value too small , we remove useful signals. But, in another side, a radius too high will increase the phase distortion in the signal (characterized by a delay in the signal and showed in the group delay plot at the end).

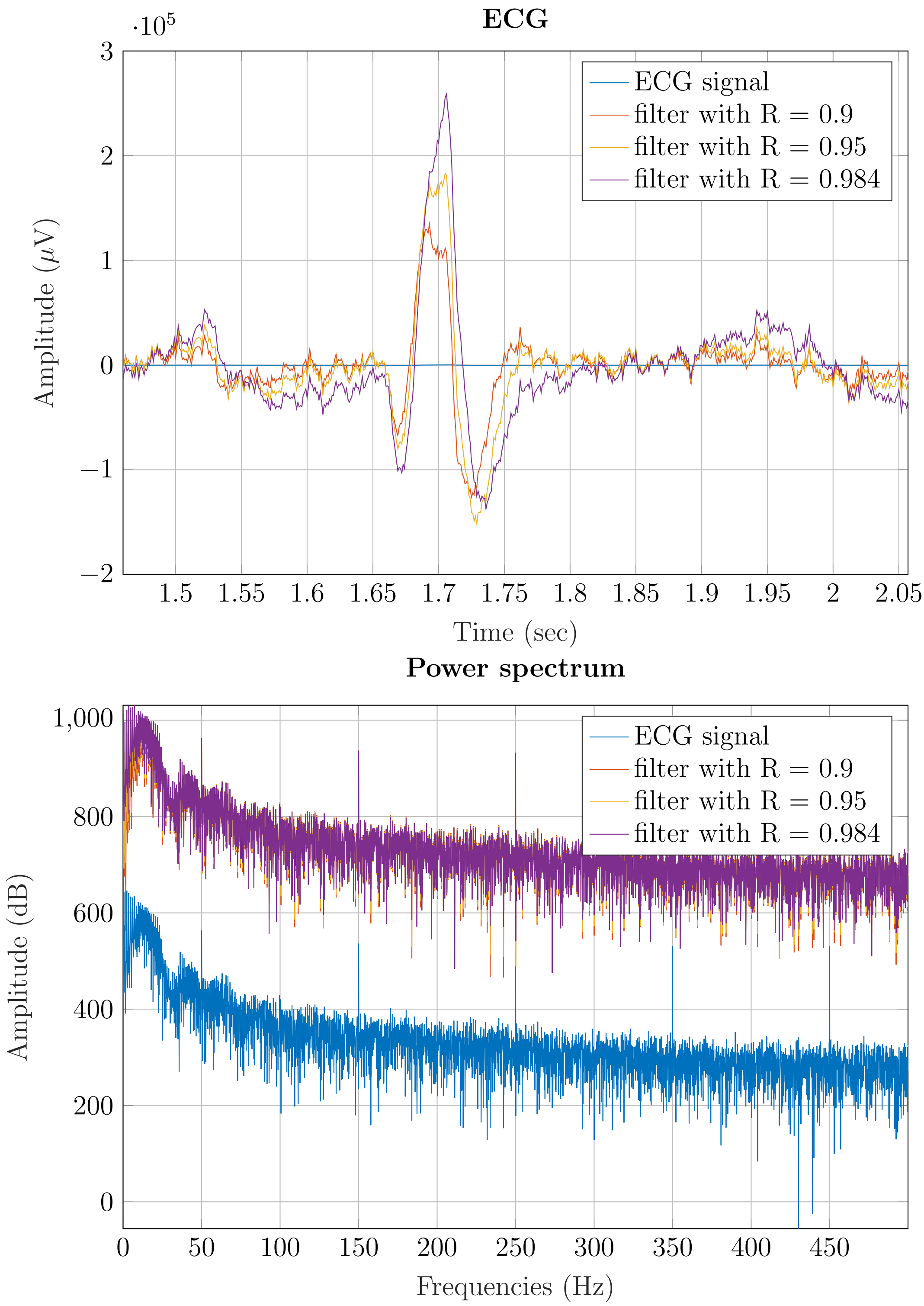

Figure 5: Comparative between noisy ECG and ECG filtered by Derivative-Based filter

4 Comb filter

As introduced before, we have a power-line noise at the frequency 50 Hz and his harmonics in 150, 250, 350 and 450 Hz. To remove it, we will place a zero at each of these frequencies. The method is very easy and explained in the book.

$$H(z) = G * \frac{(z+1)(z-z_1)(z-z_2)...(z-z_{10})}{z^{11}}$$

with

$$z_n = cos(2\pi\frac{f_n}{f_s}) \pm i * sin(2\pi\frac{f_n}{f_s})$$

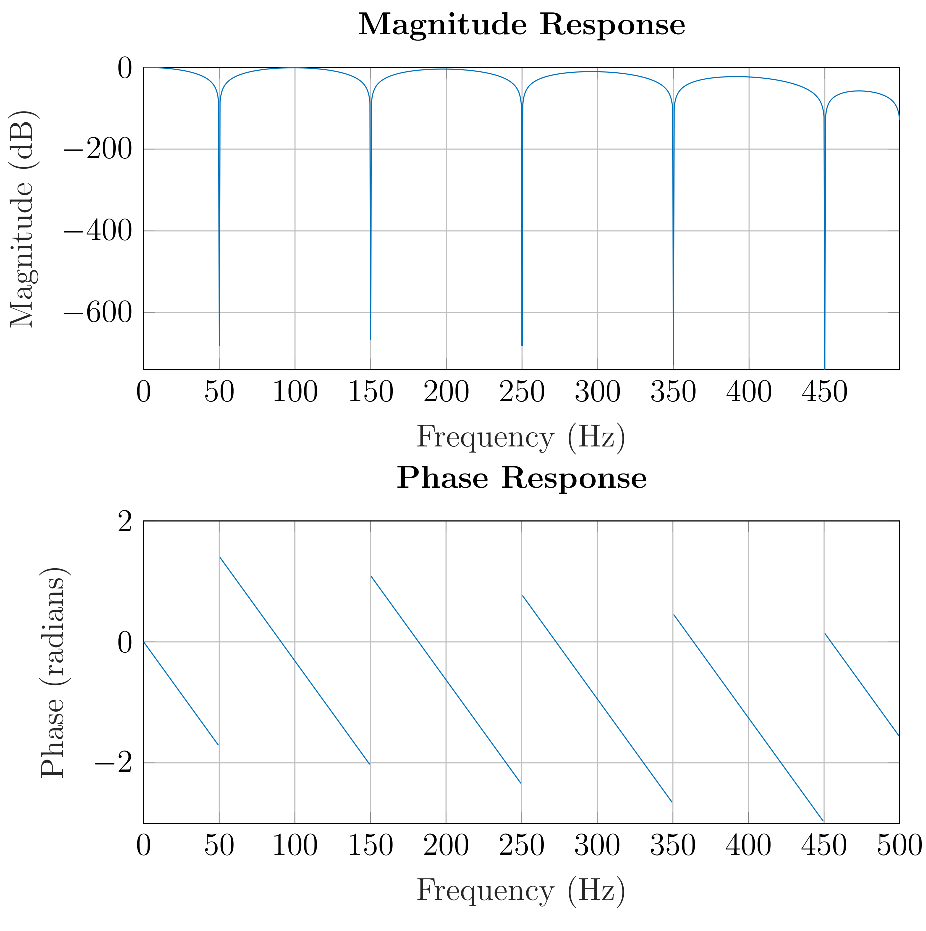

Figure 6: Transfer function of the Comb filter

As expected, chosen frequencies had been removed from the signal. The phase is non-linear which is specific to this kind of filter and bring a time delay in the filtered signal due to the phase distortion. Furthermore, the comb filter had been modified to add an extra zero placed in z = -1. A zero placed here mean that we want to cut the frequency $f_0 = \frac{f_s}{2}$ i.e. adds a low pass filter. The 3 dB cut-off of this low pass is in 113 Hz. In the same way as for the Hanning filter we see on the magnitude graph and the Power spectrum of the filtered ECG a slow decrease after this frequency.

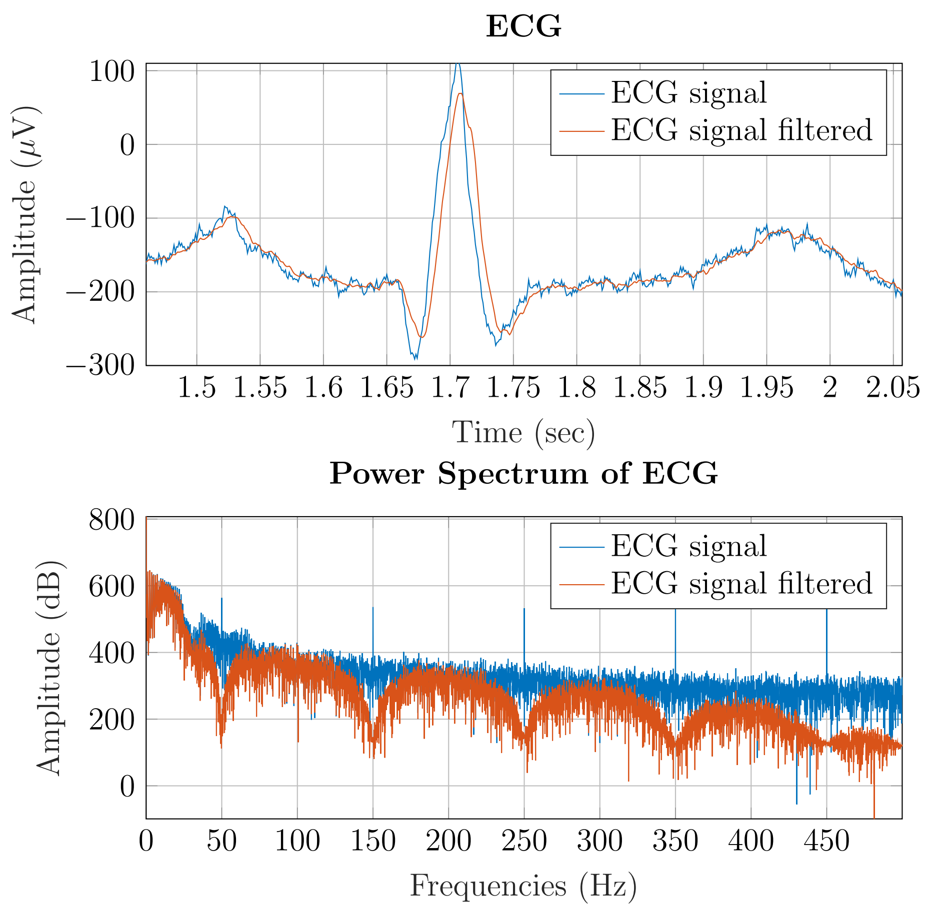

Figure 7: Comparative between ECG given in input and ECG filtered by the Comb filter

5 Combined filter

After separate study, we chose to apply in serial :

- Hanning filter : $H(z) = \frac{1}{4}(z + 2z^{-1} + z^{-2})$ with a 3 dB cut-off frequency equal to 180 Hz
- Derivative-based filters : $H(z) = f_s \frac{(1-z^{-1})}{1-r*z^{-1}}$ which amplifies the signal after 8 Hz
- Comb filter: $H(z) = G * \frac{(z+1)(z-z_1)(z-z_2)...(z-z_{10})}{z^{11}}$ which remove 5 specific power-line frequencies in addition to a low pass with a 3 dB cut-off frequency equal to 113 Hz

In serial, the new equation give a filter described in the figure below.

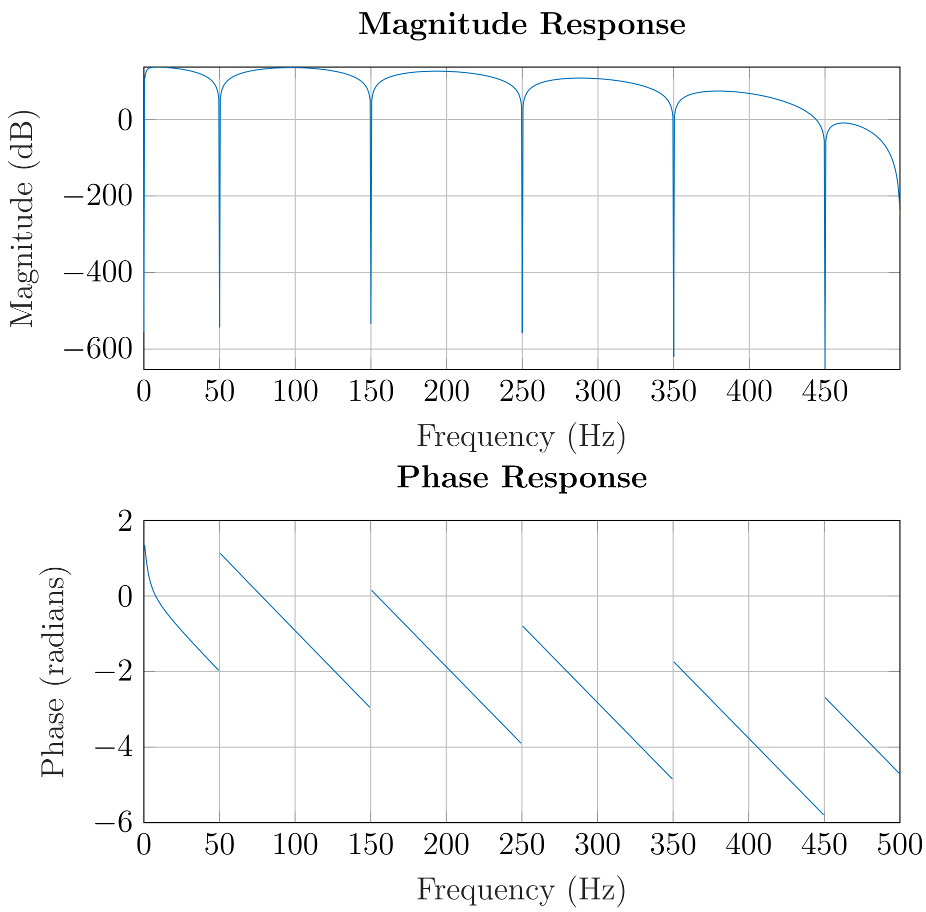

Figure 8: Transfer function of the combined filter

The filter still amplifies the signal after 8 Hz until 429 Hz, the new 3 dB cutoff frequency. The DC component and the high frequency are removed from the signal.

We obtain a signal amplified without power-line noise, baseline drift and with less high-frequency noise. We continue to see the slow P and the T wave. The segment RS can now clearly be measured.

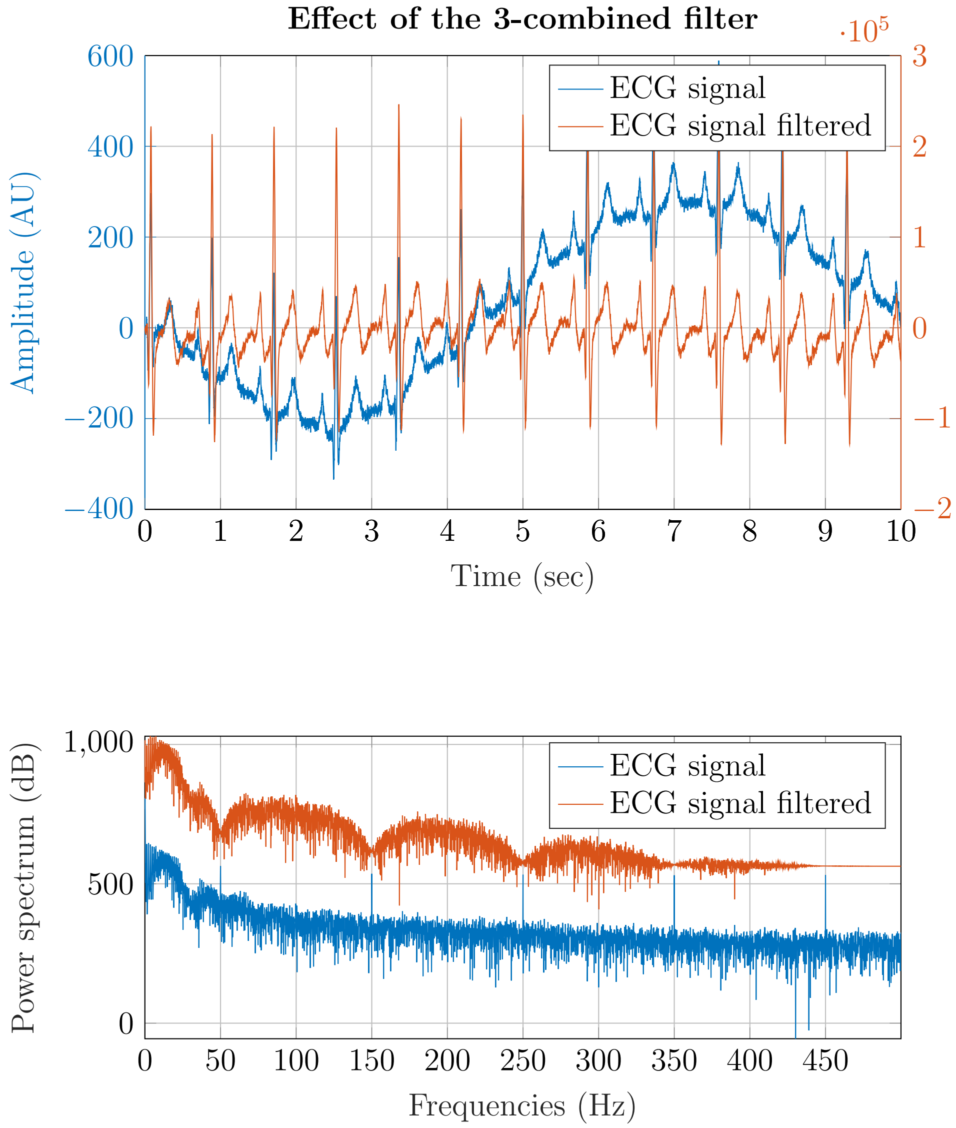

Figure 9: Comparative between ECG given in input and ECG filtered by the combined filter

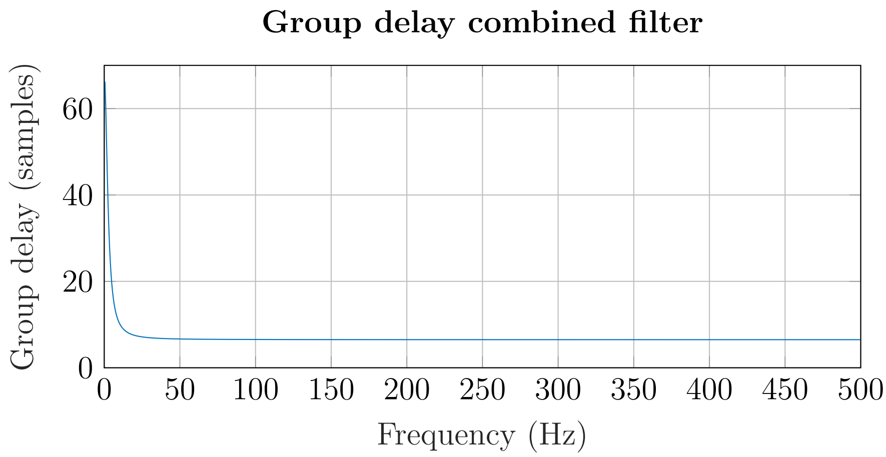

Figure 10: Group delay of the combined filter

To finish, we studied the Group Delay of the filter.
It is important to look at it when we don't have a linear phase in the filter. A nonlinear phase creates delays on the time response, especially during attack transients (as spike PQR for example). In that case, we still have a time delay in low-frequency (from the derivative-based filter) but not after.

6 Conclusion

This assignment allows us to use 3 filters applied on a noisy ECG signal. This practice put the accent on the choice of cutoff frequency to eliminate the noise without reduce the signal and on the result when they are all combined. It is also interesting to have as a result an ECG readable. To go further, there are a lot other ways to process an ECG. For example, the power-line interference (PLI) is not always known as well as in this assignment. Some research had been done to associate a PLI detection module with an adaptive filter [2]. In another hand, a pole radius varying notch filter [4], for example, had been developed to reduce better the baseline interference. We didn't have the time to do a full state of art of the question but our research show us that the subject had been often explored to create solutions more efficient to process an ECG.

References

[1] Biomedical Signal Processing assistant . EX1: Time-domain filtering and notch filtering. 2018.

[2] Ramesh Rajagopalan, Adam Dahlstrom. A pole radius varying notch filter with transient suppression for electrocardiogram.

[3] M.Rangayyan Rangaraj. *Biomedical Signal Analysis*. IEEE Press, 2015.

[4] Yue-Der Lin, Yu Hen Hu*. Power-Line Interference Detection and Suppression in ECG Signal Processing. 2014.