Differential and Integral Calculus - Exercises

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0.1 Some rules and relevant algebra

Rewriting roots as rational exponents

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

1 Simple derivatives

1.1 Mini test: $f(x) = \frac{x^5}{e^x}$ (using quotient rule)

$f'(x) = \frac{(x^5)' \cdot e^x - x^5 \cdot (e^x)'}{(e^x)^2}$

$f'(x) = \frac{5x^4 \cdot e^x - x^5 \cdot e^x}{e^{2x}}$

$f'(x) = \frac{e^x(5x^4 - x^5)}{e^{2x}}$

$f'(x) = \frac{5x^4 - x^5}{e^x}$

1.2 i) $f(x) = x^5 \cdot \sqrt[3]{x^4}$

*Solution.*1

$f'(x) = 5x^4 \cdot x^{\frac{4}{3}} + x^5 \cdot \frac{4}{3}x^{\frac{1}{3}}$

$f'(x) = 5x^{\frac{16}{3}} + \frac{4}{3}x^{\frac{16}{3}}$

$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$

*Solution.*2

$f'(x) = (x^{\frac{19}{3}})'$

$f'(x) = \frac{19}{3} x^{\frac{19}{3}-1}$

$f'(x) = \frac{19}{3} x^{\frac{16}{3}}$

1.3 l) $f(x) = \frac{6cosx}{11sinx+x}$

$f'(x) = \frac{-6sinx \cdot (11sinx+x) - 6cosx \cdot (11cosx+1)}{(11sinx+x)^2}$

$f'(x) = \frac{-66sin^2x - 6sin^2x - 66cos^2x - 6cosx}{(11sinx+x)^2}$

$f'(x) = \frac{-6(11sin^2x + 11cos^2x + xsinx + cosx)}{(11sinx+x)^2}$

$Pythagorean.identity : cos^2(x) + sin^2(x) = 1$
$-66(cos^2x + sin^2x) = -66(1) = -66$

$f'(x) = \frac{-6(xsinx + cosx + 11)}{(11sinx+x)^2}$

1.4 o) $f(x) = \frac{x}{e^x}$

$f'(x) = \frac{(x)' \cdot e^x - x \cdot (e^x)'}{(e^x)^2}$

$f'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2}$

$f'(x) = \frac{(1-x)e^x}{(e^x)^2}$

$f'(x) = \frac{1-x}{e^x}$

2 Vježbe 2