

Differential and Integral Calculus - Exercises

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0.1 Some rules and relevant algebra

Rewriting roots as rational exponents

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$$

0.2 Graphs

Things to do:

1. Determine the domain of the function.
2. Determine the zero points of the function. ($f(x) = 0$)
3. Determine whether the function is even or odd or neither.
 $f(-x) = f(x)$ — EVEN
 $f(-x) = -f(x)$ — ODD
4. Find the stationary points, 1st level (first derivative)
5. Find the stationary points, 2nd level (second derivative)
6. Find the asymptotes

1 Simple derivatives

1.1 Mini test: $f(x) = \frac{x^5}{e^x}$ (using quotient rule)

$$f'(x) = \frac{(x^5)' \cdot e^x - x^5 \cdot (e^x)'}{(e^x)^2}$$

$$f'(x) = \frac{5x^4 \cdot e^x - x^5 \cdot e^x}{e^{2x}}$$

$$f'(x) = \frac{e^x(5x^4 - x^5)}{e^{2x}}$$

$$f'(x) = \frac{5x^4 - x^5}{e^x}$$

1.2 a) $f(x) = x^3 + 5x + 7$

$$f'(x) = 3x^2 + 5 + 0$$

$$f'(x) = 3x^2 + 5$$

1.3 b) $f(x) = (x + 5)(x - 4)$

$$f'(x) = (x + 5) \cdot (x - 4)$$

$$f'(x) = (x + 5)' \cdot (x - 4) + (x + 5) \cdot (x - 4)'$$

$$f'(x) = 1 \cdot (x - 4) + (x + 5) \cdot 1$$

$$f'(x) = 2x + 1$$

1.4 c) $f(x) = (x + 5)^2(x - 4)$

$$f'(x) = ((x + 5)^2)' \cdot (x - 4) + (x + 5)^2 \cdot (x - 4)'$$

$$f'(x) = 2(x + 5) \cdot (x - 4) + (x + 5)^2 \cdot 1$$

$$f'(x) = 2x^2 - 4x + 5x - 20 + x^2 + 25$$

$$f'(x) = 3x^2 + x + 5$$

$$\mathbf{1.5 \quad d) } f(x) = \frac{x}{e} + \pi$$

$$f'(x) = \left(\frac{x}{e}\right)' + 0$$

$$f'(x) = \frac{(x)' \cdot e - x \cdot (e)'}{(e)^2}$$

$$f'(x) = \frac{1 \cdot e - x \cdot 0}{e^2}$$

$$f'(x) = \frac{e}{e^2}$$

$$f'(x) = -\frac{1}{e}$$

$$\mathbf{1.6 \quad e) } f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$f'(x) = (x^{\frac{1}{2}})' + (x^{-\frac{1}{2}})'$$

$$f'(x) = \left(\frac{1}{2}x^{\frac{1}{2}-1}\right) + \left(-\frac{1}{2}x^{-\frac{1}{2}-1}\right)$$

$$f'(x) = \left(\frac{1}{2}x^{-\frac{1}{2}}\right) + \left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$$

$$f'(x) = \left(\frac{1}{2} \cdot \frac{1}{\sqrt{x}}\right) + \left(-\frac{1}{2} \cdot \frac{1}{x^{\frac{3}{2}}}\right)$$

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$$

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$$

$$\mathbf{1.7 \quad i) } f(x) = x^5 \cdot \sqrt[3]{x^4}$$

Solution.1

$$f'(x) = 5x^4 \cdot x^{\frac{4}{3}} + x^5 \cdot \frac{4}{3}x^{\frac{1}{3}}$$

$$f'(x) = 5x^{\frac{16}{3}} + \frac{4}{3}x^{\frac{16}{3}}$$

$$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$$

Solution.2

$$f'(x) = \left(x^{\frac{19}{3}}\right)'$$

$$f'(x) = \frac{19}{3}x^{\frac{19}{3}-1}$$

$$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$$

$$1.8 \quad 1) \quad f(x) = \frac{6\cos x}{11\sin x + x}$$

$$f'(x) = \frac{-6\sin x \cdot (11\sin x + x) - 6\cos x \cdot (11\cos x + 1)}{(11\sin x + x)^2}$$

$$f'(x) = \frac{-66\sin^2 x - 6\sin^2 x - 66\cos^2 x - 6\cos x}{(11\sin x + x)^2}$$

$$f'(x) = \frac{-6(11\sin^2 x + 11\cos^2 x + x\sin x + \cos x)}{(11\sin x + x)^2}$$

$$\text{Pythagorean identity : } \cos^2(x) + \sin^2(x) = 1 \\ -66(\cos^2 x + \sin^2 x) = -66(1) = -66$$

$$f'(x) = \frac{-6(x\sin x + \cos x + 11)}{(11\sin x + x)^2}$$

$$1.9 \quad \text{o) } f(x) = \frac{x}{e^x}$$

$$f'(x) = \frac{(x)' \cdot e^x - x \cdot (e^x)'}{(e^x)^2}$$

$$f'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2}$$

$$f'(x) = \frac{(1-x)e^x}{(e^x)^2}$$

$$f'(x) = \frac{1-x}{e^x}$$

2 Vježbe 2