Differential and Integral Calculus - Exercises

Author

October 2022 - February 2023

0.1 Some rules and relevant algebra

Rewriting roots as rational exponents

$\sqrt{x} = x^{\frac{1}{2}}$

$\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$

0.2 Graphs

1. Determine the domain of the function.

2. Determine the zero points of the function. ($f(x) = 0$)

3. Determine whether the function is even or odd or neither.
$f(-x) = f(x)$ — EVEN
$f(-x) = -f(x)$ — ODD

4. Find the stationary points, 1st level (first derivative)

5. Find the stationary points, 2nd level (second derivative)

6. Find the asymptotes
- Vertical / horizontal asymptote:
Just L'Hopital I guess.

- Diagonal asymptote:
$k = \lim_{x \to \alpha} f(x)$
$l = f(x) - kx$
$y = kx + l$

1 Simple derivatives

1.1 Mini test: $f(x) = \frac{x^5}{e^x}$ (using quotient rule)

$f'(x) = \frac{(x^5)' \cdot e^x - x^5 \cdot (e^x)'}{(e^x)^2}$

$f'(x) = \frac{5x^4 \cdot e^x - x^5 \cdot e^x}{e^{2x}}$

$f'(x) = \frac{e^x(5x^4 - x^5)}{e^{2x}}$

$f'(x) = \frac{5x^4 - x^5}{e^x}$

1.2 a) $f(x) = x^3 + 5x + 7$

$f'(x) = 3x^2 + 5 + 0$

$f'(x) = 3x^2 + 5$

1.3 b) $f(x) = (x + 5)(x - 4)$

$f'(x) = (x + 5) \cdot (x - 4)$

$f'(x) = (x + 5)' \cdot (x - 4) + (x + 5) \cdot (x - 4)'$

$f'(x) = 1 \cdot (x - 4) + (x + 5) \cdot 1$

$f'(x) = 2x + 1$

1.4 c) $f(x) = (x + 5)^2(x - 4)$

$f'(x) = ((x + 5)^2)' \cdot (x - 4) + (x + 5)^2 \cdot (x - 4)'$

$f'(x) = 2(x + 5) \cdot (x - 4) + (x + 5)^2 \cdot 1$

$f'(x) = 2x^2 - 4x + 5x - 20 + x^2 + 25$

$f'(x) = 3x^2 + x + 5$

1.5 d) $f(x) = \frac{x}{e} + \pi$

$f'(x) = (\frac{x}{e})' + 0$

$f'(x) = \frac{(x)' \cdot e - x \cdot (e)'}{(e)^2}$

$f'(x) = \frac{1 \cdot e - x \cdot 0}{e^2}$

$f'(x) = \frac{e}{e^2}$

$f'(x) = -\frac{1}{e}$

1.6 e) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

$f'(x) = (x^{\frac{1}{2}})' + (x^{-\frac{1}{2}})'$

$f'(x) = (\frac{1}{2}x^{\frac{1}{2}-1}) + (-\frac{1}{2}x^{-\frac{1}{2}-1})$

$f'(x) = (\frac{1}{2}x^{-\frac{1}{2}}) + (-\frac{1}{2}x^{-\frac{3}{2}})$

$f'(x) = (\frac{1}{2} \cdot \frac{1}{\sqrt{x}}) + (-\frac{1}{2} \cdot \frac{1}{x^{\frac{3}{2}}})$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}}$

1.7 i) $f(x) = x^5 \cdot \sqrt[3]{x^4}$

*Solution.*1

$f'(x) = 5x^4 \cdot x^{\frac{4}{3}} + x^5 \cdot \frac{4}{3}x^{\frac{1}{3}}$

$f'(x) = 5x^{\frac{16}{3}} + \frac{4}{3}x^{\frac{16}{3}}$

$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$

*Solution.*2

$f'(x) = (x^{\frac{19}{3}})'$

$f'(x) = \frac{19}{3}x^{\frac{19}{3}-1}$

$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$

1.8 l) $f(x) = \frac{6cosx}{11sinx+x}$

$f'(x) = \frac{-6sinx\cdot(11sinx+x)-6cosx\cdot(11cosx+1)}{(11sinx+x)^2}$

$f'(x) = \frac{-66sin^2x-6sin^2x-66cos^2x-6cosx}{(11sinx+x)^2}$

$f'(x) = \frac{-6(11sin^2x+11cos^2x+xsinx+cosx)}{(11sinx+x)^2}$

$Pythagorean.identity : cos^2(x) + sin^2(x) = 1$
$-66(cos^2x + sin^2x) = -66(1) = -66$

$f'(x) = \frac{-6(xsinx+cosx+11)}{(11sinx+x)^2}$

1.9 o) $f(x) = \frac{x}{e^x}$

$f'(x) = \frac{(x)'\cdot e^x - x\cdot(e^x)'}{(e^x)^2}$

$f'(x) = \frac{1\cdot e^x - x\cdot e^x}{(e^x)^2}$

$f'(x) = \frac{(1-x)e^x}{(e^x)^2}$

$f'(x) = \frac{1-x}{e^x}$

2 Vježbe 2