

Differential and Integral Calculus - Exercises

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1 Simple derivatives

1.1 Mini test: $f(x) = \frac{x^5}{e^x}$ (using quotient rule)

$$f'(x) = \frac{(x^5)' \cdot e^x - x^5 \cdot (e^x)'}{(e^x)^2}$$

$$f'(x) = \frac{5x^4 \cdot e^x - x^5 \cdot e^x}{e^{2x}}$$

$$f'(x) = \frac{e^x(5x^4 - x^5)}{e^{2x}}$$

$$f'(x) = \frac{5x^4 - x^5}{e^x}$$

1.2 i) $f(x) = x^5 \cdot \sqrt[3]{x^4}$

Solution.1

$$f'(x) = 5x^4 \cdot x^{\frac{4}{3}} + x^5 \cdot \frac{4}{3}x^{\frac{1}{3}}$$

$$f'(x) = 5x^{\frac{16}{3}} + \frac{4}{3}x^{\frac{16}{3}}$$

$$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$$

Solution.2

$$f'(x) = (x^{\frac{19}{3}})'$$

$$f'(x) = \frac{19}{3}x^{\frac{19}{3}-1}$$

$$f'(x) = \frac{19}{3}x^{\frac{16}{3}}$$

1.3 1) $f(x) = \frac{6\cos x}{11\sin x + x}$

$$f'(x) = \frac{-6\sin x \cdot (11\sin x + x) - 6\cos x \cdot (11\cos x + 1)}{(11\sin x + x)^2}$$

$$f'(x) = \frac{-66\sin^2 x - 6\sin^2 x - 66\cos^2 x - 6\cos x}{(11\sin x + x)^2}$$

$$f'(x) = \frac{-6(11\sin^2 x + 11\cos^2 x + x\sin x + \cos x)}{(11\sin x + x)^2}$$

$$\text{Pythagorean identity : } \cos^2(x) + \sin^2(x) = 1 \\ -66(\cos^2 x + \sin^2 x) = -66(1) = -66$$

$$f'(x) = \frac{-6(x\sin x + \cos x + 11)}{(11\sin x + x)^2}$$

$$\mathbf{1.4 \quad o) \quad} f(x) = \frac{x}{e^x}$$

$$f'(x) = \frac{(x)' \cdot e^x - x \cdot (e^x)'}{(e^x)^2}$$

$$f'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2}$$

$$f'(x) = \frac{(1-x)e^x}{(e^x)^2}$$

$$f'(x) = \frac{1-x}{e^x}$$

2 Vježbe 2