Advanced Logic Homework 6

S5460484

1. Soundness and completeness

This exercise is about the soundness lemma for the normal modal logic K_σ. Consider the following tableau in K_σ, which contains only one branch which we call b:

$$1: \quad \neg(\Box p \supset \Box\Box p), 0$$

a) The branch is (open and) complete. Every rule that can be applied has been applied, including the symmetric (σ) restriction.

b) b) Branch b is *open*. Consider the interpretation $\mathcal{I} = \langle W, R, v\rangle$ with:

- $W = \{w_0, w_1\}$
- $R = \{\langle w_0, w_1\rangle, \langle w_1, w_0\rangle\}$
- $v_{w_0}(p) = 1$ and $v_{w_1}(p) = 0$

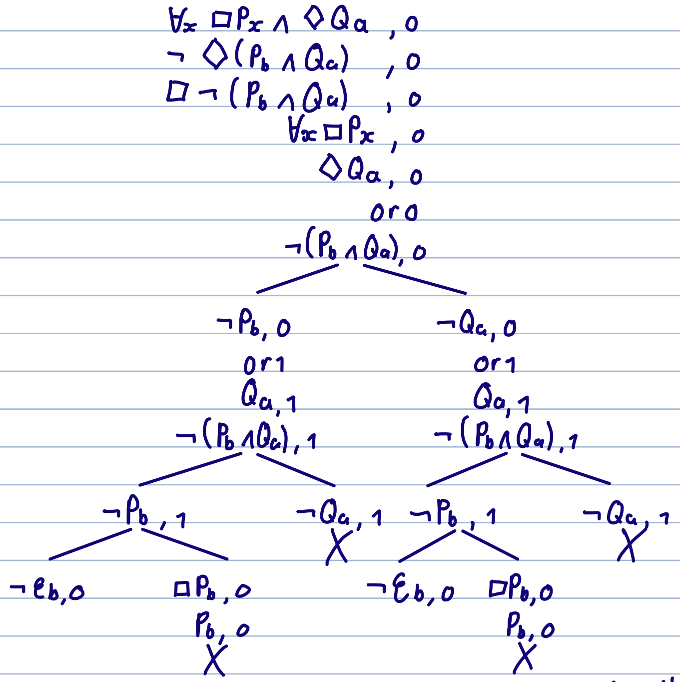
Moreover, consider partial function f from the natural numbers to W such that $f(1) = w_0$ and $f(0) = f(2) = w_1$.

Question: Does f show that $\mathcal{I}$ is *faithful* to b?
If so, show this step by step, considering each line from b. If not, provide a counterexample and show that it is one.

$f(1) = w_0$
$f(0) = f(2) = w_1$

→ I have no idea how to interpret the partial functions

2) $\forall x \Box Px \wedge \Diamond Qa \vdash_{VK_\rho} \Diamond(Pb \wedge Qa)$



There are two open and complete branches. → to the best of my knowledge. IDK what to do with that $\neg E_b$