Strategic Asset Allocation:
CAPM Alpha & Mean-Variance Optimization

Mario Innocente

February 11, 2026

Abstract

This document details the quantitative framework used to construct an optimal equity portfolio within the US Tech sector. The project utilizes a two-step approach: first, a factor analysis using the **Capital Asset Pricing Model (CAPM)** to isolate systematic risk (β) from idiosyncratic performance (α). Second, a structural optimization based on **Modern Portfolio Theory (MPT)** to construct the Efficient Frontier and identify the Tangency Portfolio (Max Sharpe Ratio) under long-only constraints.

1 Methodological Overview

The core objective of Portfolio Management is to maximize expected utility. This project solves the allocation problem by processing historical price data for a universe of large-cap technology stocks ($S = \{AAPL, MSFT, NVDA, AMZN, GOOGL\}$) relative to the market benchmark (SPY).

2 Part 1: Factor Decomposition (CAPM)

Before optimization, we analyze the return drivers of each asset. We utilize the Capital Asset Pricing Model to decompose returns into systematic and idiosyncratic components via Ordinary Least Squares (OLS) regression:

$$(R_{i,t} - R_f) = \alpha_i + \beta_i(R_{m,t} - R_f) + \varepsilon_{i,t} \tag{1}$$

Where:

- $R_{i,t}, R_{m,t}$: Log-returns of asset i and market m.
- β_i: **Systematic Risk**. Measures sensitivity to the market. A $\beta > 1$ implies the asset is more volatile than the benchmark (Aggressive), while $\beta < 1$ implies distinctiveness (Defensive).
- α_i: **Jensen's Alpha**. Represents the excess return generated beyond what is predicted by market risk. In active management, $\alpha > 0$ is the proxy for stock-picking skill or mispricing.

3 Part 2: Mean-Variance Optimization (Markowitz)

Modern Portfolio Theory posits that investors are risk-averse. The goal is to minimize portfolio variance σ_p^2 for a given level of expected return μ_p.

3.1 The Covariance Matrix

The central engine of the optimizer is the covariance matrix Σ, which captures the co-movements between assets.

$$\sigma_p^2 = w^T \Sigma w = \sum_i \sum_j w_i w_j \sigma_{ij} \tag{2}$$

The off-diagonal elements (σ_{ij}) quantify the diversification benefit. Even if individual assets are volatile, a portfolio can be stable if the correlation ρ_{ij} is low.

3.2 Optimization Problem 1: Global Minimum Variance (GMV)

The GMV portfolio represents the lowest possible risk achievable with the given assets, regardless of return. It is the only point on the frontier that does not rely on expected return estimates (which are notoriously noisy), making it a robust "defense" portfolio.

$$\begin{aligned} \min_w \quad & w^T \Sigma w \\ \text{s.t.} \quad & \sum w_i = 1, \quad 0 \leq w_i \leq 1 \quad \text{(Long-Only)} \end{aligned} \tag{3}$$

3.3 Optimization Problem 2: Tangency Portfolio (Max Sharpe)

The Tangency portfolio identifies the optimal trade-off between risk and reward. It maximizes the Sharpe Ratio, effectively finding the point where the Capital Allocation Line (CAL) touches the Efficient Frontier.

$$\max_w \frac{w^T \mu - R_f}{\sqrt{w^T \Sigma w}} \tag{4}$$

Computational Note: The optimization is solved using the **SLSQP** (Sequential Least Squares Programming) algorithm from the `scipy.optimize` library to handle the inequality constraints (bounds $0 \leq w \leq 1$).

4 Key Insights & Limitations

The analysis implemented in the Python notebook highlights several critical aspects of quantitative allocation:

1. **Diversification limits:** In a sector-specific universe (Tech), correlations tend to be high ($\rho > 0.6$). This steepens the Efficient Frontier, offering fewer diversification benefits compared to a multi-asset portfolio.
2. **Parameter Sensitivity:** The Markowitz optimizer is sensitive to input vectors (μ). A small change in expected returns can lead to large shifts in optimal weights (the "Corner Solution" problem).
3. **Constraint Impact:** The "Long-Only" constraint ($w_i \geq 0$) reflects real-world mutual fund mandates, preventing the model from taking theoretical short positions that might be costly or restricted in practice.

Created as part of the Quantitative Finance Portfolio by Mario Innocente.