Simulation 3

Calculations

Simulation 3

$$\begin{cases} \Delta u = 0 & 0<x<1 \\ u(0,y)=0,\quad u(1,y)=y^2-101y+50 & 0<y<1 \\ u(x,0)=50x,\quad u(x,1)=-50x & \end{cases}$$

Discretization: $x_0 \quad \dots \quad x_N$ $N+1$ points, $h=\frac{1}{N}$. $x_i = ih \quad y_j = jh$.

We use the 5-point formula: $-\Delta u(x_i,y_j) \simeq \dfrac{-u_{i-1,j}-u_{i+1,j}+4u_{i,j}-u_{i,j-1}-u_{i,j+1}}{h^2}$

$$-\Delta u(x_1,y_1) = \frac{-\overset{0}{\cancel{u_{0,1}}} - u_{2,1} + 4u_{1,1} - \overset{50x_1}{\cancel{u_{1,0}}} - u_{1,2}}{h^2} = 0$$

$$\rightarrow \boxed{-u_{2,1}+4u_{1,1}-u_{1,2}=50x_1}$$

$$-\Delta u(x_1,y_j) = \frac{-\overset{0}{\cancel{u_{0,j}}} - u_{2,j} + 4u_{1,j} - u_{1,j-1} - u_{1,j+1}}{h^2} = 0$$

$$\rightarrow \boxed{-u_{2,j}+4u_{1,j}-u_{1,j-1}-u_{1,j+1}=0 \qquad 2\leq j\leq N-2}$$

$$-\Delta u(x_1,y_{N-1}) = \frac{-\overset{0}{\cancel{u_{0,N-1}}} - u_{2,N-1} + 4u_{1,N-1} - u_{1,N-2} - \overset{-50x_1}{\cancel{u_{1,N}}}}{h^2} = 0$$

$$\rightarrow \boxed{-u_{2,N-1}+4u_{1,N-1}-u_{1,N-2}=-50x_1}$$

$$-\Delta u(x_i,y_{N-1}) = \frac{1}{h^2}\left(-u_{i-1,N-1}-u_{i+1,N-1}+4u_{i,N-1}-\overset{-50x_i}{\cancel{u_{i,N}}}-u_{i,N-2}\right)$$

$$\rightarrow \boxed{-u_{i-1,N-1}-u_{i+1,N-1}+4u_{i,N-1}-u_{i,N-2}=-50x_i \qquad 2\leq i\leq N-2}$$

$$-\Delta u(x_{N-1},y_{N-1}) = \frac{-u_{N-2,N-1}-\overset{y^2-101y+50}{\cancel{u_{N,N-1}}}+4u_{N-1,N-1}-u_{N-1,N-2}-\overset{-50x_{N-1}}{\cancel{u_{N-1,N}}}}{h^2}=0$$

$$\rightarrow \boxed{-u_{N-2,N-1}+4u_{N-1,N-1}-u_{N-1,N-2}=y_{N-1}^2-101y_{N-1}+50-50x_{N-1}}$$

$$-\Delta u(x_{N-1},y_j) = \frac{-u_{N-2,j}-u_{N,j}+4u_{N-1,j}-u_{N-1,j-1}-u_{N-1,j+1}}{h^2}=0$$

$$\rightarrow \boxed{-u_{N-2,j}+4u_{N-1,j}-u_{N-1,j-1}=y_j^2-101y_j+50 \qquad 2\leq j\leq N-2}$$

$$-\Delta u(x_{N-1}, y_1) = \frac{1}{h^2}\left(-u_{N-2,1} - \cancelto{y_1^2-101y_1+50}{u_{N,1}} + 4u_{N-1,1} - \cancelto{50x_{N-1}}{u_{N-1,0}} - u_{N-1,2}\right)$$

$$\rightarrow \boxed{-u_{N-2,1} + 4u_{N-1,1} - u_{N-1,2} = y_1^2 - 101y_1 + 50 + 50x_{N-1}} \quad \square$$

$$-\Delta u(x_i, y_1) = \frac{1}{h^2}\left(-u_{i-1,1} - u_{i+1,1} + 4u_{i,1} - \cancelto{50x_i}{u_{i,0}} - u_{i,2}\right)$$

$$\rightarrow \boxed{-u_{i-1,1} - u_{i+1,1} + 4u_{i,1} - u_{i,2} = 50x_i \quad 2 < i \leq N-2} \quad \square$$

These equations can be written in matrix form, as follows:

u, the numerical solution vector is defined as $u = \begin{bmatrix} u_{1,1} \\ u_{2,1} \\ \vdots \\ u_{N-1,1} \\ u_{1,2} \\ \vdots \\ u_{N-1,N-1} \end{bmatrix}$

and F is the vector of data, given by the boundary conditions: $F = \begin{bmatrix} F_{1,1} \\ \vdots \\ F_{N-1,1} \\ \vdots \\ F_{N-1,N-1} \end{bmatrix}$

In this case we have:

$F_{1,1} = 50x_1 \quad F_{i,N-1} = -50x_i \quad F_{1,N-1} = -50x_1.$

$F_{N-1,N-1} = y_{N-1}^2 - 101y_{N-1} + 50 - 50x_{N-1} \quad F_{N-1,j} = y_j^2 - 101y_j + 50$

$F_{1,j} = 0 \quad F_{i,1} = 50x_i \quad F_{N-1,1} = y_1^2 - 101y_1 + 50 + 50x_{N-1}$

with $2 \leq i,j \leq N-2$. We construct the matrix L as follows:

Let $A = \text{tridiag}_{N-1}(-1, 4, -1) = \begin{pmatrix} 4 & -1 & & \\ -1 & 4 & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 4 \end{pmatrix}$

Then $L = \begin{pmatrix} A & -I & & \\ -I & A & \ddots & \\ & \ddots & \ddots & -I \\ & & -I & A \end{pmatrix}$ with L being of size $(N-1)^2 \times (N-1)^2$.

Hence we need to solve $Lu = F$ for the solution.

Kronecker

$$M \otimes N = \begin{bmatrix} m_{11}N & m_{12}N \\ m_{21}N & m_{22}N \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & & 0 \\ -1 & 4 & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 4 \end{bmatrix}_{(N-1)\times(N-1)} \qquad L = \begin{bmatrix} A & -\mathbb{I} & & [0] \\ -\mathbb{I} & A & \ddots & \\ & \ddots & \ddots & -\mathbb{I} \\ [0] & & -\mathbb{I} & A \end{bmatrix}$$

$$L = A \otimes \mathbb{I}_{N-1} + \mathbb{I}_{N-1} \otimes A$$

Demonstration for $N-1=2$

$$A \otimes \mathbb{I}_{N-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ c & 0 & d & 0 \\ 0 & c & 0 & d \end{pmatrix}$$

$$\mathbb{I}_{N-1} \otimes A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & c & d \end{pmatrix}$$

$$L = \begin{pmatrix} 2a & b & b & 0 \\ c & a+d & 0 & b \\ c & 0 & 2a & b \\ 0 & c & d & a+b \end{pmatrix} \overset{?}{=} \begin{bmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{bmatrix}$$

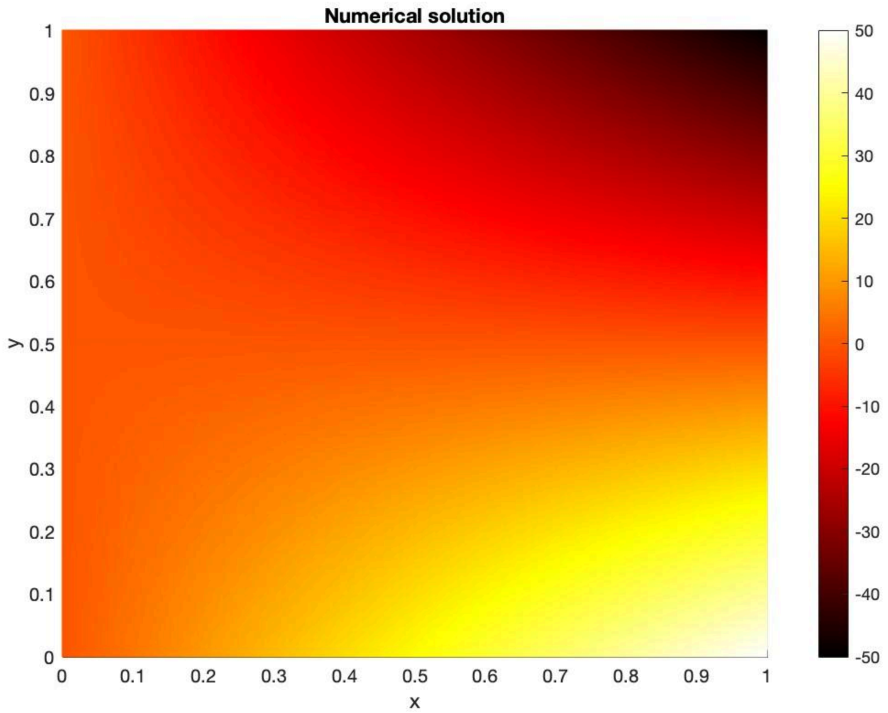
$a = d = 2$

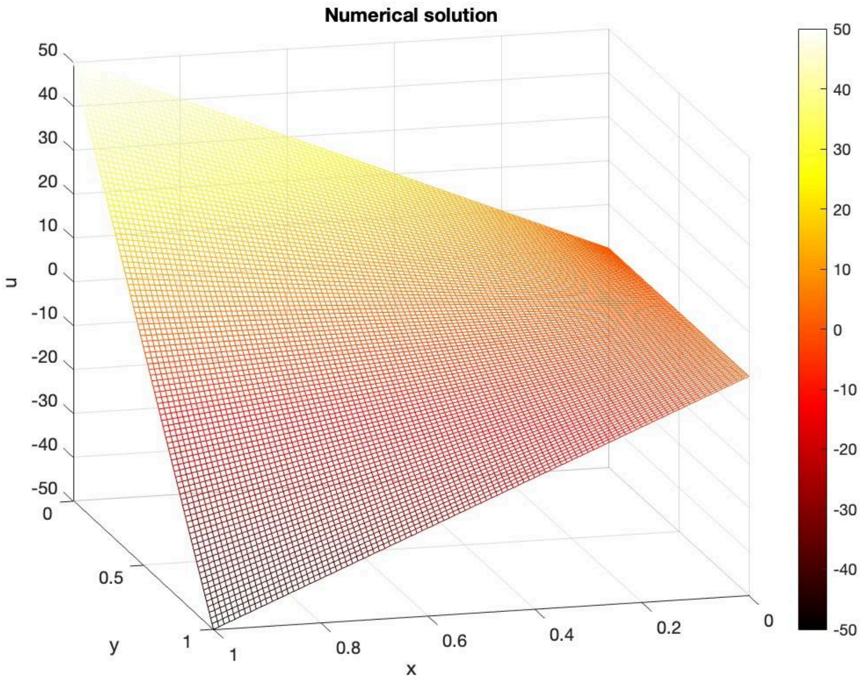
$b = c = -1$

$$F_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{N-1} \otimes F\_button_{N-1} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{N-1} \otimes F\_Top_{N-1} + F\_right_{N-1} \otimes \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{N-1}$$

$$F_{button} = 50x_i \;/\; F_{\cancel{top}\ right} = y_j^2 - 101 y_j + 50 \;/\; F_{top} = -50x_i$$

Plot





Simulation 4

Calculations

Simulation 4 Laplace problem

Same h. $N+1$ points in x. $M+1$ points in y.

$x_0, x_1 \dots x_N$

$y_0, y_1 \dots y_M$

2/10/24.

$$\begin{cases} -\Delta u = 0 & 0<x<2,\ 0<y<1 \\ u(0,y)=0,\ u(2,y)=100 & 0<y<1 \\ u_y(x,0)=-20,\ u_y(x,1)=40 & 0<x<2 \end{cases}$$

$\boxed{\begin{matrix} 0<x<2 \\ 0<y<1 \end{matrix}}$

5 point formula: $-\Delta u(x_i,y_j) \simeq \frac{1}{h^2}\left(-u_{i-1,j}-u_{i+1,j}+4u_{i,j}-u_{i,j+1}-u_{i,j-1}\right)$

$u_y(x,0)$ at $y=0$, y_0

$\boxed{u_{i,1}-u_{i,0}=-20h}$

$$\frac{\partial u}{\partial y} = \frac{u_{i,j+1}-u_{i,j}}{h} \longrightarrow \frac{u_{i,1}-u_{i,0}}{h} = -20 \longrightarrow \boxed{u_{i,1}=-20h+u_{i,0}}$$

$$-\Delta u(x_i,y_1) = \frac{-u_{i-1,1}-u_{i+1,1}+4u_{i,1}-u_{i,2}-u_{i,0}}{h^2}$$

$$\longrightarrow \boxed{-u_{i-1,1}-u_{i+1,1}+3u_{i,1}-u_{i,2}=20h}$$

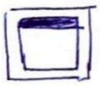
$$\frac{\partial u}{\partial y} \approx \frac{u_{i,j}-u_{i,j-1}}{h}$$

$u_{i,M-1}-u_{i,M}=-40h$

$$\text{at } y=1,\ y_M \quad \frac{\partial u}{\partial y} \approx \frac{u_{i,M}-u_{i,M-1}}{h} = 40 \longrightarrow \boxed{u_{i,M}-u_{i,M-1}=40h}$$

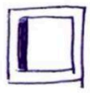
$$-\Delta u(x_i,y_{M-1}) = \frac{-u_{i-1,M-1}-u_{i+1,M-1}+4u_{i,M-1}-u_{i,M}-u_{i,M-2}}{h^2} = 0$$

$$\longrightarrow \boxed{-u_{i-1,M-1}-u_{i+1,M-1}+3u_{i,M-1}-u_{i,M-2}=40h}$$



$$-\Delta u(x_1,y_j) = \frac{-\cancel{u_{0,j}}^{\,0}-u_{2,j}+4u_{1,j}-u_{1,j-1}-u_{1,j+1}}{h^2} = 0$$

$$\longrightarrow \boxed{-u_{2,j}+4u_{1,j}-u_{1,j-1}-u_{1,j+1}=0}$$



$$-\Delta u(x_{N-1},y_j) = \frac{-u_{N-2,j}-\cancel{u_{N,j}}^{\,100}+4u_{N-1,j}-u_{N-1,j-1}-u_{N-1,j+1}}{h^2} = 0$$

$$\longrightarrow \boxed{-u_{N-2,j}+4u_{N-1,j}-u_{N-1,j-1}-u_{N-1,j+1}=100}$$

$$\boxed{u_{i,1} - u_{i,0} = -20h} \qquad \boxed{u_{i,M-1} - u_{i,M} = -40h}$$

Special points

$$\boxed{u(0,y) = 0 \quad u(2,y) = 100}$$

$$-\Delta u(x_1, y_1) = \frac{-\cancelto{0}{u_{0,1}} - u_{2,1} + 4u_{1,1} - u_{1,0} - u_{1,2}}{h^2} = 0$$

$$\longrightarrow \boxed{-u_{2,1} + 3u_{1,1} - u_{1,2} = 20h}$$

$$-\Delta u(x_1, y_{M-1}) = \frac{-\cancelto{0}{u_{0,M-1}} - u_{2,M-1} + 4u_{1,M-1} - u_{1,M-2} - u_{1,M}}{h^2}$$

$$\longrightarrow \boxed{-u_{2,M-1} + 3u_{1,M-1} - u_{1,M-2} = 40h}$$

$$-\Delta u(x_{N-1}, y_{M-1}) = \left(-u_{N-2,M-1} - \cancelto{100}{u_{N,M-1}} + 4u_{N-1,M-1} - u_{N-1,M-2} - u_{N-1,M}\right)/h^2$$

$$\longrightarrow \boxed{-u_{N-2,M-1} + 3u_{N-1,M-1} - u_{N-1,M-2} = 100 + 40h}$$

$$-\Delta u(x_{N-1}, y_1) = \frac{-u_{N-2,1} - \cancelto{100}{u_{N,1}} + 4u_{N-1,1} - u_{N-1,0} - u_{N-1,2}}{h^2} = 0$$

$$\longrightarrow \boxed{-u_{N-2,1} + 3u_{N-1,1} - u_{N-1,2} = 100 + 20h}$$

In this case we have
$$A = \begin{bmatrix} 3 & -1 & & \\ -1 & 4 & & \\ & & \ddots & -1 \\ & & -1 & 3 \end{bmatrix} = \mathrm{tridiag}_{N-1}(-1,4,-1) - \begin{bmatrix} 1 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}.$$

Then we need the matrix L to have $M-1$ rows and colums, where each element is an $(N-1)\times(N-1)$ matrix

$$L = \begin{pmatrix} A & -I & & \\ -I & A & & \\ & & \ddots & -I \\ & & -I & A \end{pmatrix}. \quad L \text{ is a } (N-1)(M-1)\times(N-1)(M-1) \text{ matrix.} \quad \boxed{Lu = F}$$

and the vectors are defined as in Simulation 3, but with the modified boundary conditions: $F_{1,1} = 20h$, $F_{N-1,M-1} = 100 + 40h$, $F_{1,M-1} = 40h$, $F_{N-1,1} = 100 + 20h$, $F_{N-1,j} = 100$, $F_{i,1} = 20h$, $F_{i,M-1} = 40h$, $F_{i,j} = 0$

with $2 \leq i \leq N-2$, $2 \leq j \leq M-1$

Kronecker

$$L = \underset{N_y-1}{A} \otimes \mathbb{I}_{N_x-1} + \mathbb{I}_{N_y-1} \otimes A_{N_x-1} - \overset{(1)}{C_1 \otimes \mathbb{I}_{N_x-1}} - \overset{(2)}{C_2 \otimes \mathbb{I}_{N_x-1}}$$

$$A = \begin{bmatrix} 2 & -1 & & 0 \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ 0 & & -1 & 2 \end{bmatrix}_{\cancel{(N_x-1)\times(N_x-1)}} \qquad C_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & \\ \vdots & & \ddots & \\ 0 & & & 0 \end{bmatrix}_{N_y-1} \qquad C_2 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & \ddots & \\ 0 & & & 1 \end{bmatrix}_{N_y-1}$$

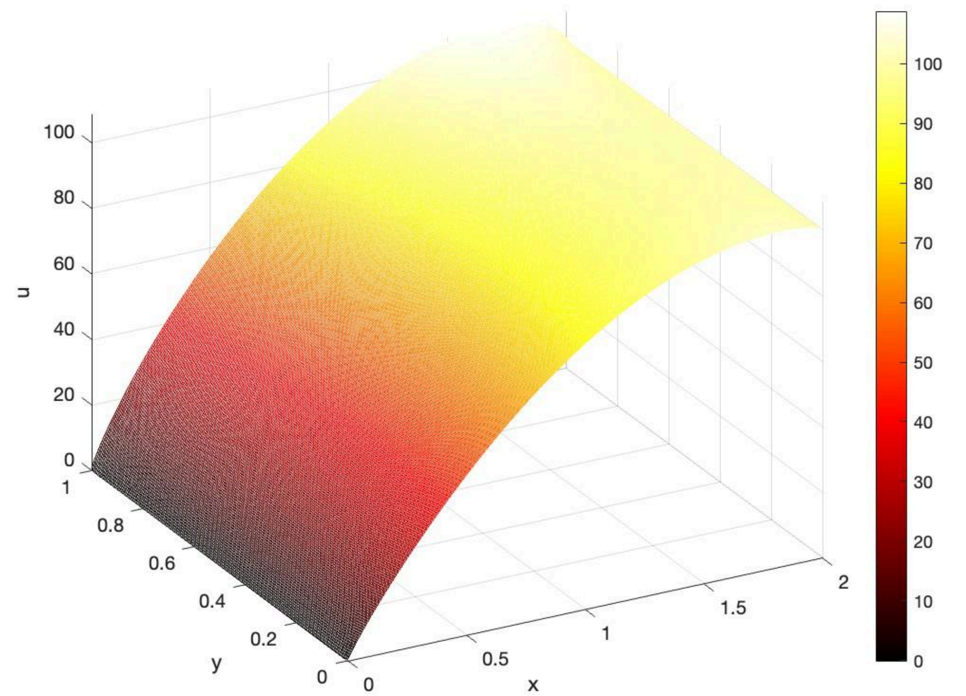
(1) ≡ For the Neumann boundary condition $u_y(x,0) = -20$

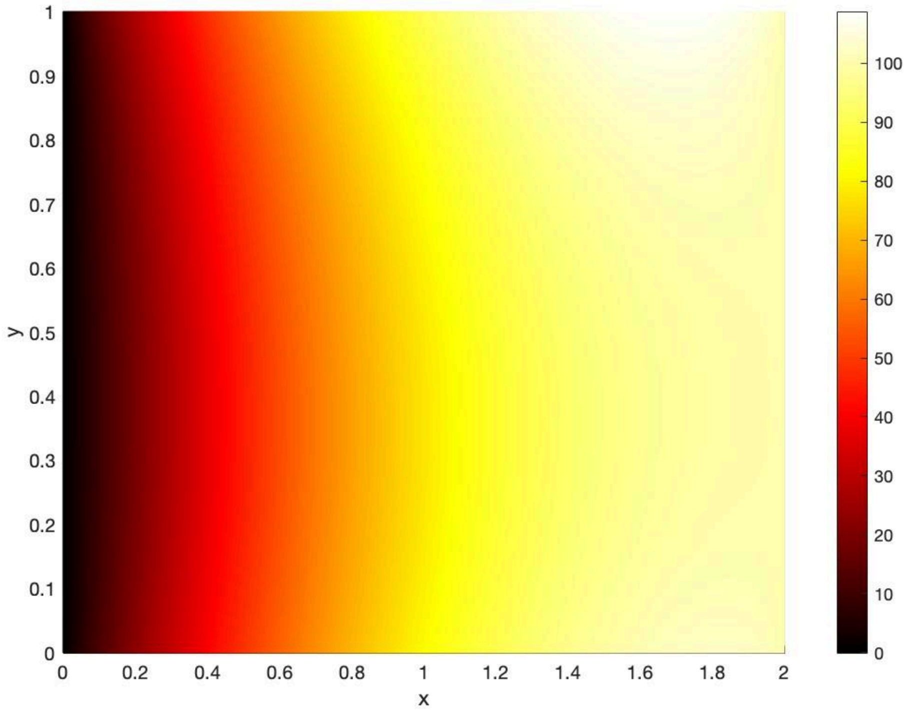
(2) ≡ For the Neumann boundary condition $u_y(x,1) = 40$

$$F_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{N_y-1} \otimes F\_bottom_{N_x-1} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}_{N_y-1} \otimes F\_top_{N_x-1} + F\_right_{N_y-1} \otimes \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}_{N_x-1}$$

$F_{bottom} = 20 \cdot h$ / $F_{right} = 100$ / $F_{top} = 40 \cdot h$

Plot





y
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
10
20
30
40
50
60
70
80
90
100

Simulation 5

Calculations

Simulation 5

N x points

M y points.

$$\begin{cases} \Delta u = 0 & 0<x<2 \quad 0<y<1 \\ u_x(0,y) = -50 \quad u(2,y) = 100 & 0<y<1 \\ u_y(x,0) = -50 \quad u_y(x,1) = -50 & 0<x<2 \end{cases}$$

Five point formula: $-\Delta u(x_i,y_j) = \left(-u_{i-1,j} - u_{i+1,j} + 4u_{i,j} - u_{i,j-1} - u_{i,j+1}\right)/h^2.$

left $x=0$ ⊛

$$-\Delta u(x_1,y_j) = \frac{1}{h^2}\left(-u_{0,j} - u_{2,j} + 4u_{1,j} - u_{1,j-1} - u_{1,j+1}\right) \qquad \boxed{u_{N,j}=100}$$

$$\frac{\partial u}{\partial x} \simeq \frac{u_{i+1,j}-u_{i,j}}{h} \longrightarrow u_x(0,y) \simeq \frac{u_{1,j}-u_{0,j}}{h} = -50 \longrightarrow \boxed{u_{1,j}-u_{0,j} = -50h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{u_{i,1}-u_{i,0}}{h} = -50 \longrightarrow \boxed{u_{i,1}-u_{i,0} = -50h}$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=1} = \frac{u_{i,M}-u_{i,M-1}}{h} = -50 \longrightarrow \boxed{u_{i,M}-u_{i,M-1} = -50h}$$

$$\circledast \longrightarrow \boxed{-u_{2,j} + 3u_{1,j} - u_{1,j-1} - u_{1,j+1} = 50h}$$

Top $y=1$

$$-\Delta u(x_i,y_{M-1}) = \frac{-u_{i-1,M-1} - u_{i+1,M-1} + 4u_{i,M-1} - u_{i,M-2} - u_{i,M}}{h^2}$$

$$\longrightarrow \boxed{-u_{i-1,M-1} - u_{i+1,M-1} + 3u_{i,M-1} - u_{i,M-2} = -50h}$$

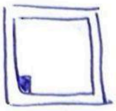
Right $x=2$

$$-\Delta u(x_{N-1},y_j) = \frac{1}{h^2}\left(-u_{N-2,j} - \overset{100}{\cancel{u_{N,j}}} + 4u_{N-1,j} - u_{N-1,j-1} - u_{N-1,j+1}\right)$$

$$\longrightarrow \boxed{-u_{N-2,j} + 4u_{N-1,j} - u_{N-1,j-1} - u_{N-1,j+1} = 100}$$

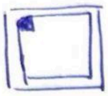
Bottom $y=0$ $\quad -\Delta u(x_i,y_1) \simeq \frac{1}{h^2}\left(-u_{i-1,1} - u_{i+1,1} + 4u_{i,1} - u_{i,0} - u_{i,2}\right)$

$$\longrightarrow \boxed{-u_{i-1,1} - u_{i+1,1} + 3u_{i,1} - u_{i,2} = 50h}$$



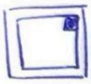
$$-\Delta u(x_1,y_1)=\frac{1}{h^2}\left(-\cancel{u_{0,1}}-u_{2,1}+4u_{1,1}-\cancel{u_{1,0}}-u_{1,2}\right)=0$$

$$\longrightarrow \boxed{-u_{2,1}+2u_{1,1}-u_{1,2}=100h}$$



$$-\Delta u(x_1,y_{M-1})=\frac{1}{h^2}\left(-\cancel{u_{0,M-1}}-u_{2,M-1}+4u_{1,M-1}-\cancel{u_{1,M}}-u_{1,M-2}\right)=0$$

$$\longrightarrow \boxed{-u_{2,M-1}+2u_{1,M-1}-u_{1,M-2}=0}$$



$$-\Delta u(x_{N-1},y_{M-1})=\frac{1}{h^2}\left(-u_{N-2,M-1}-\overset{100}{\cancel{u_{N,M-1}}}+4u_{N-1,M-1}-\cancel{u_{N-1,M}}-u_{N-1,M-2}\right)$$

$$\longrightarrow \boxed{-u_{N-2,1}+3u_{N-1,1}-u_{N-1,2}=\overset{-50h}{\cancel{100+50h}}}$$

$$-\Delta u(x_{N-1},y_1)=\frac{1}{h^2}\left(-u_{N-2,1}-\cancel{u_{N,1}}+4u_{N-1,1}-\cancel{u_{N-1,0}}-u_{N-1,2}\right)$$

$$\longrightarrow \boxed{-u_{N-2,1}+3u_{N-1,1}-u_{N-1,2}=100+50h}$$

In this case we have $A=\begin{pmatrix} 2 & -1 & & \\ -1 & 3 & & \\ & & 3 \ddots & -1 \\ & & -1 & 3 \end{pmatrix}$ with A being $(N-1)\times(N-1)$

and $B=\begin{pmatrix} 3 & -1 & & \\ -1 & 4 & & \\ & & 4 \ddots & -1 \\ & & -1 & 4 \end{pmatrix}$ also $(N-1)\times(N-1)$

Then L is the matrix $L=\begin{pmatrix} A & -I & & \\ -I & B & & \\ & & B \ddots & -I \\ & & -I & A \end{pmatrix}$.

$u=(u_{1,1},u_{2,1},\dots,u_{N-1,1},\dots,u_{N-1,M-1})^t$,

$F=(F_{1,1},\dots,F_{N-1,M-1})^t$ with

$F_{1,1}=100h$ $F_{1,M-1}=0$ $F_{N-1,1}=-50h$. $F_{N-1,1}=100+50h$.

$F_{1,j}=50h$ $F_{i,M-1}=-50h$ $F_{N-1,j}=100$ $F_{i,1}=50h$

and all other entries are zero. Then $\underline{Lu=F}$

Kronecker

$$L = A_{N_y-1} \otimes \mathbb{I}_{N_x-1} + \mathbb{I}_{N_y-1} \otimes A_{N_x-1} - \overset{(1)}{C_1 \otimes \mathbb{I}_{N_x-1}} - \overset{(2)}{C_2 \otimes \mathbb{I}_{N_x-1}} - \overset{(3)}{\mathbb{I}_{N_y-1} \otimes C_3}$$

$$A = \begin{pmatrix} 2 & -1 & & \\ -1 & \ddots & \ddots & \\ & \ddots & \ddots & -1 \\ & & -1 & 2 \end{pmatrix} \quad C_1 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & & \\ \vdots & & \ddots & \\ 0 & & & 0 \end{pmatrix}_{N_y-1} \quad C_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & \ddots & \\ 0 & & & 0 \end{pmatrix}_{N_y-1} \quad C_3 = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & & \\ \vdots & & \ddots & \\ 0 & & & 0 \end{pmatrix}_{N_x-1}$$

(1) ≡ For the Neumann boundary condition $u_y(x,0) = -50$

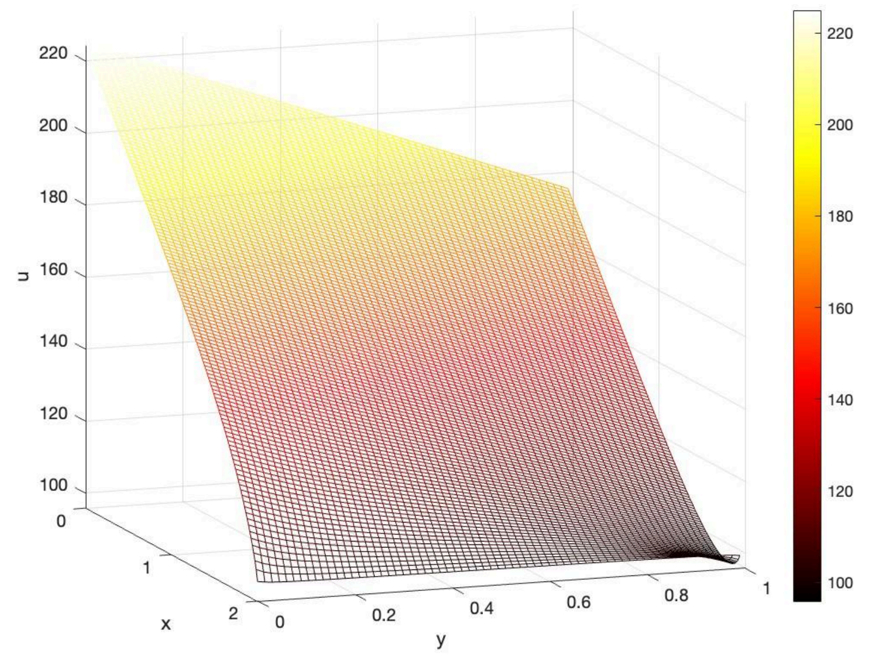
(2) ≡ For the Neumann boundary condition $u_y(x,1) = -50$

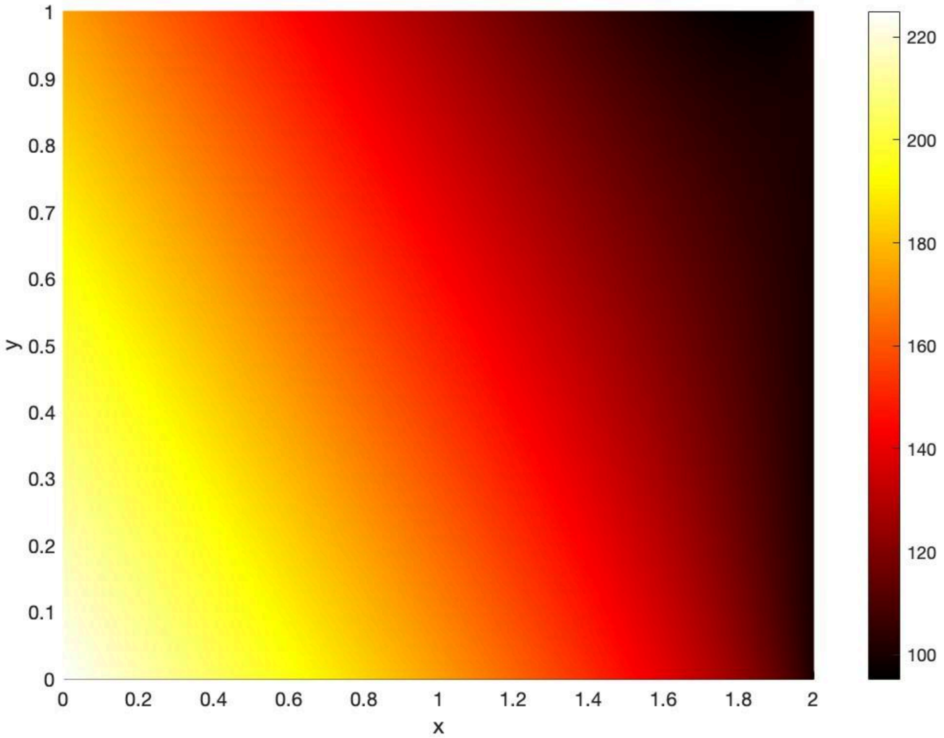
(3) ≡ For the Neumann boundary condition $u_x(0,y) = -50$

$$F_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{N_y-1} \otimes F\_bottom_{N_x-1} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \otimes F\_Top_{N_x-1} + F\_right_{N_y-1} \otimes \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}_{N_x-1} + F\_left_{N_y-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{N_x-1}$$

$F_{bottom} = 50h$ / $F_{top} = -50 \cdot h$ / $F_{right} = \cancel{50h}\ 100$ / $F_{left} = 50 \cdot h$

Plot





x
y
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
100
120
140
160
180
200
220