

# Coexistence policies in cognitive radio

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# Introduction



# **Part I**

## **Dynamic Spectrum Access**





# Background Detection of Primary User Activity in Opportunistic Spectrum Access

## 1.1 Introduction

### Motivation

One of the main problems of opportunistic spectrum access, OSA, is that current radio frequency front-ends cannot perform sensing and transmission in the same channels at the same time. In consequence, in OSA protocols, spectrum sensing and SU data transmission are performed in two separated phases. The scanning phase allows the SUs to find spectrum holes to transmit in. However, during the transmission phase, a channel occupied by an SU can be eventually used by a PU transmission (transmission overlap) causing a potentially harmful interference to the PU receiver. To solve this problem, the SU is only allowed to transmit during a limited amount of time after which it leaves the channel or performs a new sensing procedure (periodic sensing).

OSA with periodic sensing ([165], [166], [167], [168], [169], [170]) consists of a sensing period followed by a transmission period. These two separated periods are sometimes referred to as the medium access control (MAC) frame. In general, a longer sensing duration improves the sensing quality, but the higher sensing overhead in the MAC frame reduces SU throughput. The duration of the transmission period has also influence on the SU throughput and the probability of collision with PU transmissions. The throughput-collision tradeoff is present in [165] and [166], where joint designs of the sensing and access strategy are proposed. For a fixed frame duration, [167] optimizes the sensing time to improve the SU throughput under detection quality constraints. In contrast, [168] considers a fixed sensing time and optimizes the transmission duration subject to an interference power constraint. Both sensing and transmission times were jointly optimized in [169] and

[170]. One of the main contributions of the latter is to consider the impact of imperfect sensing on the throughput of secondary users as well.

The OSA model of these works constitute the base upon which we develop our proposal. We assume a system in which the sensing period is long enough to find, with a sufficiently high reliability, an available channel to transmit in. According to a given PU activity model, the length of the SU transmission period is set to the value that maximizes the SU throughput under the desired collision probability constraint. This is a worst case PU QoS criteria used in multiple previous works ([166], [170], [148], [149], [144]), which considers that any collision causes harmful interference to PU receivers. As in [170], our model also includes the effect of transmission overlap in SU performance. Additionally, we use the statistical description of the fading in the direct and interference links to evaluate this performance. The generic OSA model described is referred as *optimized OSA* in this paper, and is used as a starting point and as a benchmark for our proposal. As in [166], [148], [144], [145], [150], the transmission period is divided into time slots. Each time slot contains one SU data packet transmission.

In [171] it is assumed that, whenever an SU and a PU transmit simultaneously, both experience collision and the SU can detect the collision after the transmission. Perfect collision detection is also implicitly assumed in [172].

In this paper, we relax the perfect collision detection assumption, taking into consideration the multiple factors that hinder perfect detection: pathloss, fading effects, and decoding probability as a function of the modulation used and the signal-to-interference-and-noise ratio. To overcome these difficulties and perform an optimal collision detection, the SU receiver can make use of diverse estimations performed during the successive spectrum scanning and sensing periods: average PU received power, statistical characterization of the fading process, and PU traffic pattern. As an example, an accurate PU traffic estimator is described in [173].

## Our Contribution

We propose a collision detection mechanism that performs PU activity estimation during SU packet reception, and use it to improve the classic optimized OSA mechanism. We refer to this mechanism as *background detection* (BD). In contrast to classic spectrum sensing, where licensed channels are sensed while the SU transmitter is idle ([164]), the BD mechanism operates only at the SU receiver, during ongoing SU transmissions. In consequence, BD uses the information available at the SU receiver during packet reception: power levels, and the presence or absence of decoding errors, to decide, by means of a maximum a posteriori (MAP) estimator, if there has been overlap in each transmission slot. The MAP estimator exploits existing knowledge at the SU, such as the average received powers, the PU traffic pattern and the statistical description of the fading processes. We also evaluate the robustness of the system under inaccuracies in these data. In particular, it is shown that, while optimized OSA is very reliant on an accurate PU traffic characterization, optimized OSA with BD presents higher robustness against PU traffic misestimations.

The rest of this paper is organized as follows. Section 1.2 describes the PU and SU networks and presents the technical foundations of the proposal. Section 1.3 discloses the background detection mechanism, formulating the MAP rule and then computing the detection probabilities. Section 1.4 develops Markov models for the optimized OSA with and without BD. Combining these models with simulations, we present, in Section 1.4 a numerical evaluation of BD performance and robustness.

## 1.2 System Description

### General Considerations

The mechanism proposed in this paper is very generic, and is applicable whenever the primary and the secondary wireless networks present some general characteristics described in this section. The spectrum of the primary network is assumed to be divided into orthogonal channels, so that primary transmissions are multiplexed on these channels (e.g. TDM, FDM, OFDMA). The SU can therefore scan these channels, monitor PU activity and detect spectrum opportunities in them. The SUs are also capable of estimating the PU traffic pattern on these channels. One type of PU access networks that fit well on this description are cellular networks, but other types may be feasible for our proposal as well.

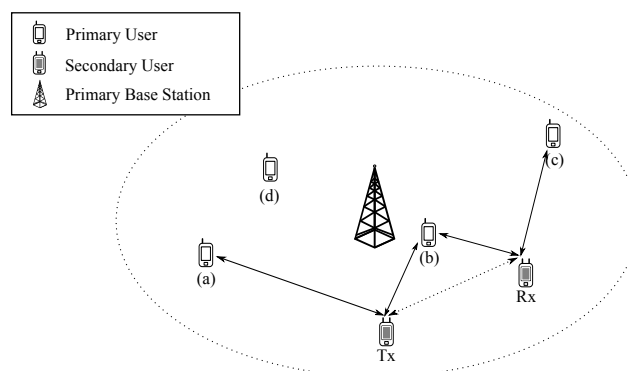


Figure 1.1: Example of the system under study.

As an example, let us consider the system in Figure 1.1, representing the coverage area (cell) of a PU base station (PU BS), where a cognitive pair (CP), consisting of an SU transmitter (SU Tx) and an SU receiver (SU Rx), tries to perform an SU transmission on a spectrum opportunity within this PU cell. Figure 1.1 also depicts the interference links between the SUs and the PU terminals. Because the SUs are within the range of the PU BS, there is also one interference link from the PU BS to each SU. Before transmitting, both the SU Tx and the SU Rx scan the licensed spectrum looking for available channels and exchange control messages containing their sensing outcomes over a dedicated control channel. Once the CP makes the access decision, the SU Tx starts the SU transmission over one or more available channels. It is assumed that the SU transmits in PU downlink channels. Although the BD scheme could be used for PU uplink channels, it is more effective

for downlink channels for the reasons explained in Section 1.5. The SU transmission is divided into time-slots. At the end of each time-slot, the SU Rx submits a control message to the SU Tx over the control channel. In general, this message would contain an ACK (or NACK) of the data transmitted on the time-slot, but our proposal will also include information about the power received at the SU Rx during the time-slot.

It is assumed that the signal power received at the SU Rx remains constant during a symbol time  $T_s$  of the SU Tx transmission. In this case, the probability of correct signal detection at the SU Rx is defined in terms of the probability that the signal to interference ratio (SINR) is above a given threshold, which is determined by the minimum acceptable bit error rate (BER) for correct decoding, the transmission rate and the modulation used. Let us consider that, given the SU data packet length, and the information and redundancy bits in it, SU packets are correctly received if the BER during the packet reception is above certain value (e.g.  $10^{-3}$ ). This BER objective is attained when the ratio  $E_b/N_0$  is above a threshold  $\gamma_b$ , where  $E_b$  is the energy per bit and  $N_0$  is the noise spectral density (including both thermal and interference noise). The threshold depends on the modulation used (e.g. for BPSK,  $\text{BER} = 10^{-3}$  for  $\gamma_b = 7$  dB). Therefore, the threshold in terms of SINR is given by  $\gamma = \gamma_b/(BT_b)$  where  $B$  is the channel bandwidth and  $T_b$  the bit duration.

## Principles of the Background Detection Scheme

Each SU data packet may be received at the SU Rx with or without PU interference. Let  $P$  denote the signal power at the SU Rx during the reception of an SU data packet. The SU signal is received with power  $Y$  at the SU Rx and, in case of transmission overlap, the signal transmitted by the PU BS is received with power  $X$  at the SU Rx. Note that  $P$ ,  $X$ , and  $Y$  are random variables. Under PU-SU transmission overlap,  $P = X + Y$  and, in absence of overlap,  $P = Y$ . Similarly, the SINR is  $\text{SINR} = \frac{Y}{N+X}$ , with overlap, and  $\text{SINR} = \frac{Y}{N}$  without overlap, where  $N$  denotes the total thermal noise power in the channel bandwidth. If  $\text{SINR} > \gamma$  the packet is correctly received and, if  $\text{SINR} \leq \gamma$  the packet is impossible to decode (error event).

After the reception of an SU data packet, the SU Rx can obtain two observations:  $P$ , and the occurrence (or absence) of an error event. The idea of the background detection (BD) mechanism is to use these observations to allow the SUs estimate the presence of PU activity during SU transmissions. If PU activity is detected, the OSA algorithm may decide to end the ongoing SU transmission to protect from interference any PU terminal in range of the SU Tx (In Figure 1.1, PU terminals a and b) in case any of them is receiving PU BS transmissions on the channel where the overlap was detected.

The BD scheme is essentially a binary hypothesis testing mechanism conducted during SU transmission to decide on the most feasible value of a random variable  $\Theta$ , which takes two possible values  $\{0, 1\}$  at each SU packet reception. The event  $\Theta = 1$  indicates that there has been PU interference during the packet reception,

and  $\Theta = 0$  indicates that the reception was free of PU interference. The probability density functions (PDF) of  $Y$  and  $X$  are  $f_Y(y)$  and  $f_X(x)$  respectively. In case that both the SU Tx and the PU BS signals experience Rayleigh fading over their respective links to the SU Rx, we have

$$f_Y(y) = \frac{1}{p_{ss}} e^{-\frac{y}{p_{ss}}} \quad (1.1)$$

$$f_X(x) = \frac{1}{p_{ps}} e^{-\frac{x}{p_{ps}}} \quad (1.2)$$

where  $p_{ss}$  is the average power received from the SU Tx and  $p_{ps}$  is the average power received from the PU BS. Both distributions, as well as the average powers can be obtained from the previous sensing history of the SUs. Similarly, this sensing history provides information about the PU traffic profile and intensity on the scanned channels. One basic metric is the probability of PU activity during the transmission time of an SU packet ( $\mathbf{P}(\Theta = \theta)$ ), with  $\theta = \{0, 1\}$ . Therefore, assuming  $f_Y(y)$ ,  $f_X(x)$  and  $\mathbf{P}(\Theta = \theta)$  are known, the hypothesis testing is based on Bayesian estimation, resulting in a maximum a posteriori probability (MAP) decision rule. In Section 1.5 we evaluate the sensitivity of the BD mechanism to the inaccuracies in the estimation of the PDFs and traffic parameters.

Summarizing, once the SU transmission has started, the SU Rx obtains, upon each packet reception, the following observation:

$$(P, E) = \begin{cases} (p_r, 0) & , \text{if } \text{SINR} > \gamma \\ (p_r, 1) & , \text{if } \text{SINR} \leq \gamma \end{cases} \quad (1.3)$$

And, upon this observation, the MAP decision rule selects the hypothesis having the maximum posterior distribution over the two possible values of  $\theta$ . Next section provides a detailed derivation for the BD MAP rule, and obtains the probabilities of correct and incorrect detection of the MAP estimator.

## 1.3 PU Activity Detection in Background

### MAP Rule

Given the observation pair,  $(P = p_r, E = e)$ , the MAP rule selects the hypothesis  $H_i$  for which the posterior probability  $\mathbf{P}(\Theta = \theta | P = p_r, E = e)$  is largest. By Bayes' rule this is equivalent to select the hypothesis maximizing  $p_{\Theta}(\theta) f_{P,E|\Theta}(p_r, e|\theta)$ . Because  $E$  is a discrete random variable, we have to consider separately each possible value of the outcome  $e$  ( $e = 0$ , if the SU packet is received without errors and  $e = 1$  otherwise) for the computation of the posterior probability. Let  $E_0$  and  $E_1$  refer to the events  $e = 0$  and  $e = 1$  respectively. Similarly, we use  $H_0$  and  $H_1$  to refer to the events  $\Theta = 0$  and  $\Theta = 1$ . Let us derive the expression of  $f_{P,E|\Theta}(p_r, e|\theta)$  for each possible combination of  $E_0$  and  $E_1$  with  $H_0$  and  $H_1$ .

**Case  $E_1, H_1$** 

Applying the definitions we have

$$\begin{aligned} f_{P,E_1|H_1}(p_r, E_1|H_1) &= \frac{\partial}{\partial p_r} \mathbf{P}(P \leq p_r, \text{SINR} \leq \gamma|H_1) \\ &= \frac{\partial}{\partial p_r} \mathbf{P}(X + Y \leq p_r, Y \leq N\gamma + X\gamma) \end{aligned} \quad (1.4)$$

We will first find the joint PDF of  $P$  and  $X$  and then integrate to find the PDF of  $P$ .

$$\begin{aligned} \mathbf{P}(X + Y \leq p_r, Y \leq N\gamma + X\gamma|X = x) &= \mathbf{P}(Y \leq p_r - x, Y \leq N\gamma + x\gamma|X = x) \\ &= \mathbf{P}(Y \leq \min(p_r - x, N\gamma + x\gamma)|X = x) \\ &= \mathbf{P}(Y \leq \min(p_r - x, N\gamma + x\gamma)) \\ &= \int_{-\infty}^{\min(p_r - x, N\gamma + x\gamma)} f_Y(y) dy \\ &= \begin{cases} \int_{-\infty}^{p_r - x} f_Y(y) dy & , \text{if } p_r \leq N\gamma + x(\gamma + 1) \\ \int_{-\infty}^{N\gamma + x\gamma} f_Y(y) dy & , \text{otherwise} \end{cases} \end{aligned} \quad (1.5)$$

where the third equality comes from the independence of the PU and SU signals,  $X$  and  $Y$ . By differentiating both sides with respect to  $p_r$  we obtain

$$f_{P,E_1|H_1,X}(p_r, E_1|H_1, x) = \begin{cases} f_Y(p_r - x) & , \text{if } p_r \leq N\gamma + x(\gamma + 1) \\ 0 & , \text{otherwise} \end{cases} \quad (1.6)$$

Then, the joint PDF of  $P$  and  $X$  is

$$\begin{aligned} f_{P,X,E_1|\Theta}(p_r, x, E_1|H_1) &= f_{X|\Theta}(x|H_1) f_{P,E_1|\Theta}(p_r, E_1|H_1) \\ &= f_X(x) f_{P,E_1|\Theta}(p_r, E_1|H_1) \\ &= \begin{cases} f_X(x) f_Y(p_r - x) & , \text{if } p_r \leq N\gamma + x(\gamma + 1) \\ 0 & , \text{otherwise} \end{cases} \end{aligned} \quad (1.7)$$

from which we finally obtain

$$\begin{aligned} f_{P,E_1|H_1}(p_r) &= \int_{-\infty}^{\infty} f_{P,X,E_1|\Theta}(p_r, x, E_1|H_1) dx \\ &= \int_{\frac{p_r - N\gamma}{(1+\gamma)}}^{\infty} f_X(x) f_Y(p_r - x) dx \end{aligned} \quad (1.8)$$

where  $f_{P,E_1|H_1}(p_r)$  denotes  $f_{P,E_1|H_1}(p_r, E_1|H_1)$  in a more compact form. When both links are characterized by Rayleigh fading,  $f_X(x)$  and  $f_Y(y)$  are exponential distributions and we have

$$f_{P,E_1|H_1}(p_r) = \begin{cases} \frac{e^{-\frac{p_r}{p_{ps}}} - e^{-\left(\frac{p_r}{p_{ss}} + \frac{(p_{ps} - p_{ss})(p_r - N\gamma)}{p_{ps}p_{ss}(1+\gamma)}\right)}}{p_{ps} - p_{ss}} & , \text{if } p_r \geq N\gamma \\ \frac{e^{-\frac{p_r}{p_{ps}}} - e^{-\frac{p_r}{p_{ss}}}}{p_{ps} - p_{ss}} & , \text{otherwise} \end{cases} \quad (1.9)$$

**Case  $E_0, H_1$** 

Proceeding similarly to the case above we have

$$\begin{aligned} f_{P,E_0|H_1}(p_r, E_0|H_1) &= \frac{\partial}{\partial p_r} \mathbf{P}(P \leq p_r, \text{SINR} > \gamma | H_1) \\ &= \frac{\partial}{\partial p_r} \mathbf{P}(X + Y \leq p_r, Y > N\gamma + X\gamma) \end{aligned} \quad (1.10)$$

To obtain the joint PDF of  $X$  and  $P$  we need the following probability

$$\begin{aligned} \mathbf{P}(X + Y \leq p_r, Y > N\gamma + X\gamma | X = x) &= \mathbf{P}(X + Y \leq p_r, Y > N\gamma + X\gamma) \\ &= \mathbf{P}(N\gamma + X\gamma < Y \leq p_r - x) \\ &= \begin{cases} \int_{N\gamma + X\gamma}^{p_r - x} f_Y(y) dy & , \text{if } x < \frac{p_r - N\gamma}{1 + \gamma} \\ 0 & , \text{otherwise} \end{cases} \end{aligned} \quad (1.11)$$

By differentiating with respect to  $p_r$  we obtain

$$f_{P,E_0|H_1,X}(p_r, E_0|H_1, x) = \begin{cases} \frac{\partial}{\partial p_r} \int_{N\gamma + X\gamma}^{p_r - x} f_Y(y) dy & , \text{if } x < \frac{p_r - N\gamma}{1 + \gamma} \\ 0 & , \text{otherwise} \end{cases} \quad (1.12)$$

Then, after obtaining the joint PDF of  $X$  and  $P$ , we integrate over  $x$  to get

$$f_{P,E_0|H_1}(p_r) = \int_{-\infty}^{\frac{p_r - N\gamma}{1 + \gamma}} \left( \frac{\partial}{\partial p_r} \int_{N\gamma + X\gamma}^{p_r - x} f_Y(y) dy \right) f_X(x) dx \quad (1.13)$$

which, for Rayleigh fading has the following form

$$f_{P,E_0|H_1}(p_r) = \begin{cases} \frac{1}{p_{ps} - p_{ss}} e^{-\frac{p_r}{p_{ss}}} \left( e^{\frac{(p_{ps} - p_{ss})(p_r - N\gamma)}{p_{ps} p_{ss} (1 + \gamma)}} - 1 \right) & , \text{if } p_r > N\gamma \\ 0 & , \text{otherwise} \end{cases} \quad (1.14)$$

**Case  $E_1, H_0$** 

In this case there is no signal received from the PU and the conditional PDF of  $P$  is defined as follows

$$\begin{aligned} f_{P,E_1|H_0}(p_r, E_1|H_1) &= \frac{\partial}{\partial p_r} \mathbf{P}(P \leq p_r, \text{SINR} \leq \gamma | H_1) \\ &= \frac{\partial}{\partial p_r} \mathbf{P}(Y \leq p_r, Y \leq N\gamma) \end{aligned} \quad (1.15)$$

The probability in the right hand side is given by

$$\begin{aligned} \mathbf{P}(Y \leq p_r, Y \leq N\gamma) &= \mathbf{P}(Y \leq \min(p_r, N\gamma)) \\ &= \begin{cases} \int_{-\infty}^{p_r} f_Y(y) dy & , \text{if } p_r \leq N\gamma \\ \int_{-\infty}^{N\gamma} f_Y(y) dy & , \text{otherwise} \end{cases} \end{aligned} \quad (1.16)$$

By differentiating with respect to  $p_r$  we obtain

$$f_{P,E_1|H_0}(p_r) = \begin{cases} f_Y(p_r) & , \text{ if } p_r \leq N\gamma \\ 0 & , \text{ otherwise} \end{cases} \quad (1.17)$$

The expression for Rayleigh fading is straightforward.

### Case $E_0, H_0$

The PDF of  $P$  is now given by

$$\begin{aligned} f_{P,E_0|H_0}(p_r, E_0|H_1) &= \frac{\partial}{\partial p_r} \mathbf{P}(P \leq p_r, \text{SINR} > \gamma | H_1) \\ &= \frac{\partial}{\partial p_r} \mathbf{P}(Y \leq p_r, Y > N\gamma) \end{aligned} \quad (1.18)$$

where  $\mathbf{P}(Y \leq p_r, Y > N\gamma)$  equals  $\mathbf{P}(N\gamma < Y \leq p_r)$  when  $p_r > N\gamma$  and is 0 otherwise. Therefore we have

$$f_{P,E_0|H_0}(p_r) = \begin{cases} \frac{\partial}{\partial p_r} \int_{N\gamma}^{p_r} f_Y(y) dy & , \text{ if } p_r > N\gamma \\ 0 & , \text{ otherwise} \end{cases} \quad (1.19)$$

The particularization for Rayleigh fading is straightforward.

Given the above expressions, the MAP rule is specified by partitioning the observation space into disjoint sets in which each of the two hypothesis is chosen. In case a reception error has occurred ( $E_1$ ), the following equation provides the threshold value(s) for the received power

$$\mathbf{P}(H_0) f_{P,E_1|H_0}(p_r) = \mathbf{P}(H_1) f_{P,E_1|H_1}(p_r) \quad (1.20)$$

And in the  $E_0$  event, the corresponding equation is

$$\mathbf{P}(H_0) f_{P,E_0|H_0}(p_r) = \mathbf{P}(H_1) f_{P,E_0|H_1}(p_r) \quad (1.21)$$

Let us determine the power thresholds for the case of Rayleigh fading. Considering exponential distributions for  $X$  and  $Y$  in (1.20), we obtain the following two equations for  $E_1$

$$lcc | \mathbf{P}(H_0) \cdot 0 = \mathbf{P}(H_1) \frac{e^{-\frac{p_r}{p_{ps}}} - e^{-\left(-\frac{p_r}{p_{ss}} + \frac{(p_{ps}-p_{ss})(p_r-N\gamma)}{p_{ps}p_{ss}(1+\gamma)}\right)}}}{p_{ps} - p_{ss}}, \text{ if } p_r > N\gamma \quad (1.22)$$

$$\mathbf{P}(H_0) \frac{e^{-\frac{p_r}{p_{ss}}}}{p_{ss}} = \mathbf{P}(H_1) \frac{e^{-\frac{p_r}{p_{ps}}} - e^{-\frac{p_r}{p_{ss}}}}{p_{ps} - p_{ss}}, \text{ if } p_r \leq N\gamma \quad (1.23)$$

The first equation (1.22) only holds for  $p_r = \infty$ . Therefore, whenever  $p_r > N\gamma$ , the MAP rule selects the hypothesis  $H_1$ , which is rational because if  $p_r > N\gamma$ , the hypothesis  $H_0$  implies that  $\text{SINR} > \gamma$  which is inconsistent with  $E_1$ . In other words,



if  $p_r > N\gamma$ ,  $E_1$  is caused by PU interference. It can be checked that the solution of second equation (1.23) is

$$p^* = \frac{p_{ss}p_{ps}}{p_{ps} - p_{ss}} \log \left( \frac{p_{ps} - p_{ss}}{p_{ss}} \frac{P(H_0)}{P(H_1)} + 1 \right) \quad (1.24)$$

If  $p^*$  is a real number and  $p^* \leq N\gamma$ , then  $p_{E_1}^* = p^*$  is the threshold below which the MAP rule selects the hypothesis  $H_0$  in the event  $E_1$ . In case (1.23) has no solution in  $(0, N\gamma]$ , then  $p_{E_1}^* = N\gamma$  if the left hand side of (1.23) is larger than the right hand side, and  $p_{E_1}^* = 0$  otherwise.

Considering Rayleigh fading, the equation for  $E_0$ , (1.21) results in the following two equations

$$I_{cc} P(H_0) \frac{e^{-\frac{p_r}{p_{ss}}}}{p_{ss}} = P(H_1) \frac{e^{-\frac{p_r}{p_{ss}}} \left( e^{\frac{(p_{ps} - p_{ss})(p_r - N\gamma)}{p_{ps}p_{ss}(1+\gamma)}} - 1 \right)}{p_{ps} - p_{ss}}, \text{ if } p_r > N\gamma \quad (1.25)$$

$$P(H_0) \cdot 0 = P(H_1) \cdot 0, \text{ if } p_r \leq N\gamma \quad (1.26)$$

The second equation (1.26) is trivially held for every  $p_r \leq N\gamma$  because it corresponds to an unfeasible event: the absence of error when  $p_r \leq N\gamma$  i.e. when  $\text{SINR} \leq \gamma$ . For  $p_r > N\gamma$  equation (1.25) has the following solution

$$p^* = \frac{p_{ss}p_{ps}(1+\gamma)}{p_{ps} - p_{ss}} \left( \log \left( \frac{p_{ps} - p_{ss}}{p_{ss}} \frac{P(H_0)}{P(H_1)} + 1 \right) + \frac{N\gamma(p_{ps} - p_{ss})}{p_{ss}p_{ps}(1+\gamma)} \right) \quad (1.27)$$

If  $p^*$  is a real number and  $p^* > N\gamma$ , then  $p_{E_0}^* = p^*$  is the threshold above which the MAP rule selects the hypothesis  $H_1$  in the event  $E_0$ . Otherwise one of the two hypothesis always holds. The threshold is defined as  $p_{E_0}^* = N\gamma$  if  $H_1$  always holds or as  $p_{E_0}^* = \infty$  if  $H_0$  always holds.

We have therefore determined four disjoint sets in the observation space:

$$I_{cc} R_{H_0 E_0} = \{(p_r, e) : p_r \leq p_{E_0}^*, e = 0\} \quad (1.28)$$

$$R_{H_1 E_0} = \{(p_r, e) : p_r > p_{E_0}^*, e = 0\} \quad (1.29)$$

$$R_{H_0 E_1} = \{(p_r, e) : p_r \leq p_{E_1}^*, e = 1\} \quad (1.30)$$

$$R_{H_1 E_1} = \{(p_r, e) : p_r > p_{E_1}^*, e = 1\} \quad (1.31)$$

When  $X$  and  $Y$  are not described by exponential distributions, the procedure to determine the  $R_{H_i E_j}$  sets is analogous to the one described. These sets are not only useful to establish the MAP rule, but also to estimate the probabilities of correct or incorrect detection.

## Detection Probability

For each possible combination of  $H_0$ ,  $H_1$  and  $E_0$ ,  $E_1$ , two outcomes of the MAP estimation are possible,  $\hat{H}_0$ ,  $\hat{H}_1$ , corresponding to the hypothesis  $\hat{\theta} = 0$  and  $\hat{\theta} = 1$  respectively. Selecting  $\hat{H}_1$  when  $H_0$  is true is generally referred to as a Type I error, and selecting  $\hat{H}_0$  when  $H_1$  is true is a Type II error.

Let us first consider the analysis of the probabilities for  $H_1$ , i.e. when transmission overlap is present. The probability of a Type II error when the SU packet is received correctly is defined as

$$P(\hat{H}_0, E_0|H_1) = P((P, E) \in R_{H_0E_0}|H_1), \quad (1.32)$$

the probability of correct PU activity detection when the SU packet is received correctly is

$$P(\hat{H}_1, E_0|H_1) = P((P, E) \in R_{H_1E_0}|H_1), \quad (1.33)$$

the Type II error probability when the SU packet received is erroneous is

$$P(\hat{H}_0, E_1|H_1) = P((P, E) \in R_{H_0E_1}|H_1), \quad (1.34)$$

and the probability of correct PU activity detection when the SU packet received is erroneous is

$$P(\hat{H}_1, E_1|H_1) = P((P, E) \in R_{H_1E_1}|H_1), \quad (1.35)$$

Let us obtain these probabilities for sets  $R_{H_iE_j}$  defined with two thresholds ( $p_{E_0}^*$  and  $p_{E_1}^*$ ) and then particularize for Rayleigh fading. The probability  $P(\hat{H}_0, E_0|H_1)$  is given by

$$\begin{aligned} P(\hat{H}_0, E_0|H_1) &= P(P \leq p_{E_0}^*, \text{SINR} > \gamma|H_1) \\ &= P(X + Y \leq p_{E_0}^*, Y > N\gamma + X\gamma) \\ &= P(N\gamma + X\gamma < Y \leq p_{E_0}^* - X) \\ &= \int_0^{x^*} \int_{N\gamma + x\gamma}^{p_{E_0}^* - x} f_Y(y) f_X(x) dy dx \end{aligned} \quad (1.36)$$

where

$$x^* = \frac{p_{E_0}^* - N\gamma}{\gamma + 1} \quad (1.37)$$

Solving for  $X$  and  $Y$  exponentially distributed we obtain the following probability

$$P(\hat{H}_0, E_0|H_1) = e^{-\frac{x^*}{p_{ps}}} p_{ss} \frac{e^{\frac{p_{E_0}^*}{p_{ss}}} (e^{-\frac{x^*}{p_{ss}}} - e^{-\frac{x^*}{p_{ps}}})}{p_{ss} - p_{ps}} + \frac{e^{-\frac{(N+x^*)\gamma}{p_{ss}}} (e^{x^* (\frac{1}{p_{ps}} + \frac{\gamma}{p_{ss}})} - 1)}{p_{ss} - p_{ps}\gamma} \quad (1.38)$$

The probability of correct PU detection at the  $E_0$  event is obtained as follows

$$\begin{aligned} P(\hat{H}_1, E_0|H_1) &= P(P > p_{E_0}^*, \text{SINR} > \gamma|H_1) \\ &= P(X + Y > p_{E_0}^*, Y > N\gamma + X\gamma) \\ &= P(Y > \max(p_{E_0}^* - X, N\gamma + X\gamma)) \\ &= \int_0^{x^*} \int_{p_{E_0}^* - x\gamma}^{\infty} f_Y(y) f_X(x) dy dx + \int_{x^*}^{\infty} \int_{N\gamma + x\gamma}^{\infty} f_Y(y) f_X(x) dy dx \end{aligned} \quad (1.39)$$

The expression for Rayleigh fading is

$$P(\hat{H}_1, E_0|H_1) = \frac{e^{-\frac{p_{E_0}^*}{p_{ss}}} \left( -1 + e^{\left(\frac{1}{p_{ss}} - \frac{1}{p_{ps}}\right)x^*} \right) p_{ss}}{p_{ps} - p_{ss}} + \frac{e^{-\frac{x^*}{p_{ss}} - \frac{\gamma(N+x^*)}{p_{ss}}} p_{ss}}{p_{ss} + p_{ps}\gamma} \quad (1.40)$$

The probability Type II error at the  $E_1$  event is given by

$$\begin{aligned}
 P(\hat{H}_0, E_1|H_1) &= P(P \leq p_{E_1}^*, \text{SINR} \leq \gamma|H_1) \\
 &= P(X + Y \leq p_{E_1}^*, Y \leq N\gamma + X\gamma) \\
 &= P(Y \leq \min(N\gamma + X\gamma, p_{E_1}^* - X)) \\
 &= P(Y \leq p_{E_1}^* - X) \\
 &= \int_0^{p_{E_1}^*} \int_0^{p_{E_1}^* - x} f_Y(y) f_X(x) dy dx
 \end{aligned} \tag{1.41}$$

where the last equality comes from the fact that  $p_{E_1}^* \leq N\gamma$ . The expression for Rayleigh fading is

$$P(\hat{H}_0, E_1|H_1) = \frac{p_{ps} \left(1 - e^{-\frac{p_{E_1}^*}{p_{ps}}}\right) + \left(-1 + e^{-\frac{p_{E_1}^*}{p_{ss}}}\right) p_{ss}}{p_{ps} - p_{ss}} \tag{1.42}$$

The probability correct PU activity detection at  $E_1$  is

$$\begin{aligned}
 P(\hat{H}_1, E_1|H_1) &= P(P > p_{E_1}^*, \text{SINR} \leq \gamma|H_1) \\
 &= P(X + Y > p_{E_1}^*, Y \leq N\gamma + X\gamma) \\
 &= P(p_{E_1}^* - X < Y \leq N\gamma + X\gamma) \\
 &= \int_0^{p_{E_1}^*} \int_{p_{E_1}^* - x}^{N\gamma + x\gamma} f_Y(y) f_X(x) dy dx + \int_{p_{E_1}^*}^{\infty} \int_0^{N\gamma + x\gamma} f_Y(y) f_X(x) dy dx
 \end{aligned} \tag{1.43}$$

For Rayleigh fading the expression of this probability is

$$P(\hat{H}_1, E_1|H_1) = \frac{e^{-\frac{p_{E_1}^*}{p_{ps}}} p_{ps}}{p_{ps} - p_{ss}} + \frac{e^{-\frac{p_{E_1}^*}{p_{ss}}} p_{ss}}{-p_{ps} + p_{ss}} - \frac{e^{-\frac{N\gamma}{p_{ss}}} p_{ss}}{p_{ss} + p_{ps}\gamma} \tag{1.44}$$

It can be checked that  $\sum_{i:0,1} \sum_{j:0,1} P(\hat{H}_i, E_j|H_1) = 1$ . Figure 1.2 illustrates the integration regions for the detection probabilities under transmission overlap.

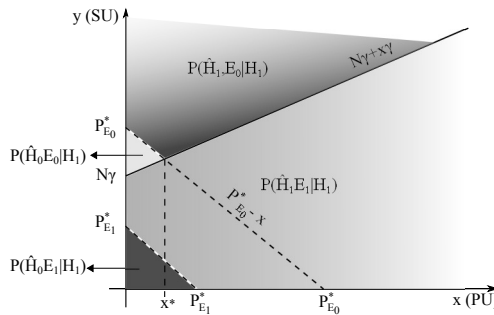


Figure 1.2: Integration regions to compute probabilities of correct detection, type I and type II errors under the  $H_1$  event (transmission overlap).

Let us now obtain the probabilities for  $H_0$ , i.e. in absence of transmission overlap. The probabilities are defined in the following way

$$P(\hat{H}_0, E_0|H_0) = P((P, E) \in R_{H_0 E_0}|H_0) \quad (1.45)$$

$$P(\hat{H}_1, E_0|H_0) = P((P, E) \in R_{H_1 E_0}|H_0) \quad (1.46)$$

$$P(\hat{H}_0, E_1|H_0) = P((P, E) \in R_{H_0 E_1}|H_0) \quad (1.47)$$

$$P(\hat{H}_1, E_1|H_0) = P((P, E) \in R_{H_0 E_1}|H_0) \quad (1.48)$$

We now obtain these probabilities as in the  $H_1$  case. The correct estimation probability  $P(\hat{H}_0, E_0|H_0)$  is given by

$$\begin{aligned} P(\hat{H}_0, E_0|H_0) &= P(P \leq p_{E_0}^*, \text{SINR} > \gamma|H_0) \\ &= P(Y \leq p_{E_0}^*, Y > N\gamma) \\ &= P(N\gamma < Y \leq p_{E_0}^*) \\ &= \int_{N\gamma}^{p_{E_0}^*} f_Y(y) dy \end{aligned} \quad (1.49)$$

which, in the case of Rayleigh fading equals  $e^{-N\gamma/p_{ss}} - e^{-p_{E_0}^*/p_{ss}}$ . The Type I error  $P(\hat{H}_0, E_0|H_0)$  is

$$\begin{aligned} P(\hat{H}_1, E_0|H_0) &= P(P > p_{E_0}^*, \text{SINR} > \gamma|H_0) \\ &= P(Y > p_{E_0}^*, Y > N\gamma) \\ &= P(Y > \max(p_{E_0}^*, N\gamma)) \\ &= P(Y > p_{E_0}^*) \\ &= \int_{p_{E_0}^*}^{\infty} f_Y(y) dy \end{aligned} \quad (1.50)$$

For Rayleigh fading  $P(\hat{H}_1, E_0|H_0) = e^{-p_{E_0}^*/p_{ss}}$ . The correct estimation probability at the  $E_1$  event is

$$\begin{aligned} P(\hat{H}_0, E_1|H_0) &= P(P \leq p_{E_1}^*, \text{SINR} \leq \gamma|H_0) \\ &= P(Y \leq p_{E_1}^*, Y \leq N\gamma) \\ &= P(Y \leq \min(N\gamma + X\gamma, p_{E_1}^*)) \\ &= P(Y \leq p_{E_1}^*) \\ &= \int_0^{p_{E_1}^*} f_Y(y) dy \end{aligned} \quad (1.51)$$

which for Rayleigh fading is  $1 - e^{-p_{E_1}^*/p_{ss}}$ . Finally the Type I error probability at the  $E_1$  event is

$$\begin{aligned} P(\hat{H}_1, E_1|H_0) &= P(P > p_{E_1}^*, \text{SINR} \leq \gamma|H_0) \\ &= P(Y > p_{E_1}^*, Y \leq N\gamma) \\ &= P(p_{E_1}^* < Y \leq N\gamma) \\ &= \int_{p_{E_1}^*}^{N\gamma} f_Y(y) dy \end{aligned} \quad (1.52)$$

which equals  $e^{-p_{E_1}^*/p_{ss}} - e^{-N\gamma/p_{ss}}$  for Rayleigh fading.

## 1.4 Markov Model for Optimized OSA

In this section, we analyze the use of the proposed background detection (BD) mechanism in an OSA system maximizing the SU throughput subject to constraints on the collision probability with PU transmissions. We assume single-channel transmission capabilities for the SU, leaving for future research the generalization to the multiple-channel case. An SU transmission period consists of one scanning slot followed by  $N$  consecutive time-slots. The SU detects at least one available channel during the scanning slot, otherwise this scanning slot does not correspond to a scanning slot. After the scanning slot, the SU transmits one data packet in each one of the  $N$  remaining time-slots of the SU transmission period. Note that  $N$  denotes, in general, a random variable. However, as later explained, this is only the case for OSA with BD. Two parameters characterize the system performance: the SU throughput ( $T$ ), defined as the expected number of correctly received SU packets per time-slot in an SU transmission period, and the collision probability  $P_c$ , defined as the PU-SU transmission overlap probability per time-slot in a SU transmission period.

PU inter arrival time follows a geometric distribution as well as the channel holding time, i.e. the equivalent of a Poisson traffic model for discrete-time. In every time-slot, a channel free of PU traffic is occupied by a PU transmission with probability  $p$ , and an ongoing PU transmission ends with probability  $q$ . The SU uses estimations of  $p$  and  $q$  to optimize the duration of the transmission period under collision probability constraints. The effect that the estimation inaccuracies in both the traffic intensity and in the traffic model have in the OSA performance is addressed in Section 1.5.

Considering the  $n$ -th time-slot of an SU transmission period,  $\phi_n$  denotes the probability vector for PU activity in the channel during this time-slot. In the single-channel traffic model considered,  $\phi_n$  contains two elements:  $\phi_n(1)$  and  $\phi_n(2)$ , corresponding to no PU activity and PU activity in the channel respectively. The probability vector in the scanning slot is  $\phi_0 = (1, 0)$  if the detection is perfect. With imperfect sensing,  $\phi_0 = (1 - p_{\bar{n}}, p_{\bar{n}})$ , where  $p_{\bar{n}}$  denotes the probability of false negative (the sensing outcome indicates that the channel is free of PU activity when it is not). Given the transition matrix of the Markov model for the PU activity in the channel,  $M = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$ , the probability vector for the  $n$ -th time-slot is given by  $\phi'_n = M^n \phi'_0$ , where  $'$  stands for the transpose operation.

### OSA Without Background Detection

In this case, after the scanning slot, the SU transmits in the channel during a fixed number of slots. In an optimized OSA, this number maximizes  $T$  while  $P_c$  remains below a given value  $\alpha$ . Let  $T(n)$  and  $P_c(n)$  denote  $T$  and  $P_c$  as functions of  $n$ . The optimized OSA selects a number of time-slots solving the following problem

$$\max_n T(n) \text{ subject to } P_c(n) \leq \alpha \quad (1.53)$$

which is solved for nonnegative integers  $n$  up to a maximum value  $n_m$ , which stands for avoiding unbounded  $n$  in case  $T(n)$  attains its maximum at  $n \rightarrow \infty$  and  $P_c(n)$

remains under  $\alpha$  for every  $n$ . Note that, provided that  $\phi_0 = (1, 0)$ ,  $P_c(n)$  is a monotonically increasing sequence bounded by  $p/(p+q)$ . If several values of  $n$  attain the maximum, the OSA algorithm picks the one with the lowest  $P_c$ . We refer to this value as  $n^*$ .

The expression for  $P_c(n)$  is given by

$$P_c(n) = \frac{1}{n+1} \sum_{i=1}^n \phi_i(2) \quad (1.54)$$

Defining  $\mathbf{P}(E_0|H_0)$  as the probability of correctly receiving an SU packet in the absence of collision, and  $\mathbf{P}(E_0|H_1)$  as the probability of  $E_0$  in a time-slot with collision,  $T(n)$  is given by the following equation

$$T(n) = \frac{1}{n+1} \left[ \sum_{i=1}^n \phi_i(1) \mathbf{P}(E_0|H_0) + \sum_{i=1}^n \phi_i(2) \mathbf{P}(E_0|H_1) \right] \quad (1.55)$$

The probability  $\mathbf{P}(E_0|H_0)$  is obtained as follows

$$\begin{aligned} \mathbf{P}(E_0|H_0) &= \mathbf{P}(\text{SINR} > \gamma | H_0) \\ &= \mathbf{P}(Y > N\gamma) \\ &= \int_{N\gamma}^{\infty} f_Y(y) dy \end{aligned} \quad (1.56)$$

and  $\mathbf{P}(E_0|H_1)$  is given by

$$\begin{aligned} \mathbf{P}(E_0|H_1) &= \mathbf{P}(\text{SINR} > \gamma | H_1) \\ &= \mathbf{P}(Y > N\gamma + X\gamma) \\ &= \int_0^{\infty} \int_{N\gamma+x\gamma}^{\infty} f_Y(y) f_X(x) dy dx \end{aligned} \quad (1.57)$$

For Rayleigh fading we have

$$1c/\mathbf{P}(E_0|H_0) = e^{-\frac{N\gamma}{\rho_{ss}}} \quad (1.58)$$

$$\mathbf{P}(E_0|H_1) = \frac{\rho_{ss} e^{-\frac{N\gamma}{\rho_{ss}}}}{\rho_{ss} + \rho_{ps}\gamma} \quad (1.59)$$

## OSA With Background Detection

In contrast to previous scheme, when OSA includes BD, the duration of the SU transmission period is not deterministic. Because BD can detect PU activity simultaneously to SU packet reception, the SU transmission can be aborted at any time-slot. Figure 1.3 compares a transmission period without BD with two possible outcomes of the transmission period using BD. Case (a) is the standard SU transmission period when the BD scheme does not detect PU activity in any slot. Case (b) corresponds to the case when the BD scheme detects a PU transmission is a slot, resulting in the immediate termination of the SU transmission in this channel.

In order to compute  $T$  and  $P_c$  we develop a Markov-reward model of the SU transmission process, consisting in a discrete-time Markov chain (DTMC) in which

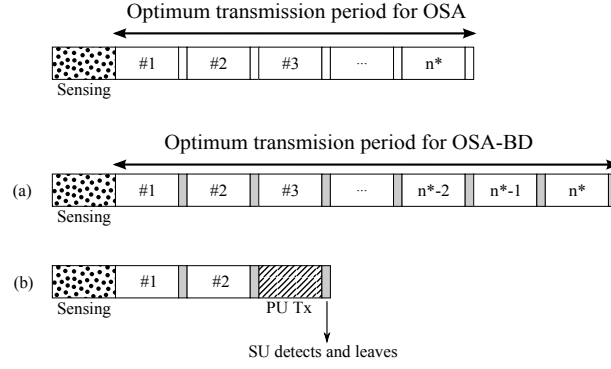


Figure 1.3: Transmission periods with and without BD. When using BD the transmission period can be aborted if PU activity is detected during an SU transmission slot. Inter-slot periods are required for signaling.

each state is associated to a reward defined in terms of  $T$  of  $P_c$ , depending on what is being computed. The DTMC, denoted by  $Z_k$ , characterizes a process of consecutive SU transmissions, where  $k$  enumerates time-slots. The integer value  $i$  taken by  $Z_k$ . i.e. the state of the DTMC at the  $k$ -th time-slot, has the following interpretation:

- If  $i$  is odd, the channel is free of PU activity ( $H_0$ ) at the  $\lceil i/2 \rceil$ -th time-slot.
- If  $i$  is even, the channel presents PU activity ( $H_1$ ) at the  $i/2$ -th time-slot.

After the scanning time-slot ( $i = 0$ ), the SU transmits with probability 1, and enters state  $i = 1$  with probability  $1 - p$  and state  $i = 2$  with probability  $p$ . The DTMC advances to a higher state if the BD does not detect PU activity ( $\hat{H}_0$ ), and returns to state 0 otherwise, which means the end of the ongoing SU transmission. Therefore, the DTMC transition probabilities are defined as

$$\begin{aligned}
 p_{0,1} &= 1 - p \\
 p_{0,2} &= p \\
 p_{2k-1,2k+1} &= (1 - p)P(\hat{H}_0|H_0) \quad , \text{ for } 0 < k < n \\
 p_{2k-1,2k+2} &= pP(\hat{H}_0|H_0) \quad , \text{ for } 0 < k < n \\
 p_{2k-1,0} &= 1 - P(\hat{H}_0|H_0) \quad , \text{ for } 0 < k < n \\
 p_{2k,2k+1} &= qP(\hat{H}_0|H_1) \quad , \text{ for } 0 < k < n \\
 p_{2k,2k+2} &= (1 - q)P(\hat{H}_0|H_1) \quad , \text{ for } 0 < k < n \\
 p_{2k,0} &= 1 - P(\hat{H}_0|H_1) \quad , \text{ for } 0 < k < n \\
 p_{n,0} &= 1
 \end{aligned} \tag{1.60}$$

where  $P(\hat{H}_0|H_0)$  and  $P(\hat{H}_0|H_1)$  can be obtained as follows

$$\begin{aligned} P(\hat{H}_0|H_0) &= P(\hat{H}_0, E_0|H_0) + P(\hat{H}_0, E_1|H_0) \\ P(\hat{H}_0|H_1) &= P(\hat{H}_0, E_0|H_1) + P(\hat{H}_0, E_1|H_1) \end{aligned} \quad (1.61)$$

where  $P(\hat{H}_0, E_0|H_1)$ ,  $P(\hat{H}_0, E_1|H_1)$ ,  $P(\hat{H}_0, E_0|H_0)$ , and  $P(\hat{H}_0, E_1|H_0)$ , are obtained by (1.36), (1.41), (1.49), and (1.51) respectively. Figure 1.4 depicts the graph of the DTMC.

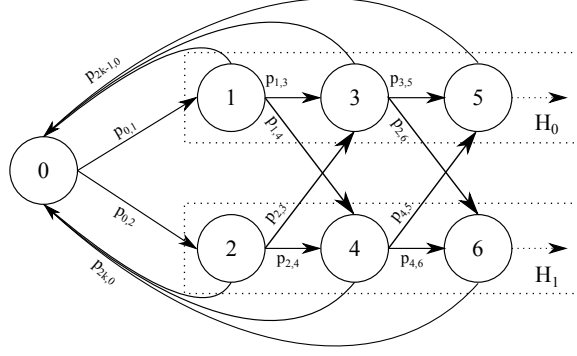


Figure 1.4: Graph of the DTMC characterizing the SU transmission process.

For a generic reward  $r(i)$ , the expected average reward is defined as  $\bar{r} = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{k=0}^K r(Z_k)$ . For an ergodic, finite-state Markov chain, and provided that  $|r(i)|$  is bounded for every  $i$ , the total expected average reward  $\bar{r}$  is  $\bar{r} = \sum_{i=0}^n r(i) \pi_i$ , where  $\pi_i$  are the steady state probabilities of the DTMC obtained by solving the equilibrium equations  $\boldsymbol{\pi} = \boldsymbol{\pi} \mathbf{T}$ ,  $\|\boldsymbol{\pi}\| = \sum_{i=0}^n \pi_i = 1$ , where  $\mathbf{T}$  is the transition matrix containing the probabilities defined in (1.60). Let  $r_T(i)$  and  $r_{P_c}(i)$  denote the per-state reward for computing  $T$  and  $P_c$  respectively. According to the definitions of  $T$  and  $P_c$ , and because the DTMC models consecutive SU transmissions,  $\bar{r}_T = T$  and  $\bar{r}_{P_c} = P_c$ . We will use the terms  $\bar{r}_T(n)$  and  $\bar{r}_{P_c}(n)$  to refer to the SU throughput and collision probability when using BD and the maximum length of SU transmissions is set to  $n$  time-slots. The optimal  $n$  is obtained by solving the following optimization problem

$$\max_n \bar{r}_T(n) \text{ subject to } \bar{r}_{P_c}(n) \leq \alpha \quad (1.62)$$

which is solved for integers  $n \in \{0, 1, \dots, n_m\}$ . Note that the trivial values for  $n = 0$  are  $\bar{r}_T(0) = 0$  and  $\bar{r}_{P_c}(0) = 0$ . If there are more than one solution, the optimal transmission limit  $n^*$  is the lowest one attaining the maximum throughput.

Let us see how to obtain  $r_T(i)$  and  $r_{P_c}(i)$  for every  $i$ . The per-stage throughput  $r_T(i)$  is defined as the expected number of SU packets correctly received at the  $i$ -th SU transmission time-slot. By definition, for every  $i > 0$ , the SU transmits one packet, therefore  $r_T(i) = P(E_0|i) \cdot 1$ , where  $P(E_0|i)$  is the probability of the  $E_0$  event at the  $i$ -th time-slot, and is given by

$$P(E_0|i) = \begin{cases} P(E_0|H_0) & , \text{ if } i \text{ odd} \\ P(E_0|H_1) & , \text{ if } i \text{ even} \end{cases} \quad (1.63)$$



for  $i > 0$ , where  $\mathbf{P}(E_0|H_0)$  and  $\mathbf{P}(E_0|H_1)$  are given by (1.56) and (1.57) respectively. Similarly, the per-stage collision probability is  $r_{P_c}(i) = 0$  when  $i$  is odd, and  $r_{P_c}(i) = 1$  when  $i$  is even. For  $i = 0$  we have  $r_T(0) = 0$  and  $r_{P_c}(0) = 0$ .

The specific structure of matrix  $\mathbf{T}$  allows an efficient computation of the steady-state probability vector  $\boldsymbol{\pi}$ . Let us express  $\mathbf{T}$  in a block-matrix form

$$\mathbf{T} = \begin{pmatrix} \mathbf{B}_{0,0} & \mathbf{B}_{0,1} & \mathbf{0} & \mathbf{0} & \cdots \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{A} & \mathbf{0} & \cdots \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{A} & \cdots \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \mathbf{A} \end{pmatrix} \quad (1.64)$$

where  $\mathbf{B}_{0,0} = (0, 0)$ ,  $\mathbf{B}_{0,1} = ((1-p), p)$ , and

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} (1-p)\mathbf{P}(\hat{H}_0|H_0) & p\mathbf{P}(\hat{H}_0|H_0) \\ q\mathbf{P}(\hat{H}_0|H_1) & (1-q)\mathbf{P}(\hat{H}_0|H_1) \end{pmatrix} \\ \mathbf{B} &= \begin{pmatrix} 1 - \mathbf{P}(\hat{H}_0|H_0) \\ 1 - \mathbf{P}(\hat{H}_0|H_1) \end{pmatrix} \\ \mathbf{1} &= \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{aligned} \quad (1.65)$$

Let us define  $\boldsymbol{\pi}_k = (\pi_{2k-1}, \pi_{2k})$ , denoting the probability of the events  $H_0$  and  $H_1$  at the  $k$ -th transmission slot. Applying the equilibrium equations we have  $\boldsymbol{\pi}_0 = \sum_{j=1}^{n-1} \boldsymbol{\pi}_j \mathbf{B} + \boldsymbol{\pi}_n \mathbf{1}$ ,  $\boldsymbol{\pi}_1 = \boldsymbol{\pi}_0 \mathbf{B}_{0,1}$ , and  $\boldsymbol{\pi}_k$  can be expressed in the matrix-geometric form  $\boldsymbol{\pi}_k = \boldsymbol{\pi}_1 \mathbf{A}^{k-1} = \boldsymbol{\pi}_0 \mathbf{B}_{0,1} \mathbf{A}^{k-1}$ . The normalization condition is  $\boldsymbol{\pi}_0 + \sum_{k=1}^{n-1} \boldsymbol{\pi}_k \mathbf{1} + \boldsymbol{\pi}_n \mathbf{1} = 1$ , which combined with the equilibrium equations results in the following expression for  $\boldsymbol{\pi}_0$

$$\boldsymbol{\pi}_0 = \frac{1}{2 + \sum_{k=1}^{n-1} \mathbf{B}_{0,1} \mathbf{A}^{k-1} (\mathbf{1} - \mathbf{B})} \quad (1.66)$$

Because  $\mathbf{A}$  is strictly positive we know, by the Perron-Frobenius theorem, that it has one positive eigenvalue, with multiplicity 1, which is strictly higher than all other eigenvalues. Since the dimension of  $\mathbf{A}$  is  $2 \times 2$ , it has only two eigenvalues and thus we conclude that  $\mathbf{A}$  has two distinct eigenvalues,  $\lambda_1$  and  $\lambda_2$  (with their corresponding eigenvectors  $\mathbf{e}_1$  and  $\mathbf{e}_2$ ). This implies that  $\mathbf{A}$  can be expressed as  $\mathbf{A} = \mathbf{M} \boldsymbol{\Lambda} \mathbf{M}^{-1}$  where  $\mathbf{M} = [\mathbf{e}_1 \mathbf{e}_2]$  and  $\boldsymbol{\Lambda}$  is a diagonal matrix containing  $(\lambda_1, \lambda_2)$  in its diagonal. Then

$$\mathbf{A}^{k-1} = \mathbf{M} \begin{pmatrix} \lambda_1^{k-1} & 0 \\ 0 & \lambda_2^{k-1} \end{pmatrix} \mathbf{M}^{-1} \quad (1.67)$$

which allows us to express (1.66) in the following closed form

$$\boldsymbol{\pi}_0 = \left( 2 + \mathbf{B}_{0,1} \mathbf{M} \begin{pmatrix} \frac{1-\lambda_1^{n-1}}{1-\lambda_1} & 0 \\ 0 & \frac{1-\lambda_2^{n-1}}{1-\lambda_2} \end{pmatrix} \mathbf{M}^{-1} (\mathbf{1} - \mathbf{B}) \right)^{-1} \quad (1.68)$$

which enables an efficient computation of  $\boldsymbol{\pi}_k = \boldsymbol{\pi}_0 \mathbf{B}_{0,1} \mathbf{A}^{k-1}$ , for  $k = 1, \dots, n$ .

Parameter	Assigned value
$p_{ss}$	-72.9 dBm
$p_{ps}$	-70.5 dBm
$N$	-103 dBm
$\gamma$	7 dBm
$p$	$10^{-3}$
$q$	$2 \cdot 10^{-3}$
$P(H_1)$	1/3
$p_{E_0}^*$	-61.8 dBm
$p_{E_1}^*$	-95.9 dBm
$P(\hat{H}_1 H_0)$	$2.65 \cdot 10^{-6}$
$P(\hat{H}_0 H_1)$	0.1026

Table 1.1: Parameter setting of the reference scenario used in numerical evaluations

## 1.5 Numerical Results

### Parameter Setting

To assess the impact of using BD on the performance and, especially, on the robustness of the OSA scheme, we consider a cognitive pair located within PU BS cell. The distance between the of PU BS and the SU Rx is  $d_{ps} = 1000$  m. The PU BS transmission power on each channel,  $p_p$ , is 20 dBm. The transmission antenna height is  $h_t = 10$  m and the receiver is at  $h_r = 1.5$  m. The transmission and reception antenna gains are  $g_t = 4$  dB and  $g_r = 2$  dB respectively. With these parameters, the average interference power at the SU receiver due to PU BS transmission is given by the pathloss equation

$$p_{ps} = \frac{p_p(h_t h_r)^2 g_t g_r}{d_{ps}^{-4}} \quad (1.69)$$

which equals -70.5 dBm. Assuming that the SU Tx antenna height and gain are 1.5 m and 4 dB respectively and that the distance between SU Tx and the SU Rx is  $d_{ss} = 250$  m, we obtain  $p_{ss} = -72.9$  dBm. Considering a channel bandwidth  $B = 2$  MHz, and an SU Rx noise figure equal to 18 dB, the total noise power is  $N = -103$  dBm. The cognitive pair transmits at a rate of 2 Mbit/s using BPSK modulation and, as explained in Section 1.2, each packet is assumed to arrive correctly if  $\text{BER} < 10^{-3}$ , therefore the SINR threshold is  $\gamma = 7$  dB. The PU traffic parameters are  $p = 10^{-3}$  and  $q = 2 \cdot 10^{-3}$ , resulting in an occupation probability  $P(H_1) = \frac{1}{3}$ . With these parameters we obtain the following thresholds for Rayleigh fading:  $p_{E_0}^* = -61.8$  dBm and  $p_{E_1}^* = -95.9$  dBm. The type I and type II error probabilities are  $P(\hat{H}_1|H_0) = 2.65 \cdot 10^{-6}$  and  $P(\hat{H}_0|H_1) = 0.1026$  respectively. The main parameters are summarized in Table 1.1.

Before evaluating the performance and robustness of optimally configured OSA with and without BD, it is interesting to assess how the performance parameters

(throughput  $T$  and collision probability  $P_c$ ) vary with the optimization parameter  $n$  (in time-slots). Figure 1.5 shows the throughput and collision performance versus  $n$  for both implementations. The values of  $T$  and  $P_c$  are obtained numerically and by simulation also, to validate the model. We can appreciate one of the main advantages of using BD: while  $P_c$  increases almost linearly with  $n$  when BD is not used, it does not surpass a notably low level when BD is used. The reason is that PU activity can be detected during SU transmission, implying the end of the transmission. Therefore, although  $n$  determines the maximum duration of a SU transmission when BD is used, the expected duration is determined by the PU activity detection. When BD is not used, larger values of  $n$  imply higher  $P_c$  and lower  $T$ , because transmission overlap also degrades SU performance. Seeing this figure, one can anticipate that the BD implementation may be more robust against parameter estimation inaccuracies, than the one without BD.

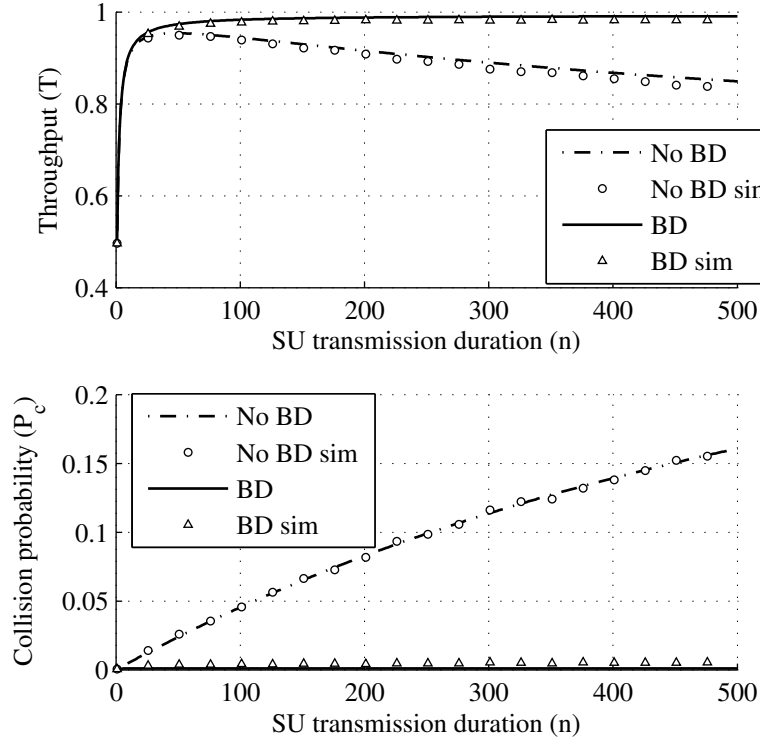


Figure 1.5: Throughput and collision performance with and without BS vs. the number of time-slots ( $n$ ) of the SU transmission

### Sensitivity to Estimation Inaccuracies

In this subsection we study how the optimized implementations of OSA and OSA with BD perform under drifts of the real values of some critical parameters. By doing this, we evaluate the robustness of each implementation against inaccuracies on the parameter estimation. With the parameters described in previous subsection, and setting the collision probability threshold  $\alpha = 0.025$ , the optimal  $n$  value for OSA without BD is 49. With BD,  $T(n)$  increases monotonically with  $n$ , while  $P(n)$

remains below 0.025. In this case we select  $n = 100$  which provides a throughput very close to the maximum, and would result in, at most, 0.05 collision probability in case of a complete malfunction of the BD mechanism.

Both schemes are heavily reliant on the characterization of the PU traffic in the channel. To evaluate the impact of traffic estimation inaccuracies, let us consider that the arrival traffic intensity  $p$  differs from the estimated one  $p_{\text{est}}$ , which is used for the computation of  $n^*$  in both schemes and for the threshold computation in BD. Figure 1.6 shows the performance versus the ratio  $p/p_{\text{est}}$ . We see that in both cases, the  $T$  decreases and  $P_c$  increases but, for the BD case the degradation is notably less severe than the no BD case, showing the higher robustness of BD against error in traffic estimation.

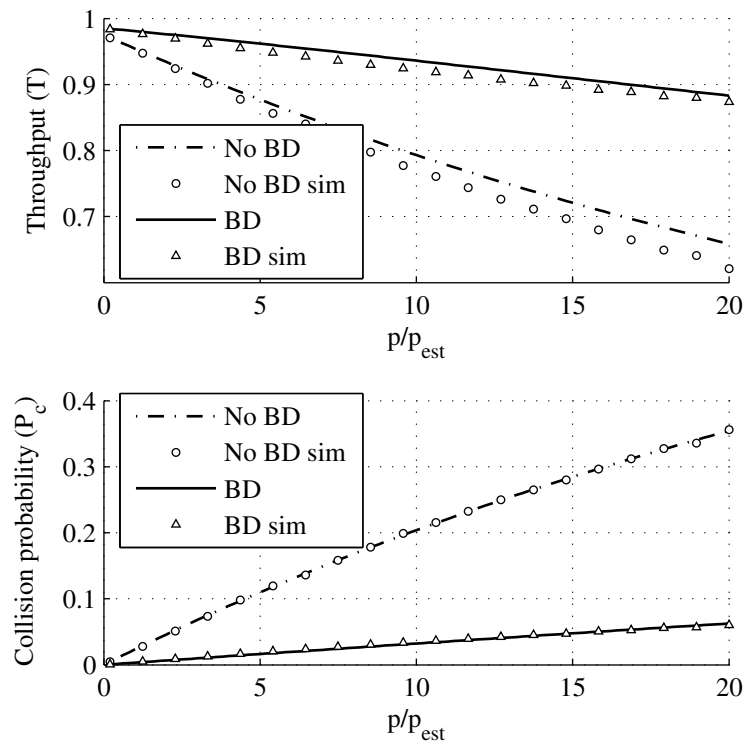


Figure 1.6: Throughput and collision performance with and without BS vs. the inaccuracy on the estimation of the traffic intensity

Another critical aspect in the BD scheme is the channel characterization. In the example developed, The MAP estimator is designed for a Rayleigh fading model for both PU and SU signals, assuming a previous knowledge of the average signal powers at the receiver. However it is possible that the SU misestimates the average power, especially for the PU signal ( $p_{ps}$ ), or even the channel statistical description itself. To assess the effect of these misestimations, we have simulated an OSA system with BD configured for Rayleigh fading, to obtain the performance under different  $p_{ps}$  estimation errors when the fading is Rayleigh, Ricean with  $K = 10$ , and Nakagami with  $m = 2$ . We also compare these results with the performance of an OSA system without BD in the same scenarios. As can be seen in Figure 1.7, OSA with BD maintains its advantage respect to the non BD system in almost every

case. The statistical characterization of the fading shows a moderate impact on BD performance. In contrast, overestimating  $p_{ps}$  shows to have the most harmful effect on the collision probability. The reason is that as the real  $p_{ps}$  decreases respect the estimated value, the probability of Type II error approaches 1 and, in consequence the OSA with BD tends to operate as an OSA without BD. In Figure 1.7, OSA BD is set to  $n = 100$  which, as explained previously, results in a  $P_c$  near 0.05 in case of BD malfunctioning. If OSA and OSA BD are both configured with the same  $n$  value, they present similar performance under  $p_{ps}$  overestimation in the scenarios of Figure 1.7.

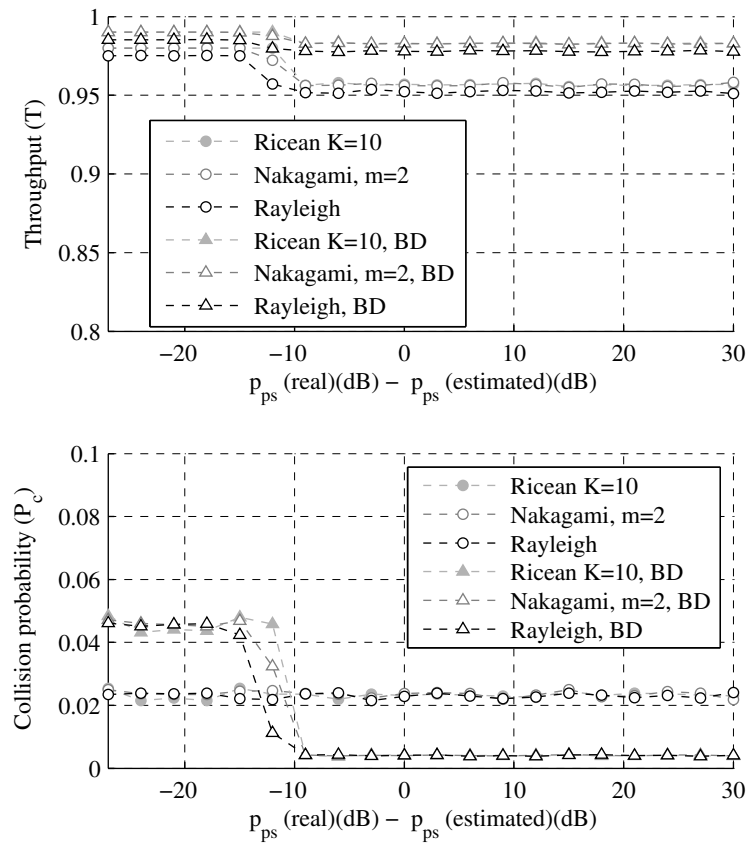


Figure 1.7: Throughput and collision performance with and without BS vs. the inaccuracy on the estimation of the traffic intensity

## 1.6 Conclusions



# Response Surface Methodology for Efficient Spectrum Reuse in Cellular Networks

## 2.1 Introduction

In this chapter we move on from mechanisms that ensure the proper operation of opportunistic access to develop a full DSA framework particularized for cellular networks.

Approximately every decade, a new cellular access technology is developed and introduced in the market. However, because not all users upgrade their terminals at the same pace, the operators have to keep their legacy networks working, and the spectrum assigned to these old networks becomes gradually more and more underused.

One way to increase the spectrum efficiency is to allow the users of the newest networks to opportunistically access the spectrum of the legacy networks. Applying the concepts of cognitive radio [174], the terminals of the newest network would be the secondary users (SUs), and the terminals of the legacy network would be the primary users (PUs). The secondary access to the spectrum of the primary (legacy) network should not cause noticeable degradation to PU transmissions.

The operator owns and control both the primary and the secondary networks, and therefore can establish the rules for secondary access and monitor the impact of these rules on the performance of both SU and PU transmissions. However, it is more profitable for the operator *not making any modifications on the legacy network*, implying that the operation of the secondary network is *transparent* to the primary network.

In this chapter, we propose a semi-decentralized secondary access scheme allowing each SU to access a set of primary frequency bands using either temporal or spatial spectrum holes (opportunities). By giving more access options to each SU, this approach can outperform previous, more limited, mechanisms. We consider

that the SUs establish point-to-point connections among them (cognitive pairs) in an ad-hoc fashion.

To fully exploit its potential, we describe an on-line algorithm that, with very small computational and signaling overhead, allows the system to learn the optimal SU access policy in terms of capacity.

## Related Work

Other works have proposed the combination of temporal and spatial sensing by the SUs [175], [176], [177], [178]. Works like [175] and [176] consider a single secondary transmitter and exploit information from spatial sensing to improve the performance of temporal sensing. The single transmitter model is also applied in [177] for vehicular networks. In [178] the capacity is optimized by randomizing the access strategies, but for a single PU, single SU scenario. However, the motivational scenario of our work comprises multiple secondary transmitters with multiple primary base stations. Thus, we must consider relevant features that are not captured by the single transmitter model, such as the inter-SU interference and the frequency reuse in a cellular structure.

In cellular networks, previous works like [179] and [180] have addressed spatial spectrum reuse by a secondary network. However, these works assume that the secondary users only scan one frequency. As pointed out by their authors, scanning all the frequencies would improve the performance. In contrast, we follow this latter approach, mixing it with the exploitation of temporal opportunities.

## Contribution

Our contribution in this chapter is summarized as follows:

- We develop a semi-distributed mechanism for opportunistic spectrum access combining access over temporal and spatial opportunities, in which *all the spectrum of the system can be made available to each SU*.
- Our method is especially conceived for cellular networks, considering multiple PU transmitters within a cell structure. The system is evaluated in a realistic setting considering irregular cell shapes.
- The most important feature, 3) *the system is capable of learning the optimal probability distribution over the frequency bands of the cellular network* to maximize SU capacity. The learning algorithm applies the response surface methodology (RSM) [183], which is a novel and promising approach to address interference management problems.

In the following section we describe the system. Section 2.3 formulates the design problem and Section 2.4 presents the RSM algorithm to solve it. Finally, Sections 2.5 and 2.6 presents the numerical results and the conclusions of this work.



## 2.2 System description

The system considered comprises: a legacy cellular network (primary network), a secondary network establishing ad-hoc point-to-point links, and a secondary access controller (SAC) which monitors the system performance and broadcasts the operation parameters of the SU access strategy, with small signaling overhead, as we discuss in Section 2.4. The primary network contains base stations (PBSs) and primary users (PUs). Each PBS covers a certain geographical area (primary cell) and is assigned a frequency band different from its adjacent cells.

We assume a frequency reuse scheme of 7 frequency bands (reuse factor 7), denoted by  $f_1, \dots, f_7$ . Nevertheless, the proposed method can be applied to other reuse factors as well. Each frequency band is divided into  $2N$  orthogonal channels ( $N$  downlink and  $N$  uplink channels). Secondary access is constrained to downlink channels. Time is divided into equal duration time-slots, which is usual in most cellular systems. The data transmitted over a single PU channel in a time-slot is referred to as *packet*.

The secondary network consists of pairs of secondary users (SUs) entering and leaving the system in an ad-hoc fashion. This network model can characterize femtocells, terminals acting as relays, or any short-range transmission using available spectral resources of the legacy cellular network.

The SAC is associated to the network to which the SUs belong. In particular, we assume that the SUs are the users of a new generation cellular network. Because both networks (the new generation network and the legacy one) belong to the same operator, we consider that the SAC can retrieve some information from the legacy network.

The SUs can detect the power of the pilot tones of the neighboring PBSs. With this information (and possibly with the aid of the SAC) each SU can infer its position with respect to the surrounding cells and therefore be aware of the PBS power levels from each frequency band. Indeed, the SU does not need to estimate exactly these PBS power levels, it just needs to establish an ordering of the frequencies  $f_1, \dots, f_7$  from lower to higher PBS power level. For a generic SU, let  $\phi = (\phi_1, \dots, \phi_7)$  be the vector of frequencies ordered in increasing received power, which clearly depends on the location of the SU. Note that  $\phi_7$  corresponds, in general, to the frequency band of the primary cell where the SU is located.

**Example.** In Figure 2.1, any SU located in area A has  $\phi_1 = f_1$  (similarly, any SU in area B has  $\phi_1 = f_2$ ). But because each SU is closer to a different PBS,  $\phi_7 = f_5$  for SU 1, and  $\phi_7 = f_6$  for SU 2. These SUs have also different values of  $\phi_2$  and  $\phi_4$ .

When an SU transmits over any band  $\phi_i$ , it may cause some interference to the PUs of the closest cells using this band (and the same channel within this band). In general, the level of SU interference caused at PU receivers is proportional to the ordering index  $i$  of the selected band  $\phi_i$ , *i.e.* a single SU transmission over  $\phi_1$  causes generally less interference than an SU transmission over  $\phi_2$ , and so on.

In our system, the classic strategy of occupying exclusively *spatial opportunities* implies that each SU only tries to access over  $\phi_1$ . However, what really matters

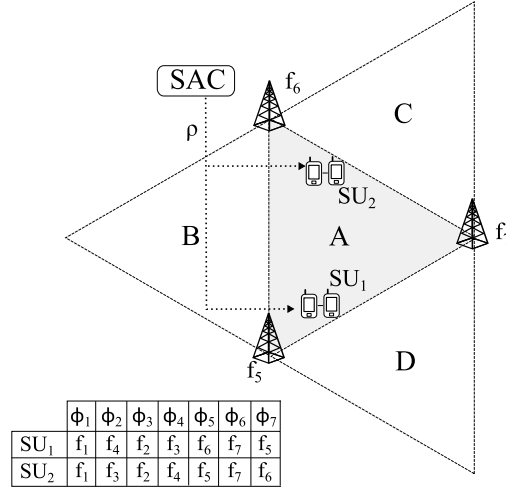


Figure 2.1: Example of the system with 2 SU pairs. Every SU in area A associates  $f_1$  to  $\phi_1$ . The remaining  $f_j$ - $\phi_i$  associations for each SU depend on how close the SU is to areas B, C or D. The SAC broadcasts a vector of operation parameters  $\rho$  to the SUs.

is the aggregate interference at the receivers. Note that, in accordance to the ad-hoc nature of the secondary network (small SU transmission range, low antenna heights, indoor locations sometimes) the SINR at the PU receivers can still be acceptable even if the interference from some SUs is a bit higher, *i.e.* if some SUs select other bands different from  $\phi_1$ . The potential benefit of giving more frequency band options to the SUs is to decrease the inter-SU interference, increasing the capacity of the secondary network.

Besides, when an SU tries to access the band of a close PBS, *i.e.* a band associated to a higher interference power such as  $\phi_7$ , the SU can perform *PU activity detection* on the channels within the band [174]. This activity detection allows the SU to detect free channels before starting transmission, and to stop SU transmission when detecting overlap with PU transmissions as our mechanism in chapter 1 does. The closer the SU is to the PBS, the more reliable is the PU activity detection and therefore the less impact on PU communication. The free channels in  $\phi_7$  are identified as *temporal opportunities*.

The typical strategies of using only spatial opportunities ( $\phi_1$  channels) or only temporal ones ( $\phi_7$  free channels) are not optimal as we show in Fig. 2.2. The achievable capacity for SUs using these strategies is limited, because of inter-SU interference and the limitation in the number of available channels when using temporal opportunities.

Regarding PU degradation, note that the interference from the SUs using  $\phi_7$  or  $\phi_6$  may be very intense but, thanks to the PU activity detection in closer cells, it should be infrequent and last short periods of time (depending on the detection accuracy). On the other hand, the interference from the SUs using bands that usually do not allow PU activity detection (*e.g.*,  $\phi_1$  or  $\phi_2$ ) will be sustained over time, but will be less intense (similar to the co-channel interference from other PBSs). To properly adjust the vector  $\rho$  we should combine the effects of these two

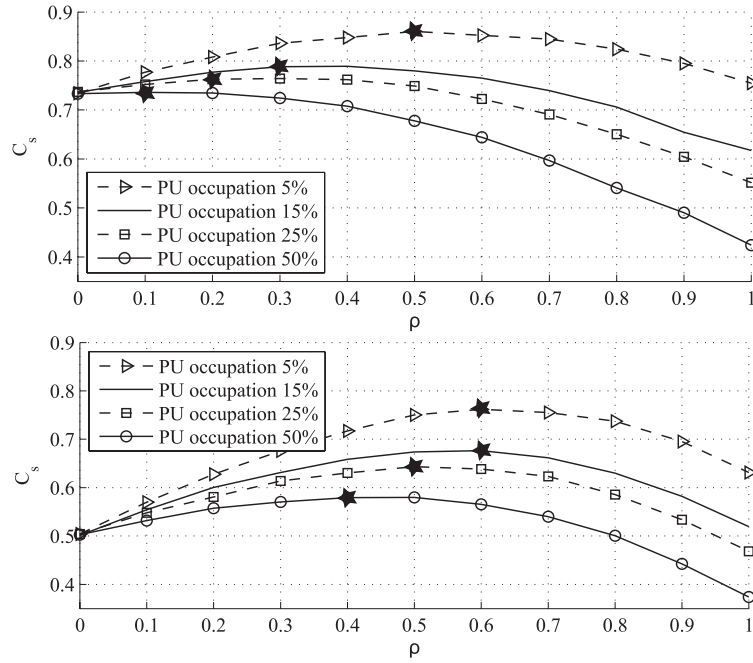


Figure 2.2: SU capacity versus  $\rho$ , the probability of using a temporal opportunity (and a spatial opportunity otherwise) under different PU traffic intensities. The number of new SU cognitive pairs at each time slot is generated by a binomial distribution characterized by 20 trials of 0.1 (top figure) and 0.2 (bottom figure) probability each. The rest of this reference scenario is described in section 2.5. The stars indicate the optimum SU capacity for each PU occupation.

types of interference into a single performance metric for the PUs: the probability of correct packet reception per time-slot.

Let  $R$  be the number of bands that an SU can use. We define the  $R$ -dimensional vector  $\boldsymbol{\rho} = (\rho_1, \dots, \rho_R)$ , where  $\rho_i$  denotes the probability of using band  $\phi_i$ , except for  $\rho_R$  which always corresponds to the probability of using band  $\phi_7$  (pure temporal access). Therefore,  $0 \leq \rho_i \leq 1$  for  $i = 1, \dots, R$ , and  $\sum_{i=1}^R \rho_i = 1$ . Let  $\mathcal{P}$  denote the set of  $R$ -dimensional vectors fulfilling these conditions.

The following steps summarize the **system operation**:

1. The SAC periodically updates the vector  $\boldsymbol{\rho}$  and sends it to the SUs.<sup>1</sup>
2. Each SU builds its own  $\boldsymbol{\phi}$  vector by scanning the PBS pilot tones on each frequency band. This vector only needs to be updated when the SU changes its location.
3. Before transmitting, an SU randomly selects one band,  $\phi_i$  from its  $\boldsymbol{\phi}$  vector, according to the probability distribution  $\boldsymbol{\rho}$ , i.e.  $P(\text{select band } \phi_i) = \rho_i$ .

<sup>1</sup>It is advisable to divide the system into relatively homogeneous regions in terms of PBS density, traffic intensity and type of terrain, so that a suitable  $\boldsymbol{\rho}$  can be found for each region.

4. If the SU is capable of detecting enough PBS power in  $\phi_i$  to perform PU activity detection, then the SU tries to access over temporal opportunities, occupying only channels free of PU activity. Otherwise, the SU will access any channel with a sufficiently low SINR (considering also the interference from other SUs).
5. Periodically, the SAC retrieves information from both the primary and the secondary networks, particularly performance measures, which are taken into account to update vector  $\boldsymbol{\rho}$ .

This mechanism allows a semi-decentralized resource allocation. The SAC only needs to *announce* the vector  $\boldsymbol{\rho}$  and retrieve small pieces of information measured at the terminals. The signaling overhead is therefore very low, especially compared to a centralized channel allocation by the SAC. If  $\boldsymbol{\rho}$  is properly adjusted, the SUs will allocate themselves over the legacy spectrum, autonomously, efficiently, and with a low impact on the primary network. Moreover,  $\boldsymbol{\rho}$  can be learned by the SAC with the RSM scheme explained in Section 2.4.

Next section formalizes the problem in terms of SU achieved capacity and PU probability of correct packet reception.

## 2.3 Problem Formulation

In this section we define the performance metrics for both the PUs and SUs and formulate the problem that the SAC needs to solve to determine the  $\boldsymbol{\rho}$  vector.

Let us consider a secondary network with a given traffic intensity characterized by its arrival rate per area unit, and a random transmission time. As stated above, the vector  $\boldsymbol{\rho}$  determines how the SUs distribute themselves over the available spectrum and the proportion of SUs using PU activity detection. In consequence, the SINR at each PU or SU receiver depends on  $\boldsymbol{\rho}$ . Let  $N_t^{SU}$  and  $N_t^{PU}$  denote the number of SUs and PUs in the system, respectively, at time-slot  $t = 1, 2, \dots$ . The SINR over time at SU receiver  $s \in \{1 \dots N_t^{SU}\}$ , is a discrete time stochastic process induced by  $\boldsymbol{\rho}$ , and denoted by  $\Gamma_t^s(\boldsymbol{\rho})$ . The expected normalized capacity per active SU pair is given by

$$C_s(\boldsymbol{\rho}) = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \sum_{s=1}^{N_t^{SU}} \frac{\log_2(1 + \Gamma_t^s(\boldsymbol{\rho}))}{N_t^{SU} C_{\max}^S} \right\} \quad (2.1)$$

where  $C_{\max}^S$  is the maximum achievable SU capacity per hertz. The expectation is taken over  $\Gamma_t^s$  and  $N_t^{SU}$ .<sup>2</sup> At a PU receiver, it is assumed that a data packet transmitted on time-slot  $t$  is correctly decoded if its SINR,  $\Gamma_t^{PU}$ , is greater than a given detection threshold,  $\gamma^{PU}$ . Then, the probability of correct detection at a PU

<sup>2</sup>Note that the traffic intensity in a cellular communication network varies during a day, but if it is observed during a smaller time window, e.g. 1 hour, the traffic arrival process can be considered stationary, with constant intensity.

is defined as

$$P_c(\boldsymbol{\rho}) = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \sum_{j=1}^{N_t^{PU}} \frac{\mathbb{I}_{\{\Gamma_t^j(\boldsymbol{\rho}) > \gamma^{PU}\}}}{N_t^{PU}} \right\} \quad (2.2)$$

where  $\Gamma_t^j$  refers to the SINR process at the  $j$ -th active PU, and  $\mathbb{I}_{\{z\}}$  is an indicator function which equals 1 if condition  $z$  holds, and equals 0 otherwise.

Therefore, the objective of the SAC is to find  $\boldsymbol{\rho}$  solving the following problem:

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & C_s(\boldsymbol{\rho}) \\ \text{s.t.} \quad & P_c(\boldsymbol{\rho}) \geq P_{c,\min} \\ & \boldsymbol{\rho} \in \mathcal{P} \end{aligned} \quad (2.3)$$

where  $P_{c,\min}$  denotes the minimum acceptable  $P_c$ .

Finding an optimal  $\boldsymbol{\rho}$  is a challenging task because the (multiple) SINR stochastic processes capture the interaction of a random number of randomly located terminals with fading effects among each pair of them. Moreover, these processes and the decision vector  $\boldsymbol{\rho}$ , take values from continuous spaces, and therefore conventional dynamic programming techniques result infeasible. The problem (2.3) is, in fact, a *stochastic optimization problem with stochastic constraints* [183]. As usual in this type of problems, the SAC has to dynamically *learn* an optimal  $\boldsymbol{\rho}$ . One feasible way to address it is by means of Surface Response Methods (RSM) [184]. However, the inclusion of stochastic constraints introduces high complexity in the formulation. In the following section we develop the particular case in which  $P_{c,\min}$  is sufficiently low for condition  $P_c(\boldsymbol{\rho}) \geq P_{c,\min}$  to hold at the optimal  $\boldsymbol{\rho}$  (in numerical results,  $P_c(\boldsymbol{\rho})$  at the optimal  $\boldsymbol{\rho}$  is never less than 5% of the  $P_c$  obtained in absence of SU access). This allows us to remove the stochastic constraint regarding  $P_c(\boldsymbol{\rho})$ . We leave as future work the inclusion of constraints in the RSM formulation.

## 2.4 Response Surface Method (RSM) Algorithm

The aim of the algorithm is the maximization of the expected value of the average capacity function (2.1) on a closed convex feasible domain  $\mathcal{P} \subset \mathbb{R}^R$  for the input vector  $\boldsymbol{\rho}$ , that is, to find a  $\boldsymbol{\rho}$  approximately solving

$$\max_{\boldsymbol{\rho} \in \mathcal{P}} C_s(\boldsymbol{\rho}) \quad (2.4)$$

RSM allows us to find an approximate solution to this problem by successively estimating the gradient of the objective function and using these estimations in stochastic gradient ascent steps. At each one of these steps, numbered by  $n = 1, 2, \dots$ , the system generates one update of the input vector  $\boldsymbol{\rho}_{(n)}$  according to the following expression:

$$\boldsymbol{\rho}_{(n+1)} = \boldsymbol{\rho}_{(n)} + \alpha_{(n)} \hat{\nabla} C_s(\boldsymbol{\rho}_{(n)}) \quad (2.5)$$

where  $\hat{\nabla} C_s$  denotes the estimation of the gradient  $\nabla C_s$ , and  $\alpha_{(n)}$  is the step-size weighting factor. A standard condition for the selection of  $\alpha_{(n)}$ , assuring the convergence of  $\boldsymbol{\rho}_{(n)}$  [183], is  $\sum_{n=1}^{\infty} \alpha_{(n)} = \infty$ ,  $\sum_{n=1}^{\infty} \alpha_{(n)}^2 < \infty$ .

Let us discuss the computation of  $\hat{\nabla}C_s$ . Given  $\rho_{(n)}$  at step  $n$ , we consider a subdomain  $\mathcal{S}_{(n)}$  of the feasible domain  $\mathcal{P}$  such that  $\rho_{(n)} \in \mathcal{S}_{(n)} \subset \mathcal{P}$ . Note that  $\rho_{(n)}$  is the point at which the estimate  $\hat{\nabla}C_s$  must be computed. Therefore, we need to estimate the objective function  $C_s(\rho_{(n)})$  on  $\mathcal{S}_{(n)}$ , by taking samples  $y^{(i)}$ ,  $i = 1, \dots, p$ , of the function. For this, we need a finite set of points  $\rho_{(n,i)}$ ,  $i = 1, \dots, p$ , generally called *design points*, belonging to  $\mathcal{S}_{(n)}$ . These points are chosen by the decision maker (the SAC in our case), and can be, for example, random perturbations of  $\rho_{(n)}$ , falling within  $\mathcal{S}_{(n)}$ .

Let  $t_{(n)}$  denote the time-slot in which the update  $\rho_{(n)}$  is obtained. Given the set of  $p$  decision points, the SAC can obtain samples  $y^{(i)}$  by this simple procedure:

1. Determine  $p$  sampling instants  $t_{(n,i)} = t_{(n)} + iT$  for  $i = 1 \dots p$ , where  $T$  is a sufficiently long time period for measuring performance at the SUs.
2. At  $t_{(n,i-1)}$  (where  $t_{(n,0)} = t_{(n)}$ ), the SAC signals the design point  $\rho_{(n,i)}$  to the SUs, for  $i = 1 \dots p$ .
3. At  $t_{(n,i)}$ , the SAC obtains the capacity samples from each active SU and averages them to obtain  $y^{(i)}$ .

Thus, the estimates can be expressed as:

$$y^{(i)} = \frac{1}{T} \sum_{k=t_{(n,i-1)}}^{t_{(n,i)}} \sum_{s=1}^{N_k^{SU}} \frac{\log_2(1 + \Gamma_k^s(\rho_{(n,i)}))}{N_k^{SU} C_{\max}^S} \quad i = 1, 2, \dots, p, \quad (2.6)$$

Note that the stochastic ascent algorithm needs  $p$  periods of length  $T$  to upgrade  $\rho_{(n+1)}$ .

The objective function  $C_s$  is then approximated on  $\mathcal{S}_{(n)}$  by a polynomial response surface model  $\hat{C}_s(\rho) = \hat{C}_s(\rho|\beta_0, \beta_1 \dots \beta_R)$ . The coefficients  $\beta_j$ , are determined by least squares estimation. Therefore, the RSM-gradient estimator  $\hat{\nabla}C_s(\rho_{(n)})$  at  $\rho_{(n)}$  is defined by the gradient (with respect to  $\rho$ )

$$\hat{\nabla}C_s(\rho_{(n)}) = \nabla \hat{C}_s(\rho_{(n)}). \quad (2.7)$$

Thus,  $C_s$  is estimated on  $\mathcal{S}_{(n)}$  by the linear empirical model

$$\hat{C}_s(\rho_{(n)}) = \beta_0 + \beta_l^T (\rho - \rho_{(n)}) \quad (2.8)$$

where

$$\beta^T = (\beta_0, \beta_l^T) = (\beta_0, \beta_1, \dots, \beta_R)^T \quad (2.9)$$

is the  $(R+1)$ -vector of unknown coefficients of the linear model. Having samples  $y^{(i)}$  of the function values  $C_s(\rho_{(n,i)})$  at the design points  $\rho_{(n,i)}$ ,  $i = 1, \dots, p$ , in  $\mathcal{S}_{(n)}$ , we can obtain, by least squares, the following estimate  $\hat{\beta}$  of  $\beta$ :

$$\hat{\beta} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \mathbf{y}. \quad (2.10)$$

Here, the  $p \times (R + 1)$ -matrix  $\mathbf{W}$  and the  $p$ -dimensional vector  $\mathbf{y}$  are defined by

$$\mathbf{W} = \begin{pmatrix} 1 & \delta^{(1)} \\ 1 & \delta^{(2)} \\ \vdots & \vdots \\ 1 & \delta^{(p)} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(p)} \end{pmatrix} \quad (2.11)$$

with  $\delta^{(i)} = \boldsymbol{\rho}_{(n,i)} - \boldsymbol{\rho}_{(n)}$ , for  $i = 1, 2, \dots, p$ . Note that  $(\mathbf{W}^T \mathbf{W})$  in (2.10) is invertible whenever the columns of  $\mathbf{W}$  are linearly independent, which can be easily guaranteed by a proper selection of the design points.

In case  $C_s(\boldsymbol{\rho})$  is concave on  $\mathcal{P}$ , the RSM algorithm will approach the global optimum following the stochastic gradient ascent iterations (2.5) as it has been observed for  $R \geq 2$  dimensions in the scenarios considered in the following Section.

## 2.5 Numerical Results

### Evaluation Framework

This subsection describes the Monte-Carlo methodology and the scenario used to evaluate  $C_s$ . The primary network uses a 7-band frequency planning, as previously stated, with irregular shaped cells having an average radius  $r = 700$  m. We consider pathloss and multipath fading. To compute the pathloss attenuation over distance,  $A(d)$  (dB), we use the following piecewise dual-slope model [125]:

$$A(d) = \begin{cases} K + 10\gamma_1 \log_{10}(d/d_0) & d_0 \leq d \leq d_c \\ K + 10\gamma_1 \log_{10}(d_c/d) + 10\gamma_2 \log_{10}(d/d_c) & d > d_c \end{cases} \quad (2.12)$$

The critical distance  $d_c$  is notably smaller for SU transmission than for PBS transmissions, because the PBSs are located at high outdoor locations while SUs are, in general, located either indoors or at ground level. All the signals are assumed to experience Rayleigh fading.

The area used to generate random terminal locations is a  $4.2 \text{ km} \times 4.8 \text{ km}$  rectangle. We focus on the downlink channels of one frequency band. We consider an scenario in which the primary traffic intensity is low  $0.16 \text{ Erlangs/km}^2$ , so that the average occupation of the spectrum by the primary terminals is only 5%. In contrast, the SU traffic intensity,  $27.5 \text{ Erlangs/km}^2$ , is high in comparison.

In this case, the SU capacity is mostly determined by the inter-SU interference and the RSM algorithm is essentially performing *interference management* in the SU network. We also discuss the effect of higher PU spectrum occupation and a different SU traffic intensity.

According to the ad-hoc nature of the secondary network, the average SU link distance considered is 90 m. Table 2.1 summarizes the simulation parameters considered.

Parameter	Assigned value
<b>Primary transmitters</b>	
number of downlink channels, $N$	5
cell radius, $r$	700 m
average received power at PU	-78 dBm
SINR detection threshold at PU, $\gamma^{PU}$	-17 dB
baseline noise at PU ( $N_0 + I_{PU}$ )	-110 dBm
<b>Secondary transmitters</b>	
average SU Tx power per channel	0.5 W
SU link distance	90 m
probability of PU activity detection	0.9
probability of overlap detection	0.8
<b>Propagation parameters</b>	
pathloss exponents, $\gamma_1, \gamma_2$	2.4, 4.2
propagation factor $K$	46.7 dB
critical distance for PBS transmission	$1.2r$ m
critical distance for SU transmission	100 m
<b>RSM parameters</b>	
dimensions of $\rho, R$	1 ... 5
measuring period $T$ (in time-slots)	300
number of samples per step $p$	$3(2^{R-1} + 1)$

Table 2.1: Parameter setting of the reference scenario used in numerical evaluations

## Convergence and Usage of the Frequency Bands

Let us consider the case of 3-dimensional  $\rho$  vectors,  $R = 3$ . In this case, the  $\rho$  vector of a particular SU is  $\rho = (\rho_1, \rho_2, \rho_3)$ , being  $\rho_3$  the probability of trying to access the band  $\phi_7$  of the primary cell where the SU is in, and  $\rho_1$  and  $\rho_2$  the probabilities of accessing  $\phi_1$  and  $\phi_2$ , the bands where the SU receives less PBS power. The SU can only perform PU activity detection on  $\phi_7$  channels, while the channels of the other bands are considered spatial opportunities. Fig. 2.3 shows the values of  $\rho_1$  and  $\rho_2$  ( $\rho_3$  is simply  $1 - \rho_1 - \rho_2$ ) over consecutive update steps  $n = 1, 2, \dots$ . The initial vector is  $\rho_{(0)} = (0, 0, 1)$ . Although  $\rho_0$  is a rather poor initial guess, we see that the RSM algorithm stabilizes after 20 iterations. The final value is  $\rho = (0.29, 0.42, 0.29)$ . Fig. 2.4 shows the estimated value of  $C_s$  as a function of  $\rho_1$  and  $\rho_2$ . Consistently with the result obtained by RSM, the maximum values of  $C_s$  lie on the line  $\rho_1 + \rho_2 = 0.7$ .

Let us now consider the results for  $R = 5$ . The probability  $\rho_5$  is now associated to  $\phi_7$ , allowing PU activity detection, while  $\rho_1 \dots \rho_4$  are associated to  $\phi_1 \dots \phi_4$ , where PU activity detection is assumed to be unfeasible. The initial  $\rho$  vector is  $(0, 0, 0, 0, 1)$ . Fig. 2.5 shows that, in this case, the convergence of RSM is as fast as with  $R = 3$ . Note that, as in the case of  $R = 3$ , the algorithm is not assigning  $\rho_1$  the highest probability. The reason is that the algorithm aims to minimize inter



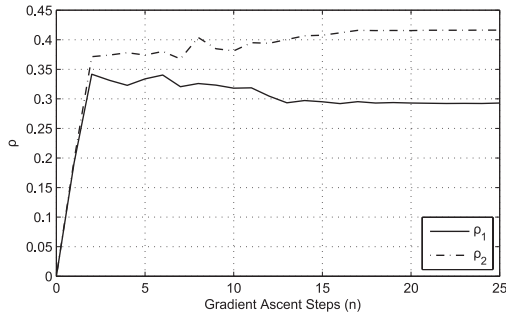


Figure 2.3: Consecutive RSM updates of  $\rho_1$  and  $\rho_2$ .

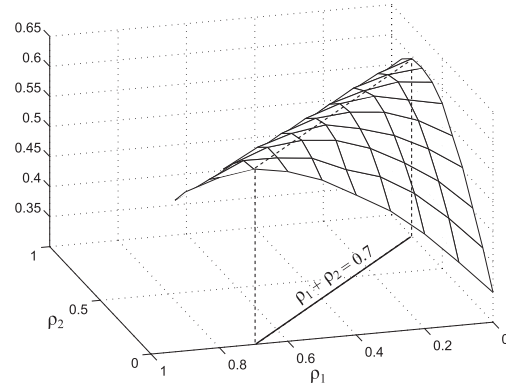


Figure 2.4: Estimation of  $C_s$  over  $\rho_1$  and  $\rho_2$ .

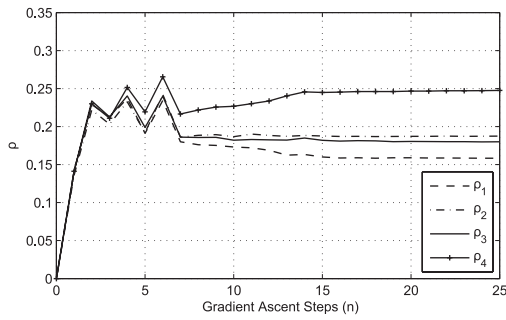


Figure 2.5: Consecutive RSM updates of  $\rho_1 \dots \rho_4$ .

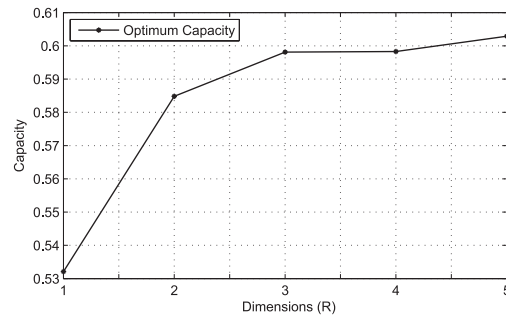


Figure 2.6: Maximum SU capacity obtained with RSM versus the number of dimensions of  $\rho$ ,  $R$ .

SU interference by separating the SU transmitters using the same band. Recall that in Fig. 2.1 all the SUs in triangular region A shared the same  $\phi_1$  but had different values for  $\phi_2$ . The aggregated probability of using spatial opportunities is  $\sum_{i=1}^4 \rho_i = 0.77$ , which is higher than in the  $R = 3$  case. This result confirms the idea that, the more spectrum is available to each SU, the more spectrum the SU exploits. Having more spectrum options per SU (higher  $R$ ) also allows the SU network to achieve a higher capacity, as illustrated by Fig. 2.6.

## Effect of the Traffic Intensity

Fig. 2.7 shows the average SU capacity attained by the RSM algorithm with  $R = 2$ , versus the average spectrum usage by the PUs, for different SU traffic intensities. As expected, the more spectrum occupied by PU traffic, the smaller the achieved capacity. Similarly, more SU traffic implies less SU capacity. The reason of the reduction on the maximum achieved capacity is the increment of the aggregate interference power at the SU receivers, from both the PBSs and other SU pairs. Compared to the typical approach of exploiting only temporal opportunities, the benefit of using RSM is noticeable for every traffic situation. In all cases, the reduction of the probability of correct detection at the PU receivers ( $P_c$ ) was less than 5%.

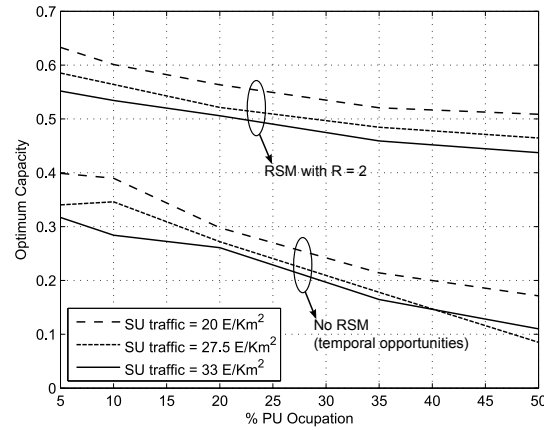


Figure 2.7: Maximum SU capacity obtained with RSM under different traffic intensities. For comparison, the figure also shows the capacity for a SU network exploiting only temporal opportunities (no RSM).

## 2.6 Conclusion

Motivated by the problem of spectrum reuse in cellular networks, we presented a semi-distributed mechanism allowing the secondary network to learn the most efficient spectrum access strategy. This mechanism exploits both spatial and temporal opportunities, and is especially effective for interference management in highly dense secondary networks. The learning approach is based on response surface methodology (RSM) which, according to our numerical results, improves notably the system capacity compared to usual strategies, and shows a fast convergence rate even when it is poorly initialized. Because of that, the system can adapt its control vector  $\rho$  to variations on the traffic intensity or user distribution. Surprisingly, to the best of our knowledge, the use of RSM on this framework had not been previously reported. Our future work is focused on incorporating performance constraints for the primary network in the RSM formulation.

## **Part II**

### **Automated spectrum trading**



## **Final conclusions**



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