

# Coexistence policies in cognitive radio

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## Abstract

Public regulatory agencies have traditionally granted access to radio-electric spectrum through fixed, long term licenses for large geographical areas. The increasing demand of wireless communication has led to an almost fully assigned spectrum. However, it is sparsely and unevenly used. A dramatic improvement on spectrum usage efficiency is possible through the operation of cognitive radios, capable of gathering information about their surrounding spectrum environment and adapting their transmission parameters accordingly. Cognitive radios allow unlicensed users to dynamically exploit unused fragments of spectrum. This thesis emphasizes the use of mathematical tools such as stochastic modeling, dynamic programming, reinforcement learning, among others, in the development of algorithms for dynamic spectrum access. Our main concern in this work is protecting the licensed users from interferences caused by unlicensed users' transmissions, and creating incentives for the licensed users to implement mechanisms for unlicensed access to spectrum. Another important guideline of our work is the focus on low-complexity algorithms that can operate at the small time-scale of the spectrum opportunities. The first part of this thesis focus on protection, through the development of several mechanisms for Opportunistic Spectrum Access, in which unlicensed users scan the spectrum to detect transmission holes so that they can transmit causing as less possible interference to licensed users. In the second part, we focus on the incentives, by developing two frameworks for spectrum trading, in which the licensed users can lease unused bandwidth to unlicensed ones, in exchange of money and relay services, respectively. We also include an extensive survey on the topic, unique in its scope.



# Introduction

## 1.1 Motivation

Wireless communications need, among other resources, radio-electric spectrum, which is finite. Government agencies, such as the FCC in the United States or the Electronic Communications Committee (ECC) through national administrations in Europe, grant access to it through fixed, long-term licenses on large geographical areas, in what is called the "command-and-control" management scheme. Licenses were given to operators in national lotteries and comparative hearings first, and later through spectrum auctions. Nevertheless, the ever-growing demand of spectrum due to the popularization of mobile services has left these traditional policies obsolete since almost all the spectrum has already been assigned.

However, several reports have been pointing out that most of the licensed spectrum show very little use. In 2002, the FCC Spectrum Policy Task Force [4] highlighted that typical channel occupancy of some public safety community, for example, could be less than 15%. The European Commission, in 2007, also stated that "some users hold large amounts of valuable spectrum that they do not use to its full capacity" [1].

From early on, experts suggested [92] it would be necessary to switch to a more flexible management to improve spectrum efficiency. In the US, the FCC started allowing license holders to lease their licenses under different constraints in 2003 [7], whereas in Europe is taking some more time: by 2011, only 9 out of 22 european countries surveyed by the Electronic Communications Committee (ECC) allowed spectrum leasing [2].

Cognitive radios [5] could elasticize spectrum management even further by enabling the automatic exploitation of unused spectrum opportunities anywhere in time, space, frequency, power and/or in CDMA codes. A *cognitive radio* has two main characteristics [3]: cognitive capability and reconfigurability. Cognitive capability refers to the ability of the radio to become aware of its surrounding radio environment. Reconfigurability refers to the ability of the radio of changing its transmission parameters dynamically.

In the most typical scenario, secondary users (SUs, unlicensed), equipped with cognitive radios, would look for unused fragments of spectrum and would use them for their transmissions subject to interference constraints for the protection of primary users (PUs, licensed). This situation corresponds to the Hierarchical Access Model, also known as "Opportunistic Spectrum Access", one of the possible coexistence approaches under Dynamic Spectrum Access (DSA) but there are others [94]. Along the lines of current regulation models is the Dynamic Exclusive Use Model, which considers leasing or selling licenses but at a smaller time-scale, even real-time and free choice of technology. Lastly, the Spectrum Commons Model assumes open sharing with no categories of users.

DSA has recently gained popularity because of the TV digitalization, in order to take advantage of the spectrum opportunities it has generated. Therefore, the last few years have seen some DSA standards such as IEEE 802.22 [9], ECMA-392 [10], 802.11af [12], 802.19.1 [11] and IEEE SCC41 [13].

No matter which model is considered, spectrum sharing can annoy licensed operators at present, because it may imply additional costs to them such as changes in their infrastructures, QoS impairments due to interferences or profit reduction due to increased competition, while they have already paid for their licenses. Protecting and creating incentives for PUs instead of forcing them avoids that actual spectrum owners as well as prospective ones feel discouraged from investing in new spectrum technologies and services: "why am I going to spend money in spectrum if the government is going to give it for free to others?".

Therefore, it is needed that these operators believe in an automated mechanism that controls their profits. But this idea alone is scary. "Is there any security on obtaining benefit"? It is hard to convince the telecommunications industry to adopt automated dynamic spectrum management mechanisms if there is no strong evidence that these algorithms do not lead to economic losses (due to disadaptation to real time, for example). Optimality, then, remains in the background.

Our<sup>1</sup> *motivation* in this thesis is, therefore, *to develop dynamic spectrum access mechanisms with a strong focus on the protection and creation of incentives to the PUs*. On the other hand, secondary users also need an incentive to request spectrum from the primary operator (e.g. customized tariffs). Otherwise, the approach will not work because they would all prefer to become primary users.

In addition, the main tendency of previous (and contemporaneous) solutions in the area has been building increasingly complex models, which deal with more issues of spectrum in more convoluted scenarios, and aiming to solve them optimally. Nonetheless, the resources required to reach such elaborated solutions have not been considered enough, specially time. Time consumption is a relevant issue because, as we have been pointing out throughout our survey, multiple important

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<sup>1</sup>I will be using the first person plural voice by default throughout my thesis. Most of the ideas and work is a team effort involving my supervisor and other authors. This should not be detrimental of showing off my autonomy, but even my personal decisions were strongly inspired by others' opinions. Meaningful research is not done (should not be done) by a single person in a vacuum. Thus, I consider artificial the use of "I" here. In other parts of this work, such first person plural will also include the reader, primarily when describing mathematical derivations, as a natural way of walking him through the process.



parameters in spectrum trading experience rapid variations over time: spectrum opportunities, demand (unplanned peaks of traffic), valuation of the spectrum (changes in channel gains, mobility of entities), etc. In consequence, we think that *time consumption should be regarded as a key feature in spectrum management algorithms*: if obtaining an optimal solution takes so long that the systems parameters vary significantly during the computation time, this solution will be not optimal when applied. In addition, the more time devoted to negotiation among the agents, the less time used in transmission. For that reason, it is more practical to design models that may not fully exploit spectrum in a particular moment (for example, by not taking into account spectrum geographical reuse) but that are capable of reducing the uncertainty due to the changes over time, and thus, providing higher guarantees on the agents satisfaction.

## 1.2 Contributions and thesis structure

Our contribution can be divided into two main blocks, coincident with the structure of the thesis. Part I features our works in the field of Opportunistic Spectrum Access. These works are focused on protecting the PUs from interference caused by SUs access to spectrum, while trying to increase the spectrum usage efficiency. The most common approach in previous literature is to study the sensing and access from the SUs perspective, that is, the algorithms the SUs follow (independently if the works are addressing the impact on PUs or on SUs). Conceptually, cognitive radios, especially when used under an OSA approach, is to use the cognitive capabilities of these users to make their access transparent (or as transparent as possible) to the PUs. However, our works introduce the novel approach of giving more power to the PU: they focus on PU spectrum management strategies. More specifically:

- Chapter ??: Background Detection of Primary User Activity in Opportunistic Spectrum Access. We propose a collision detection mechanism for the SU, simultaneous to packet reception. Our proposal exploits local characterization of the PU activity, power levels and packet errors at each time slot. Based on these data, we develop a Maximum *A Posteriori* (MAP) estimator as the basis for the collision detection system. We describe the general methodology to build this estimator, and study the system's robustness against parameter misestimations. The proposed scheme can be incorporated to existing OSA protocols. Our results show a notable throughput improvement for the SUs while assuring a lower collision probability with PU transmissions in most cases, even under severe estimation inaccuracies.
- Chapter ??: Bandwidth Reservation as a Coexistence Strategy in Opportunistic Spectrum Access Environments. We study a bandwidth reservation (BR) scheme by which the PN keeps a set of adjacent channels free of PU transmissions. These reserved channels only accommodate PU traffic when all the non-reserved channels are used, and the SUs only occupy available channels

within the reserved spectrum. From the point of view of the PN, BR entails a tradeoff between the benefits of an improved coexistence with SUs, and the capacity reduction associated to having fewer options for PU channel allocation. The SUs are characterized by a hardware limited radio, imperfect spectrum sensing, bayesian estimation of PU activity and multichannel access. By means of a Markov-reward model, we compute the expected PU capacity with and without BR, considering propagation effects, interference and random locations. The results show that, in a non-congested PN with SU activity, the interference reduction capability of BR increases the overall capacity of the PN compared to not using BR.

- Chapter ??: On the Impact of Licensed Users Management on the Efficiency of Opportunistic Spectrum Access. The main contribution of this work is to evaluate resource management strategies for licensed users that are intuitively more suitable for coexistence with OSA nodes, but not implying a direct cooperation with them. We also develop an OSA policy optimization procedure handling partial and unreliable spectrum observations at the SUs. The formulation combines SU throughput and PU performance degradation, characterized by the overlapping time with SU activity which is a more general parameter than collision probability. Although it may raise some concerns about the robustness of this approach, our results show that estimation inaccuracies have a very small effect on the quality of service attained by the PUs.
- Chapter 2. Response Surface Methodology for Efficient Spectrum Reuse in Cellular Networks. As cellular network technology evolves, the operators deploy new generation networks while maintaining their legacy networks, since not all users upgrade their terminals at the same pace. Therefore, the spectrum associated to these legacy networks becomes gradually underused. By means of cognitive radio techniques, the operator can allow its new generation terminals to reuse this spectrum. We propose a semi-decentralized scheme in which a cellular operator guides the secondary access by broadcasting some operational parameters of the access strategy. The mechanism dynamically learns the optimal parameters by means of a response surface methodology (RSM), implying a very small signalling overhead. Our results show a notable capacity improvement compared to the classical approaches of exploiting spatial or temporal opportunities.

Part II of the thesis contains those works in the field of automated spectrum trading. It is especially in this part where the our low-complexity-intention has a greater significance, since most previous works are dealing with great convoluted markets and neglecting the importance of real-time adaptation, as our survey in chapter ?? shows. Our contribution in this topic can be summarized as follows:

- Chapter ??. Automated spectrum trading mechanisms: understanding the big picture. This work presents a general view of automated spectrum trading. Our work is unique in its scope, focusing on automated spectrum trading

as the most promising mechanism to solve spectrum scarcity. We identify the specific aspects that make spectrum different from conventional goods and the impact that these features have on automatic trading. We discuss past, current approaches and future research lines, highlighting their advantages and disadvantages. Each sub-topic is presented and developed explicitly with references to previous works where each aspect has been considered or addressed. Finally, as a conclusion of the surveying effort, we highlight overlooked issues of automated spectrum trading, with a special focus on practical implementation.

- Chapter ?? An MDP Framework for Centralized Dynamic Spectrum Auction. This work addresses the design of centralized DSA MAC protocols comprising dynamic spectrum auction. We explore the possibilities of a formal design based on a Markov decision process (MDP) formulation. We propose a design framework to balance the grade-of-service (given by the blocking probability for LUs) and the expected economic revenue provided by the SUs' accepted bids. This trade-off can be managed in two ways. One consists of computing a single objective value given by a combination of the blocking probability and the expected revenue. The weights assigned to each objective determine the point in the Pareto front where the obtained policy lies. The other approach, consists of solving for one of the objectives and setting constraints on the remaining ones. Our paper focuses on centralized trading in which we explore the use of MDP and CMDP formulations to balance benefit and grade of service for LUs. The main advantages of this approach is that it assures operating at global optimum and reduces the computational effort at SUs.
- Chapter ?. A superprocess with Upper Confidence Bounds for Cooperative Spectrum Sharing. In this last work we focus on Cooperative Spectrum Sharing (CSS). CSS is an appealing approach for primary users (PUs) to share spectrum with secondary users (SUs) because it increases the transmission range or rate of the PUs. Most previous works are focused on developing complex algorithms which may not be fast enough for real-time variations such as channel availability and/or assume perfect information about the network. Instead, we develop a learning mechanism for a PU to enable CSS in a strongly incomplete information scenario with low computational overhead. Our mechanism is based on a Markovian variant of multi-armed bandits (MABs) called superprocess, enhanced with the concept of Upper Confidence Bound (UCB) from stochastic MABs. By means of Monte-Carlo simulations we show that, despite its low computational overhead, it converges to the optimal solution and outperforms previous approaches such as  $\epsilon$ -greedy. This algorithm can be extended to include more sophisticated features while maintaining its desirable properties such as low computational overhead and fast speed of convergence.

## 1.3 Methodology

# **Part I**

## **Opportunistic Spectrum Access**



# Response Surface Methodology for Efficient Spectrum Reuse in Cellular Networks

## 2.1 Introduction

In this chapter we move on from mechanisms that ensure the proper operation of opportunistic access to develop a full DSA framework particularized for cellular networks.

Approximately every decade, a new cellular access technology is developed and introduced in the market. However, because not all users upgrade their terminals at the same pace, the operators have to keep their legacy networks working, and the spectrum assigned to these old networks becomes gradually more and more underused.

One way to increase the spectrum efficiency is to allow the users of the newest networks to opportunistically access the spectrum of the legacy networks. Applying the concepts of cognitive radio [177], the terminals of the newest network would be the secondary users (SUs), and the terminals of the legacy network would be the primary users (PUs). The secondary access to the spectrum of the primary (legacy) network should not cause noticeable degradation to PU transmissions.

The operator owns and control both the primary and the secondary networks, and therefore can establish the rules for secondary access and monitor the impact of these rules on the performance of both SU and PU transmissions. However, it is more profitable for the operator *not making any modifications on the legacy network*, implying that the operation of the secondary network is *transparent* to the primary network.

In this chapter, we propose a semi-decentralized secondary access scheme allowing each SU to access a set of primary frequency bands using either temporal or spatial spectrum holes (opportunities). By giving more access options to each SU, this approach can outperform previous, more limited, mechanisms. We consider

that the SUs establish point-to-point connections among them (cognitive pairs) in an ad-hoc fashion.

To fully exploit its potential, we describe an on-line algorithm that, with very small computational and signaling overhead, allows the system to learn the optimal SU access policy in terms of capacity.

## Related Work

Other works have proposed the combination of temporal and spatial sensing by the SUs [178], [179], [180], [181]. Works like [178] and [179] consider a single secondary transmitter and exploit information from spatial sensing to improve the performance of temporal sensing. The single transmitter model is also applied in [180] for vehicular networks. In [181] the capacity is optimized by randomizing the access strategies, but for a single PU, single SU scenario. However, the motivational scenario of our work comprises multiple secondary transmitters with multiple primary base stations. Thus, we must consider relevant features that are not captured by the single transmitter model, such as the inter-SU interference and the frequency reuse in a cellular structure.

In cellular networks, previous works like [182] and [183] have addressed spatial spectrum reuse by a secondary network. However, these works assume that the secondary users only scan one frequency. As pointed out by their authors, scanning all the frequencies would improve the performance. In contrast, we follow this latter approach, mixing it with the exploitation of temporal opportunities.

## Contribution

Our contribution in this chapter is summarized as follows:

- We develop a semi-distributed mechanism for opportunistic spectrum access combining access over temporal and spatial opportunities, in which *all the spectrum of the system can be made available to each SU*.
- Our method is especially conceived for cellular networks, considering multiple PU transmitters within a cell structure. The system is evaluated in a realistic setting considering irregular cell shapes.
- The most important feature, 3) *the system is capable of learning the optimal probability distribution over the frequency bands of the cellular network* to maximize SU capacity. The learning algorithm applies the response surface methodology (RSM) [186], which is a novel and promising approach to address interference management problems.

In the following section we describe the system. Section 2.3 formulates the design problem and Section 2.4 presents the RSM algorithm to solve it. Finally, Sections 2.5 and 2.6 presents the numerical results and the conclusions of this work.



## 2.2 System description

The system considered comprises: a legacy cellular network (primary network), a secondary network establishing ad-hoc point-to-point links, and a secondary access controller (SAC) which monitors the system performance and broadcasts the operation parameters of the SU access strategy, with small signaling overhead, as we discuss in Section 2.4. The primary network contains base stations (PBSs) and primary users (PUs). Each PBS covers a certain geographical area (primary cell) and is assigned a frequency band different from its adjacent cells.

We assume a frequency reuse scheme of 7 frequency bands (reuse factor 7), denoted by  $f_1, \dots, f_7$ . Nevertheless, the proposed method can be applied to other reuse factors as well. Each frequency band is divided into  $2N$  orthogonal channels ( $N$  downlink and  $N$  uplink channels). Secondary access is constrained to downlink channels. Time is divided into equal duration time-slots, which is usual in most cellular systems. The data transmitted over a single PU channel in a time-slot is referred to as *packet*.

The secondary network consists of pairs of secondary users (SUs) entering and leaving the system in an ad-hoc fashion. This network model can characterize femtocells, terminals acting as relays, or any short-range transmission using available spectral resources of the legacy cellular network.

The SAC is associated to the network to which the SUs belong. In particular, we assume that the SUs are the users of a new generation cellular network. Because both networks (the new generation network and the legacy one) belong to the same operator, we consider that the SAC can retrieve some information from the legacy network.

The SUs can detect the power of the pilot tones of the neighboring PBSs. With this information (and possibly with the aid of the SAC) each SU can infer its position with respect to the surrounding cells and therefore be aware of the PBS power levels from each frequency band. Indeed, the SU does not need to estimate exactly these PBS power levels, it just needs to establish an ordering of the frequencies  $f_1, \dots, f_7$  from lower to higher PBS power level. For a generic SU, let  $\phi = (\phi_1, \dots, \phi_7)$  be the vector of frequencies ordered in increasing received power, which clearly depends on the location of the SU. Note that  $\phi_7$  corresponds, in general, to the frequency band of the primary cell where the SU is located.

**Example.** In Figure 2.1, any SU located in area A has  $\phi_1 = f_1$  (similarly, any SU in area B has  $\phi_1 = f_2$ ). But because each SU is closer to a different PBS,  $\phi_7 = f_5$  for SU 1, and  $\phi_7 = f_6$  for SU 2. These SUs have also different values of  $\phi_2$  and  $\phi_4$ .

When an SU transmits over any band  $\phi_i$ , it may cause some interference to the PUs of the closest cells using this band (and the same channel within this band). In general, the level of SU interference caused at PU receivers is proportional to the ordering index  $i$  of the selected band  $\phi_i$ , *i.e.* a single SU transmission over  $\phi_1$  causes generally less interference than an SU transmission over  $\phi_2$ , and so on.

In our system, the classic strategy of occupying exclusively *spatial opportunities* implies that each SU only tries to access over  $\phi_1$ . However, what really matters

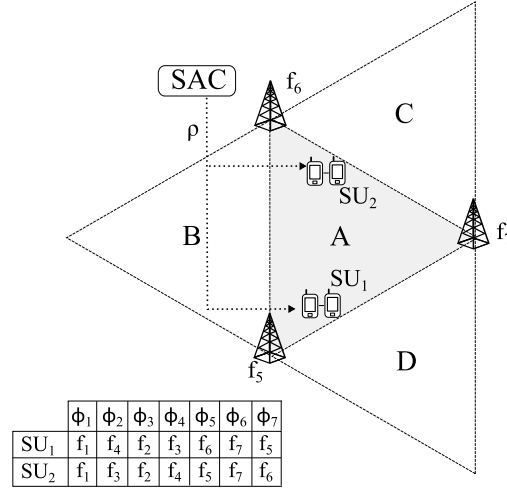


Figure 2.1: Example of the system with 2 SU pairs. Every SU in area A associates  $f_1$  to  $\phi_1$ . The remaining  $f_j$ - $\phi_i$  associations for each SU depend on how close the SU is to areas B, C or D. The SAC broadcasts a vector of operation parameters  $\rho$  to the SUs.

is the aggregate interference at the receivers. Note that, in accordance to the ad-hoc nature of the secondary network (small SU transmission range, low antenna heights, indoor locations sometimes) the SINR at the PU receivers can still be acceptable even if the interference from some SUs is a bit higher, *i.e.* if some SUs select other bands different from  $\phi_1$ . The potential benefit of giving more frequency band options to the SUs is to decrease the inter-SU interference, increasing the capacity of the secondary network.

Besides, when an SU tries to access the band of a close PBS, *i.e.* a band associated to a higher interference power such as  $\phi_7$ , the SU can perform *PU activity detection* on the channels within the band [177]. This activity detection allows the SU to detect free channels before starting transmission, and to stop SU transmission when detecting overlap with PU transmissions as our mechanism in chapter ?? does. The closer the SU is to the PBS, the more reliable is the PU activity detection and therefore the less impact on PU communication. The free channels in  $\phi_7$  are identified as *temporal opportunities*.

The typical strategies of using only spatial opportunities ( $\phi_1$  channels) or only temporal ones ( $\phi_7$  free channels) are not optimal as we show in Fig. 2.2. The achievable capacity for SUs using these strategies is limited, because of inter-SU interference and the limitation in the number of available channels when using temporal opportunities.

Regarding PU degradation, note that the interference from the SUs using  $\phi_7$  or  $\phi_6$  may be very intense but, thanks to the PU activity detection in closer cells, it should be infrequent and last short periods of time (depending on the detection accuracy). On the other hand, the interference from the SUs using bands that usually do not allow PU activity detection (*e.g.*,  $\phi_1$  or  $\phi_2$ ) will be sustained over time, but will be less intense (similar to the co-channel interference from other PBSs). To properly adjust the vector  $\rho$  we should combine the effects of these two

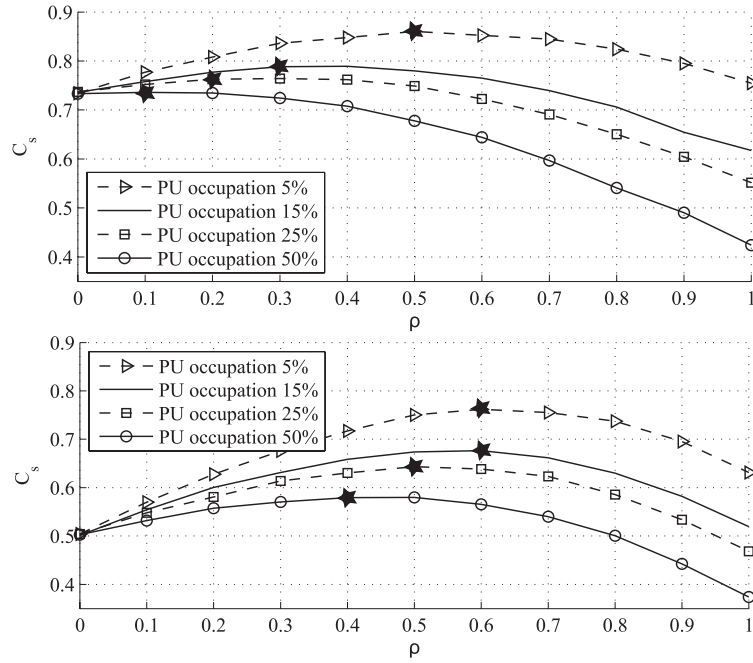


Figure 2.2: SU capacity versus  $\rho$ , the probability of using a temporal opportunity (and a spatial opportunity otherwise) under different PU traffic intensities. The number of new SU cognitive pairs at each time slot is generated by a binomial distribution characterized by 20 trials of 0.1 (top figure) and 0.2 (bottom figure) probability each. The rest of this reference scenario is described in section 2.5. The stars indicate the optimum SU capacity for each PU occupation.

types of interference into a single performance metric for the PUs: the probability of correct packet reception per time-slot.

Let  $R$  be the number of bands that an SU can use. We define the  $R$ -dimensional vector  $\boldsymbol{\rho} = (\rho_1, \dots, \rho_R)$ , where  $\rho_i$  denotes the probability of using band  $\phi_i$ , except for  $\rho_R$  which always corresponds to the probability of using band  $\phi_7$  (pure temporal access). Therefore,  $0 \leq \rho_i \leq 1$  for  $i = 1, \dots, R$ , and  $\sum_{i=1}^R \rho_i = 1$ . Let  $\mathcal{P}$  denote the set of  $R$ -dimensional vectors fulfilling these conditions.

The following steps summarize the **system operation**:

1. The SAC periodically updates the vector  $\boldsymbol{\rho}$  and sends it to the SUs.<sup>1</sup>
2. Each SU builds its own  $\boldsymbol{\phi}$  vector by scanning the PBS pilot tones on each frequency band. This vector only needs to be updated when the SU changes its location.
3. Before transmitting, an SU randomly selects one band,  $\phi_i$  from its  $\boldsymbol{\phi}$  vector, according to the probability distribution  $\boldsymbol{\rho}$ , i.e.  $P(\text{select band } \phi_i) = \rho_i$ .

<sup>1</sup>It is advisable to divide the system into relatively homogeneous regions in terms of PBS density, traffic intensity and type of terrain, so that a suitable  $\boldsymbol{\rho}$  can be found for each region.

4. If the SU is capable of detecting enough PBS power in  $\phi_i$  to perform PU activity detection, then the SU tries to access over temporal opportunities, occupying only channels free of PU activity. Otherwise, the SU will access any channel with a sufficiently low SINR (considering also the interference from other SUs).
5. Periodically, the SAC retrieves information from both the primary and the secondary networks, particularly performance measures, which are taken into account to update vector  $\boldsymbol{\rho}$ .

This mechanism allows a semi-decentralized resource allocation. The SAC only needs to *announce* the vector  $\boldsymbol{\rho}$  and retrieve small pieces of information measured at the terminals. The signaling overhead is therefore very low, especially compared to a centralized channel allocation by the SAC. If  $\boldsymbol{\rho}$  is properly adjusted, the SUs will allocate themselves over the legacy spectrum, autonomously, efficiently, and with a low impact on the primary network. Moreover,  $\boldsymbol{\rho}$  can be learned by the SAC with the RSM scheme explained in Section 2.4.

Next section formalizes the problem in terms of SU achieved capacity and PU probability of correct packet reception.

## 2.3 Problem Formulation

In this section we define the performance metrics for both the PUs and SUs and formulate the problem that the SAC needs to solve to determine the  $\boldsymbol{\rho}$  vector.

Let us consider a secondary network with a given traffic intensity characterized by its arrival rate per area unit, and a random transmission time. As stated above, the vector  $\boldsymbol{\rho}$  determines how the SUs distribute themselves over the available spectrum and the proportion of SUs using PU activity detection. In consequence, the SINR at each PU or SU receiver depends on  $\boldsymbol{\rho}$ . Let  $N_t^{SU}$  and  $N_t^{PU}$  denote the number of SUs and PUs in the system, respectively, at time-slot  $t = 1, 2, \dots$ . The SINR over time at SU receiver  $s \in \{1 \dots N_t^{SU}\}$ , is a discrete time stochastic process induced by  $\boldsymbol{\rho}$ , and denoted by  $\Gamma_t^s(\boldsymbol{\rho})$ . The expected normalized capacity per active SU pair is given by

$$C_s(\boldsymbol{\rho}) = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \sum_{s=1}^{N_t^{SU}} \frac{\log_2(1 + \Gamma_t^s(\boldsymbol{\rho}))}{N_t^{SU} C_{\max}^S} \right\} \quad (2.1)$$

where  $C_{\max}^S$  is the maximum achievable SU capacity per hertz. The expectation is taken over  $\Gamma_t^s$  and  $N_t^{SU}$ .<sup>2</sup> At a PU receiver, it is assumed that a data packet transmitted on time-slot  $t$  is correctly decoded if its SINR,  $\Gamma_t^{PU}$ , is greater than a given detection threshold,  $\gamma^{PU}$ . Then, the probability of correct detection at a PU

<sup>2</sup>Note that the traffic intensity in a cellular communication network varies during a day, but if it is observed during a smaller time window, e.g. 1 hour, the traffic arrival process can be considered stationary, with constant intensity.

is defined as

$$P_c(\boldsymbol{\rho}) = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^T \sum_{j=1}^{N_t^{PU}} \frac{\mathbb{I}\{\Gamma_t^j(\boldsymbol{\rho}) > \gamma^{PU}\}}{N_t^{PU}} \right\} \quad (2.2)$$

where  $\Gamma_t^j$  refers to the SINR process at the  $j$ -th active PU, and  $\mathbb{I}_{\{z\}}$  is an indicator function which equals 1 if condition  $z$  holds, and equals 0 otherwise.

Therefore, the objective of the SAC is to find  $\boldsymbol{\rho}$  solving the following problem:

$$\begin{aligned} \max_{\boldsymbol{\rho}} \quad & C_s(\boldsymbol{\rho}) \\ \text{s.t.} \quad & P_c(\boldsymbol{\rho}) \geq P_{c,\min} \\ & \boldsymbol{\rho} \in \mathcal{P} \end{aligned} \quad (2.3)$$

where  $P_{c,\min}$  denotes the minimum acceptable  $P_c$ .

Finding an optimal  $\boldsymbol{\rho}$  is a challenging task because the (multiple) SINR stochastic processes capture the interaction of a random number of randomly located terminals with fading effects among each pair of them. Moreover, these processes and the decision vector  $\boldsymbol{\rho}$ , take values from continuous spaces, and therefore conventional dynamic programming techniques result infeasible. The problem (2.3) is, in fact, a *stochastic optimization problem with stochastic constraints* [186]. As usual in this type of problems, the SAC has to dynamically *learn* an optimal  $\boldsymbol{\rho}$ . One feasible way to address it is by means of Surface Response Methods (RSM) [187]. However, the inclusion of stochastic constraints introduces high complexity in the formulation. In the following section we develop the particular case in which  $P_{c,\min}$  is sufficiently low for condition  $P_c(\boldsymbol{\rho}) \geq P_{c,\min}$  to hold at the optimal  $\boldsymbol{\rho}$  (in numerical results,  $P_c(\boldsymbol{\rho})$  at the optimal  $\boldsymbol{\rho}$  is never less than 5% of the  $P_c$  obtained in absence of SU access). This allows us to remove the stochastic constraint regarding  $P_c(\boldsymbol{\rho})$ . We leave as future work the inclusion of constraints in the RSM formulation.

## 2.4 Response Surface Method (RSM) Algorithm

The aim of the algorithm is the maximization of the expected value of the average capacity function (2.1) on a closed convex feasible domain  $\mathcal{P} \subset \mathbb{R}^R$  for the input vector  $\boldsymbol{\rho}$ , that is, to find a  $\boldsymbol{\rho}$  approximately solving

$$\max_{\boldsymbol{\rho} \in \mathcal{P}} C_s(\boldsymbol{\rho}) \quad (2.4)$$

RSM allows us to find an approximate solution to this problem by successively estimating the gradient of the objective function and using these estimations in stochastic gradient ascent steps. At each one of these steps, numbered by  $n = 1, 2, \dots$ , the system generates one update of the input vector  $\boldsymbol{\rho}_{(n)}$  according to the following expression:

$$\boldsymbol{\rho}_{(n+1)} = \boldsymbol{\rho}_{(n)} + \alpha_{(n)} \hat{\nabla} C_s(\boldsymbol{\rho}_{(n)}) \quad (2.5)$$

where  $\hat{\nabla} C_s$  denotes the estimation of the gradient  $\nabla C_s$ , and  $\alpha_{(n)}$  is the step-size weighting factor. A standard condition for the selection of  $\alpha_{(n)}$ , assuring the convergence of  $\boldsymbol{\rho}_{(n)}$  [186], is  $\sum_{n=1}^{\infty} \alpha_{(n)} = \infty$ ,  $\sum_{n=1}^{\infty} \alpha_{(n)}^2 < \infty$ .

Let us discuss the computation of  $\hat{\nabla}C_s$ . Given  $\rho_{(n)}$  at step  $n$ , we consider a subdomain  $\mathcal{S}_{(n)}$  of the feasible domain  $\mathcal{P}$  such that  $\rho_{(n)} \in \mathcal{S}_{(n)} \subset \mathcal{P}$ . Note that  $\rho_{(n)}$  is the point at which the estimate  $\hat{\nabla}C_s$  must be computed. Therefore, we need to estimate the objective function  $C_s(\rho_{(n)})$  on  $\mathcal{S}_{(n)}$ , by taking samples  $y^{(i)}$ ,  $i = 1, \dots, p$ , of the function. For this, we need a finite set of points  $\rho_{(n,i)}$ ,  $i = 1, \dots, p$ , generally called *design points*, belonging to  $\mathcal{S}_{(n)}$ . These points are chosen by the decision maker (the SAC in our case), and can be, for example, random perturbations of  $\rho_{(n)}$ , falling within  $\mathcal{S}_{(n)}$ .

Let  $t_{(n)}$  denote the time-slot in which the update  $\rho_{(n)}$  is obtained. Given the set of  $p$  decision points, the SAC can obtain samples  $y^{(i)}$  by this simple procedure:

1. Determine  $p$  sampling instants  $t_{(n,i)} = t_{(n)} + iT$  for  $i = 1 \dots p$ , where  $T$  is a sufficiently long time period for measuring performance at the SUs.
2. At  $t_{(n,i-1)}$  (where  $t_{(n,0)} = t_{(n)}$ ), the SAC signals the design point  $\rho_{(n,i)}$  to the SUs, for  $i = 1 \dots p$ .
3. At  $t_{(n,i)}$ , the SAC obtains the capacity samples from each active SU and averages them to obtain  $y^{(i)}$ .

Thus, the estimates can be expressed as:

$$y^{(i)} = \frac{1}{T} \sum_{k=t_{(n,i-1)}}^{t_{(n,i)}} \sum_{s=1}^{N_k^{SU}} \frac{\log_2(1 + \Gamma_k^s(\rho_{(n,i)}))}{N_k^{SU} C_{\max}^S} \quad i = 1, 2, \dots, p, \quad (2.6)$$

Note that the stochastic ascent algorithm needs  $p$  periods of length  $T$  to upgrade  $\rho_{(n+1)}$ .

The objective function  $C_s$  is then approximated on  $\mathcal{S}_{(n)}$  by a polynomial response surface model  $\hat{C}_s(\rho) = \hat{C}_s(\rho|\beta_0, \beta_1 \dots \beta_R)$ . The coefficients  $\beta_j$ , are determined by least squares estimation. Therefore, the RSM-gradient estimator  $\hat{\nabla}C_s(\rho_{(n)})$  at  $\rho_{(n)}$  is defined by the gradient (with respect to  $\rho$ )

$$\hat{\nabla}C_s(\rho_{(n)}) = \nabla \hat{C}_s(\rho_{(n)}). \quad (2.7)$$

Thus,  $C_s$  is estimated on  $\mathcal{S}_{(n)}$  by the linear empirical model

$$\hat{C}_s(\rho_{(n)}) = \beta_0 + \beta_l^T (\rho - \rho_{(n)}) \quad (2.8)$$

where

$$\beta^T = (\beta_0, \beta_l^T) = (\beta_0, \beta_1, \dots, \beta_R)^T \quad (2.9)$$

is the  $(R+1)$ -vector of unknown coefficients of the linear model. Having samples  $y^{(i)}$  of the function values  $C_s(\rho_{(n,i)})$  at the design points  $\rho_{(n,i)}$ ,  $i = 1, \dots, p$ , in  $\mathcal{S}_{(n)}$ , we can obtain, by least squares, the following estimate  $\hat{\beta}$  of  $\beta$ :

$$\hat{\beta} = (\mathbf{W}^T \mathbf{W})^{-1} \mathbf{W}^T \mathbf{y}. \quad (2.10)$$

Here, the  $p \times (R + 1)$ -matrix  $\mathbf{W}$  and the  $p$ -dimensional vector  $\mathbf{y}$  are defined by

$$\mathbf{W} = \begin{pmatrix} 1 & \delta^{(1)} \\ 1 & \delta^{(2)} \\ \vdots & \vdots \\ 1 & \delta^{(p)} \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(p)} \end{pmatrix} \quad (2.11)$$

with  $\delta^{(i)} = \boldsymbol{\rho}_{(n,i)} - \boldsymbol{\rho}_{(n)}$ , for  $i = 1, 2, \dots, p$ . Note that  $(\mathbf{W}^T \mathbf{W})$  in (2.10) is invertible whenever the columns of  $\mathbf{W}$  are linearly independent, which can be easily guaranteed by a proper selection of the design points.

In case  $C_s(\boldsymbol{\rho})$  is concave on  $\mathcal{P}$ , the RSM algorithm will approach the global optimum following the stochastic gradient ascent iterations (2.5) as it has been observed for  $R \geq 2$  dimensions in the scenarios considered in the following Section.

## 2.5 Numerical Results

### Evaluation Framework

This subsection describes the Monte-Carlo methodology and the scenario used to evaluate  $C_s$ . The primary network uses a 7-band frequency planning, as previously stated, with irregular shaped cells having an average radius  $r = 700$  m. We consider pathloss and multipath fading. To compute the pathloss attenuation over distance,  $A(d)$  (dB), we use the following piecewise dual-slope model [128]:

$$A(d) = \begin{cases} K + 10\gamma_1 \log_{10}(d/d_0) & d_0 \leq d \leq d_c \\ K + 10\gamma_1 \log_{10}(d_c/d) + 10\gamma_2 \log_{10}(d/d_c) & d > d_c \end{cases} \quad (2.12)$$

The critical distance  $d_c$  is notably smaller for SU transmission than for PBS transmissions, because the PBSs are located at high outdoor locations while SUs are, in general, located either indoors or at ground level. All the signals are assumed to experience Rayleigh fading.

The area used to generate random terminal locations is a  $4.2 \text{ km} \times 4.8 \text{ km}$  rectangle. We focus on the downlink channels of one frequency band. We consider an scenario in which the primary traffic intensity is low  $0.16 \text{ Erlangs/km}^2$ , so that the average occupation of the spectrum by the primary terminals is only 5%. In contrast, the SU traffic intensity,  $27.5 \text{ Erlangs/km}^2$ , is high in comparison.

In this case, the SU capacity is mostly determined by the inter-SU interference and the RSM algorithm is essentially performing *interference management* in the SU network. We also discuss the effect of higher PU spectrum occupation and a different SU traffic intensity.

According to the ad-hoc nature of the secondary network, the average SU link distance considered is 90 m. Table 2.1 summarizes the simulation parameters considered.

Parameter	Assigned value
<b>Primary transmitters</b>	
number of downlink channels, $N$	5
cell radius, $r$	700 m
average received power at PU	-78 dBm
SINR detection threshold at PU, $\gamma^{PU}$	-17 dB
baseline noise at PU ( $N_0 + I_{PU}$ )	-110 dBm
<b>Secondary transmitters</b>	
average SU Tx power per channel	0.5 W
SU link distance	90 m
probability of PU activity detection	0.9
probability of overlap detection	0.8
<b>Propagation parameters</b>	
pathloss exponents, $\gamma_1, \gamma_2$	2.4, 4.2
propagation factor $K$	46.7 dB
critical distance for PBS transmission	$1.2r$ m
critical distance for SU transmission	100 m
<b>RSM parameters</b>	
dimensions of $\rho, R$	1 ... 5
measuring period $T$ (in time-slots)	300
number of samples per step $p$	$3(2^{R-1} + 1)$

Table 2.1: Parameter setting of the reference scenario used in numerical evaluations

## Convergence and Usage of the Frequency Bands

Let us consider the case of 3-dimensional  $\rho$  vectors,  $R = 3$ . In this case, the  $\rho$  vector of a particular SU is  $\rho = (\rho_1, \rho_2, \rho_3)$ , being  $\rho_3$  the probability of trying to access the band  $\phi_7$  of the primary cell where the SU is in, and  $\rho_1$  and  $\rho_2$  the probabilities of accessing  $\phi_1$  and  $\phi_2$ , the bands where the SU receives less PBS power. The SU can only perform PU activity detection on  $\phi_7$  channels, while the channels of the other bands are considered spatial opportunities. Fig. 2.3 shows the values of  $\rho_1$  and  $\rho_2$  ( $\rho_3$  is simply  $1 - \rho_1 - \rho_2$ ) over consecutive update steps  $n = 1, 2, \dots$ . The initial vector is  $\rho_{(0)} = (0, 0, 1)$ . Although  $\rho_0$  is a rather poor initial guess, we see that the RSM algorithm stabilizes after 20 iterations. The final value is  $\rho = (0.29, 0.42, 0.29)$ . Fig. 2.4 shows the estimated value of  $C_s$  as a function of  $\rho_1$  and  $\rho_2$ . Consistently with the result obtained by RSM, the maximum values of  $C_s$  lie on the line  $\rho_1 + \rho_2 = 0.7$ .

Let us now consider the results for  $R = 5$ . The probability  $\rho_5$  is now associated to  $\phi_7$ , allowing PU activity detection, while  $\rho_1 \dots \rho_4$  are associated to  $\phi_1 \dots \phi_4$ , where PU activity detection is assumed to be unfeasible. The initial  $\rho$  vector is  $(0, 0, 0, 0, 1)$ . Fig. 2.5 shows that, in this case, the convergence of RSM is as fast as with  $R = 3$ . Note that, as in the case of  $R = 3$ , the algorithm is not assigning  $\rho_1$  the highest probability. The reason is that the algorithm aims to minimize inter



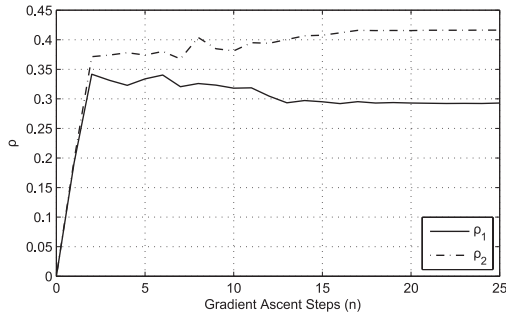


Figure 2.3: Consecutive RSM updates of  $\rho_1$  and  $\rho_2$ .

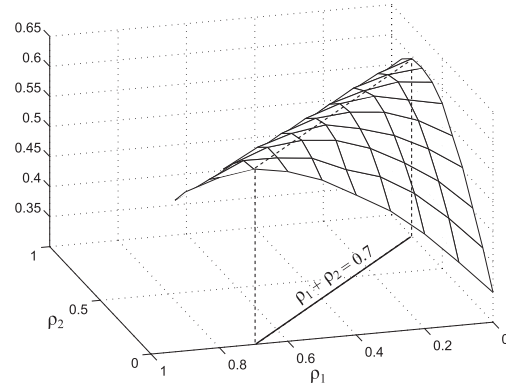


Figure 2.4: Estimation of  $C_s$  over  $\rho_1$  and  $\rho_2$ .

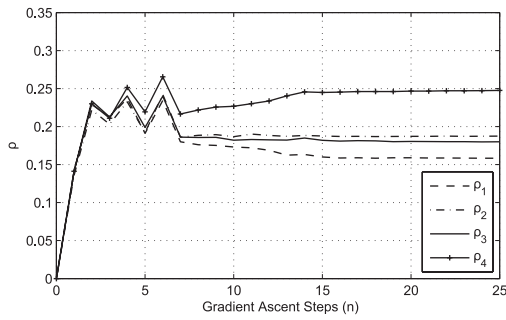


Figure 2.5: Consecutive RSM updates of  $\rho_1 \dots \rho_4$ .

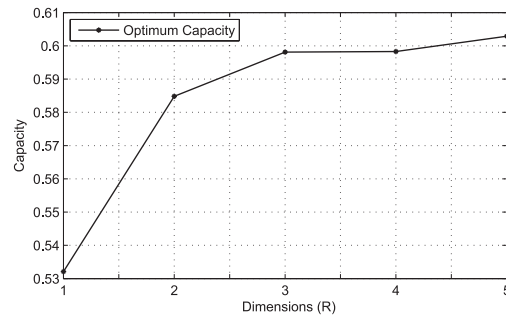


Figure 2.6: Maximum SU capacity obtained with RSM versus the number of dimensions of  $\rho$ ,  $R$ .

SU interference by separating the SU transmitters using the same band. Recall that in Fig. 2.1 all the SUs in triangular region A shared the same  $\phi_1$  but had different values for  $\phi_2$ . The aggregated probability of using spatial opportunities is  $\sum_{i=1}^4 \rho_i = 0.77$ , which is higher than in the  $R = 3$  case. This result confirms the idea that, the more spectrum is available to each SU, the more spectrum the SU exploits. Having more spectrum options per SU (higher  $R$ ) also allows the SU network to achieve a higher capacity, as illustrated by Fig. 2.6.

## Effect of the Traffic Intensity

Fig. 2.7 shows the average SU capacity attained by the RSM algorithm with  $R = 2$ , versus the average spectrum usage by the PUs, for different SU traffic intensities. As expected, the more spectrum occupied by PU traffic, the smaller the achieved capacity. Similarly, more SU traffic implies less SU capacity. The reason of the reduction on the maximum achieved capacity is the increment of the aggregate interference power at the SU receivers, from both the PBSs and other SU pairs. Compared to the typical approach of exploiting only temporal opportunities, the benefit of using RSM is noticeable for every traffic situation. In all cases, the reduction of the probability of correct detection at the PU receivers ( $P_c$ ) was less than 5%.

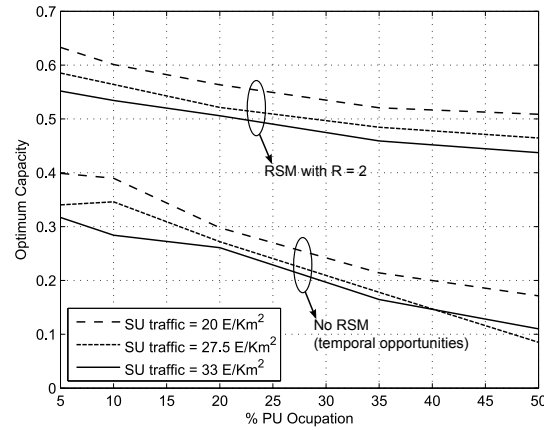


Figure 2.7: Maximum SU capacity obtained with RSM under different traffic intensities. For comparison, the figure also shows the capacity for a SU network exploiting only temporal opportunities (no RSM).

## 2.6 Conclusion

Motivated by the problem of spectrum reuse in cellular networks, we presented a semi-distributed mechanism allowing the secondary network to learn the most efficient spectrum access strategy. This mechanism exploits both spatial and temporal opportunities, and is especially effective for interference management in highly dense secondary networks. The learning approach is based on response surface methodology (RSM) which, according to our numerical results, improves notably the system capacity compared to usual strategies, and shows a fast convergence rate even when it is poorly initialized. Because of that, the system can adapt its control vector  $\rho$  to variations on the traffic intensity or user distribution. Surprisingly, to the best of our knowledge, the use of RSM on this framework had not been previously reported. Our future work is focused on incorporating performance constraints for the primary network in the RSM formulation.

## **Part II**

### **Automated spectrum trading**



## **Final conclusions**



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