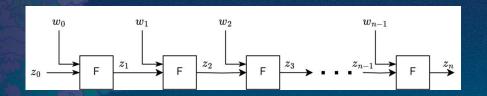




$$s_n = F^{(n)}(s_0)$$

Incrementally verifiable computations (IVC) are repetitive

- A model which infer on thousands of different data points
- 2. A VM with a fixed set of opcodes
- 3. A rollups which batch-verify signatures
- 4. A blockchain consensus



Benefits of folding based IVC schemes

Why folding schemes based IVC are cool:

- 1. No need to specify the number of steps beforehand
- 2. O(|F|) prover memory
- 3. Verifier time does not depend on n

 $W \leftarrow W_1 + r \cdot W_2$

Using folding schemes for recursion

Coined within the Nova paper (Kothapalli et al., 2021)

Intuition: if you have two witnesses satisfying a particular R1CS, a random linear combination of those two witnesses results in another satisfying unique witness.



Decider: the final step

Compressing the IVC proof is required since it is not succinct.

You can verify compressed IVC-proof onchain!





HyperNova: Recursive arguments for customizable constraint systems

Customizable constraint systems for succinct arguments

Srinath Setty*

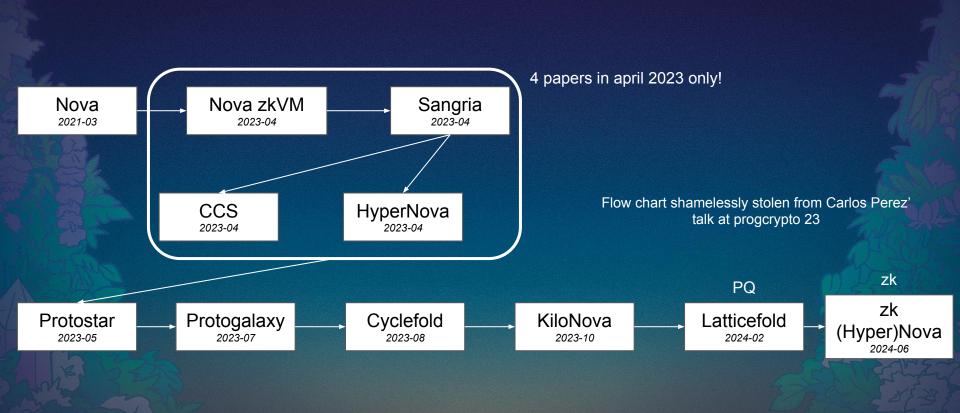
Justin Thaler[†]

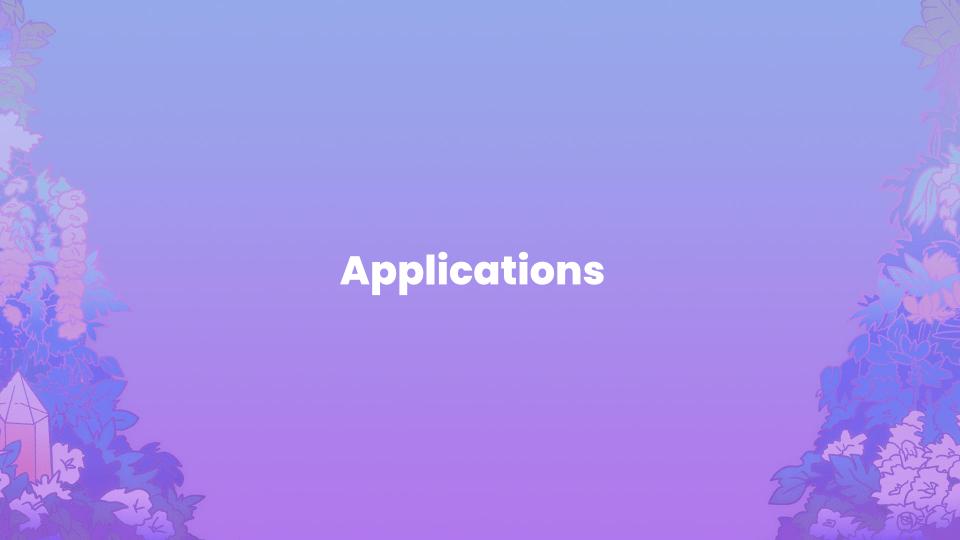
Riad Wahby[‡]

Folding schemes zoo

Many papers popped:

- Folding for plonkish circuits (e.g. sangria)
- Folding-adapted arithmetizations
- Many-to-1 folding
- Lattice based folding







Bitcoin light clients

Proving that the current block hash is the block hash obtained after N blocks.

Verify the POW for 100k blocks.

Cost (servers + tx): less than 30\$ on Optimism

https://github.com/dmpierre/sonobe-btc



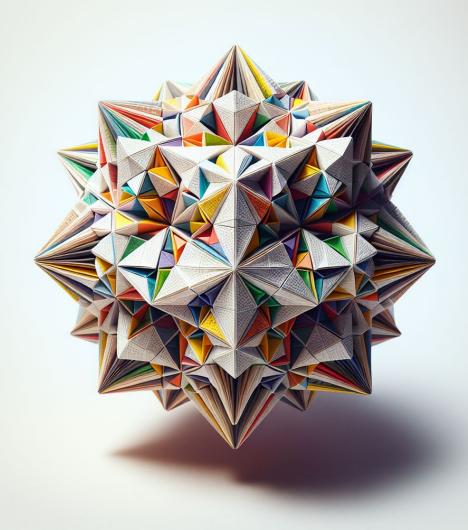
zkVMs

Nexus

Jolt (for memory checking)







Sonobe

Experimental folding schemes library implemented jointly by 0xPARC and PSE

https://github.com/privacy-scaling-explorations/sonobe

Modular library,

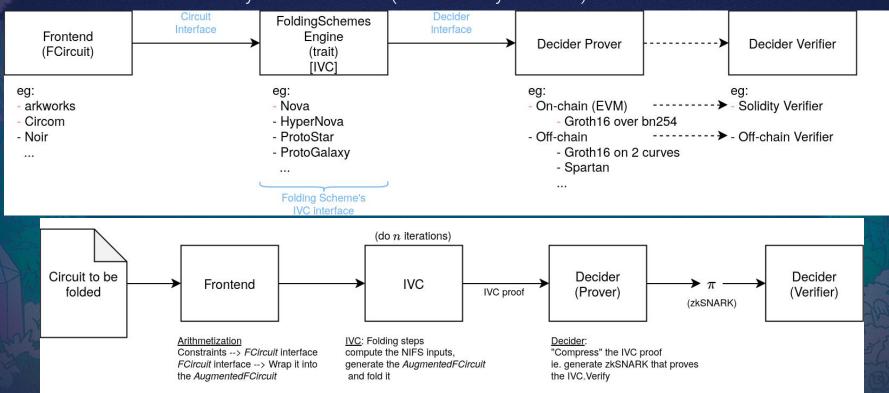
- Be able to
 - Add and test new folding schemes
 - Compare schemes 'apples-to-apples'
 - Researchers can easily add their own schemes (eg. Mova paper)
- Make it easy for devs to use folding
 - Minimal code to fold your circuits ('plug-and-fold')
 - Easy to switch between folding schemes and curves
 - Support multiple zk-circuit languages
- Achieve Onchain Verification on Ethereum

Remark: experimental & research library, unoptimized.

Dev flow:

Sonobe - Dev experience

- Define a circuit to be folded
- 2. Set which folding scheme to be used (eg. Nova with CycleFold)
- 3. Set a final decider to generate the final compressed proof (eg. Groth16 over BN254 curve)
- 4. Generate the Solidity decider verifier (EVM Solidity contract)



Status of Sonobe - schemes implemented

Implemented (fully implemented):

- **Nova**: Recursive Zero-Knowledge Arguments from Folding Schemes, Abhiram Kothapalli, Srinath Setty, Ioanna Tzialla. 2021. https://eprint.iacr.org/2021/370.pdf
- **CycleFold**: Folding-scheme-based recursive arguments over a cycle of elliptic curves, Abhiram Kothapalli, Srinath Setty. 2023. https://eprint.iacr.org/2023/1192.pdf
- **HyperNova**: Recursive arguments for customizable constraint systems, Abhiram Kothapalli, Srinath Setty. 2023. https://eprint.iacr.org/2023/573.pdf
- ProtoGalaxy: Efficient ProtoStar-style folding of multiple instances, Liam Eagen, Ariel Gabizon. 2023.
 https://eprint.iacr.org/2023/1106.pdf

Started (NIFS implemented, next: folding circuit, IVC, Decider, etc):

- **Mova**: Nova folding without committing to error terms, Nikolaos Dimitriou, Albert Garreta, Ignacio Manzur, Ilia Vlasov. 2024. https://eprint.iacr.org/2024/1220.pdf
- Ova: Reduce the accumulation verifier in Nova from 2 to just 1 group operation, Benedikt Bünz. 2024. https://eprint.iacr.org/2024/1220.pdf

Frontends - how can the dev define a circuit to be folded

- Arkworks https://github.com/arkworks-rs
- experimental: Circom, Noir, Noname.

FCircuit interface

```
/// The parameter z_i denotes the current state, and z_i+1 denotes the next state after applying
/// the step.
pub trait FCircuit(F: PrimeField): Clone + Debug {
    type Params: Debug;
    /// returns a new FCircuit instance
    fn new(params: Self::Params) -> Result(Self, Error);
    /// returns the number of elements in the state of the FCircuit, which corresponds to the
    /// FCircuit inputs.
    fn state len(&self) -> usize;
    /// returns the number of elements in the external inputs used by the FCircuit. External inputs
    /// are optional, and in case no external inputs are used, this method should return 0.
    fn external_inputs_len(&self) -> usize;
    /// computes the next state values in place, assigning z_{i+1} into z_{i}, and computing the new
   fn step_native(
        // this method uses self, so that each FCircuit implementation (and different frontends)
        // can hold a state if needed to store data to compute the next state.
       z_i: Vec<F>
        external_inputs: Vec<F>, // inputs that are not part of the state
    -> Result<Vec<F>, Error>;
    /// generates the constraints for the step of F for the given z_i
    fn generate_step_constraints(
        // this method uses self, so that each FCircuit implementation (and different frontends)
        // can hold a state if needed to store data to generate the constraints.
        cs: ConstraintSystemRef<F>
        z i: Vec<FpVar<F>>
        external_inputs: Vec<FpVar<F>>>, // inputs that are not part of the state
     -> Result<Vec<FpVar<F>>>, SynthesisError>;
```

To fold a circuit, it just needs to implement the *FCircuit* trait.

That's also an easy way to add new frontends for other zk-circuit languages.

Folding the circuit

```
let mut rng = ark_std::test_rng();
let poseidon_config = poseidon_canonical_config::<Fr>();
// set the FCircuit to be folded:
tupe FC = CubicFCircuit(Fr);
let f circuit = FC::new(())?;
tupe FS = Nova<G1. GVar1, G2, GVar2, FC, Pedersen<G1>, Pedersen<G2>, false>;,
let prep param = NovaPreprocessorParam::new(poseidon config.clone(), f circuit);
let fs_params = FS::preprocess(&mut rng, &prep_param)?;
// set the IVC's initial state
let z_0 = vec![C1::ScalarField::from(3_u32)];
// initialize the folding scheme
let mut fs = FS::init(&fs_params, F_circuit, z_0.clone())?;
// perform multiple IVC steps (internally folding)
let num_steps: usize = 100;
for _ in 0..num_steps {
    fs.prove_step(&mut rnq, vec![], None)?;
// get the IVC proof
let ivc_proof: FS::IVCProof = fs.ivc_proof();
// verify the IVCProof
FS::verify(fs_params.1.clone(), ivc_proof.clone())?;
```

Folding the circuit

```
let mut rng = ark_std::test_rng();
let poseidon_config = poseidon_canonical_config::⟨Fr⟩();
// set the FCircuit to be folded:
tupe FC = CubicFCircuit(Fr);
let f circuit = FC::new(())?;
// set Nova as the FoldingScheme to use:
type FS = Nova<G1, GVar1, G2, GVar2, FC, Pedersen<G1>, Pedersen<G2>, false>;
let prep_param = Novarreprocessor or om..new(poserdon_com.rq.clone(), f_circuit);
let fs_params = FS::preprocess(&mut rng, &prep_param)?;
// set the IVC's initial state
let z_0 = vec![C1::ScalarField::from(3_u32)];
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// verify the IVCProof
FS::verify(fs_params.1.clone(), ivc_proof.clone())?;
```

Switching between Folding Schemes & curves

```
type FS = Nova(G1, GVar1, G2, GVar2, FC, Pedersen(G1), Pedersen(G2), false);
type FS = Nova(G1, GVar1, G2, GVar2, FC, KZG('static, Bn254), Pedersen(G2), false);
type FS = HyperNova< G1, GVar1, G2, GVar2, FC, Pedersen<G1>, Pedersen<G2>, 1, 1, false, >;
type FS = ProtoGalaxy(G1, GVar1, G2, GVar2, FC, Pedersen(G1), Pedersen(G2));
```

G1 & G2 could be any cycle of curves available in artworks.

Decider: compress the IVC Proof with a zkSNARK

Setting which decider to use:

Pedersen<Projective2>, Groth16<Bn254>, FS, >;

Using the Decider:

Onchain verification

- At the end of n folding steps, we have the last IVC state and the IVC Proof
- We compress it through a zkSNARK (Decider)
- One of Sonobe's goals: verify it onchain in Ethereum

- Original Nova: wrapp the Decider checks in 2 Spartan (zkSNARK) proofs (one over each curve of the cycle of curves).
 - → 2 Spartan proofs, one on each curve
- In our case we were interested into verifying the proofs in Ethereum's EVM.

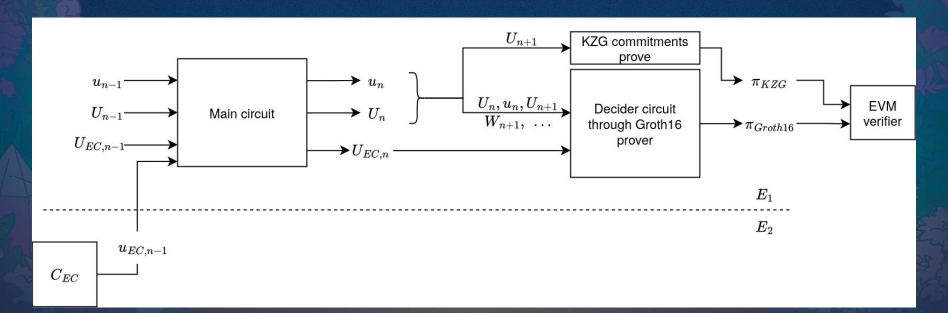
Need to do a bit of gymnastics to verify the folding proofs in Ethereum, EVM limitations:

- limited to BN254 curve
- constrained by gas costs

Onchain verification

Sonobe's Onchain verification:

- Generate a Decider proof that can be verified in Solidity
- Offer methods to generate the Solidity smart contract code that verifies the Sonobe proof



Some preliminary numbers

- Till now we've been focusing on implementing the various schemes
 - Nova, HyperNova, ProtoGalaxy, CycleFold
- Without focusing on optimization/efficiency
- So far the numbers we got look promising

Next steps: we're going to start profiling, optimizing, adding benchmarks, etc. Getting to a 'first release'.

Some preliminary numbers

- Till now we've been focusing on implementing the various schemes
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Next steps: we're going to start profiling, optimizing, adding benchmarks, etc. Getting to a 'first release'.

Recall, this proof is proving that applying n times the function F (the circuit that we're folding) to an initial state z_0 results in the state z_n .

Unoptimized preliminary numbers:

- <u>folding step</u> (the recursive iteration): **~ 300ms**
 - Folding circuit (Nova+CycleFold): ~ 50k R1CS
 constraints
- offchain Decider prove: < 1 min
- onchain Decider:
 - Circuit: ~ 10M R1CS constraints
 - < 3 minutes in a 32GB RAM 16 core laptop</p>
 - o gas costs (Decider proof verification): ~ 800k gas
 - mostly from G16, KZG10, public inputs processing
 - will be reduced by hashing the public inputs& batching the pairings check
 - expect to get it down to < 500k gas.

Repo: https://github.com/privacy-scaling-explorations/sonobe

Docs: https://privacy-scaling-explorations.github.io/sonobe-docs/



Wrap up

- Folding Schemes are not a tool that fits in all use cases, but in those where it fits it can provide significantly speed & memory improvements.
- Sonobe: experimental research modular folding schemes library
- Schemes available: Nova, HyperNova, ProtoGalaxy (all with CycleFold)
- Onchain verification available
- Preliminary benchmarks look promising
- Next steps: optimizations & first release