THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 2

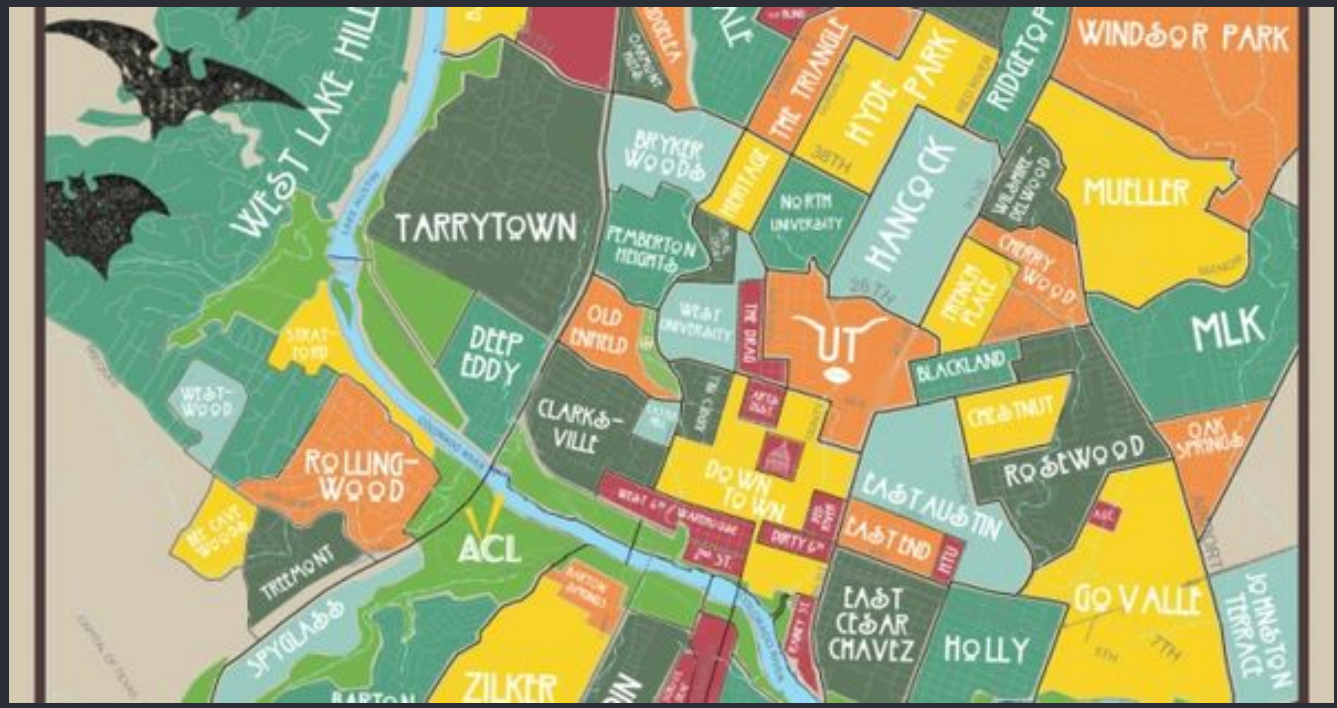
Lecture 2

STA 371G

Sample vs Population

Find out the average house price in Austin.

How would you do that?



Look at each house price?

360,000 houses in Austin!

Can we do something smarter?

Sample vs Population

A smarter approach:

- Pick n houses randomly (e.g. $n = 100$)
- Take the average of the prices of these n houses
- Hope that your estimate is close to the true price average.

Just like making polls to predict election results!

Sample vs Population

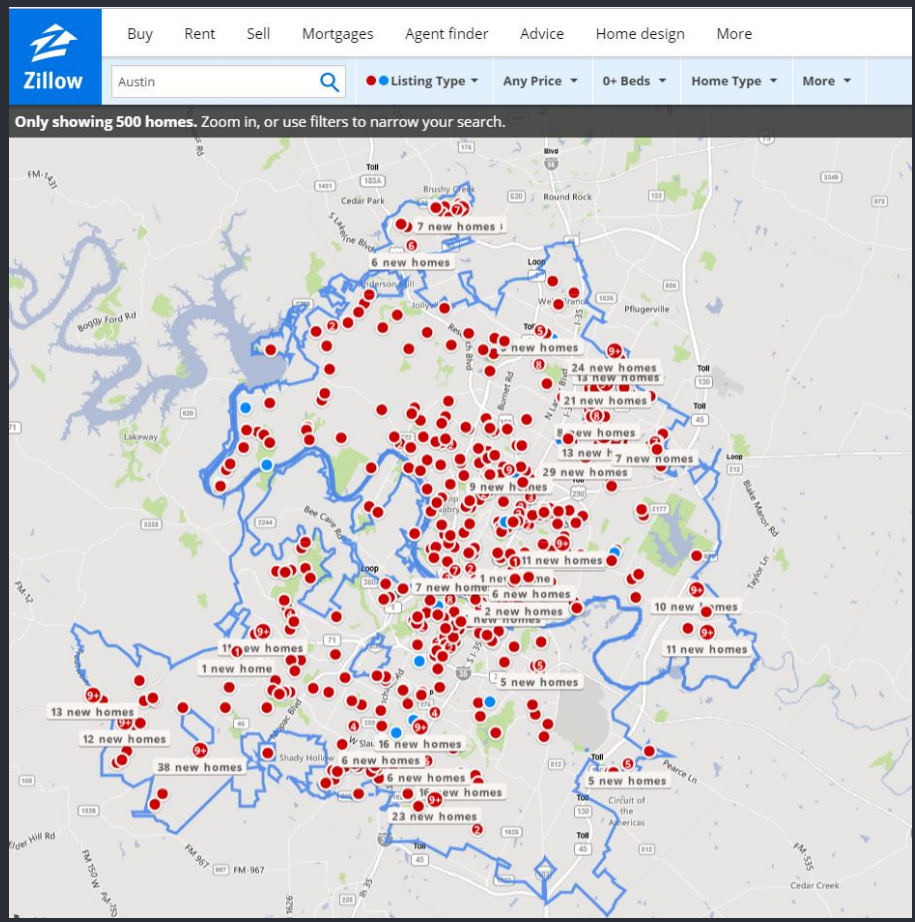
	Population	**Sample**
Members	all house prices	prices you picked
Average	population mean	sample mean
Variance	population variance	sample variance

Estimating a population parameter (population mean) based on a sample statistic (sample mean).

Collecting a sample

Zillow.com, “Austin, TX.”

- Click “More Map”
- Select 15 houses, note their prices in an R script.
- Do not discard any price, use the first 15
- Try to represent different regions



Collecting a sample

Your R script should look like this

```
# Create a vector of house prices (You should have 15 price data)
sample_house_prices <- c(327000,276000,513000)
# Calculate sample statistics
sample_mean <- mean(sample_house_prices)
sample_variance <- var(sample_house_prices)
sample_standard_deviation <- sd(sample_house_prices)
# Sample mean of first 5 houses
sample_mean_5 <- mean(sample_house_prices[1:5])
# Print them to console
cat("Sample Mean", sample_mean)
cat("Sample Variance", sample_variance)
cat("Sample Standard Deviation", sample_standard_deviation)
cat("Sample Mean of first 5 houses",sample_mean_5)
```

Sampling Distribution

On Learning Catalytics, enter your results.

And here is what they look like...

Sampling Distribution

Distribution of your answers ⟶ Sampling distribution

Statistic	**Population**	**Sample Mean**
Mean	μ	μ
Standard Deviation	σ	$\sigma/\sqrt{n}$

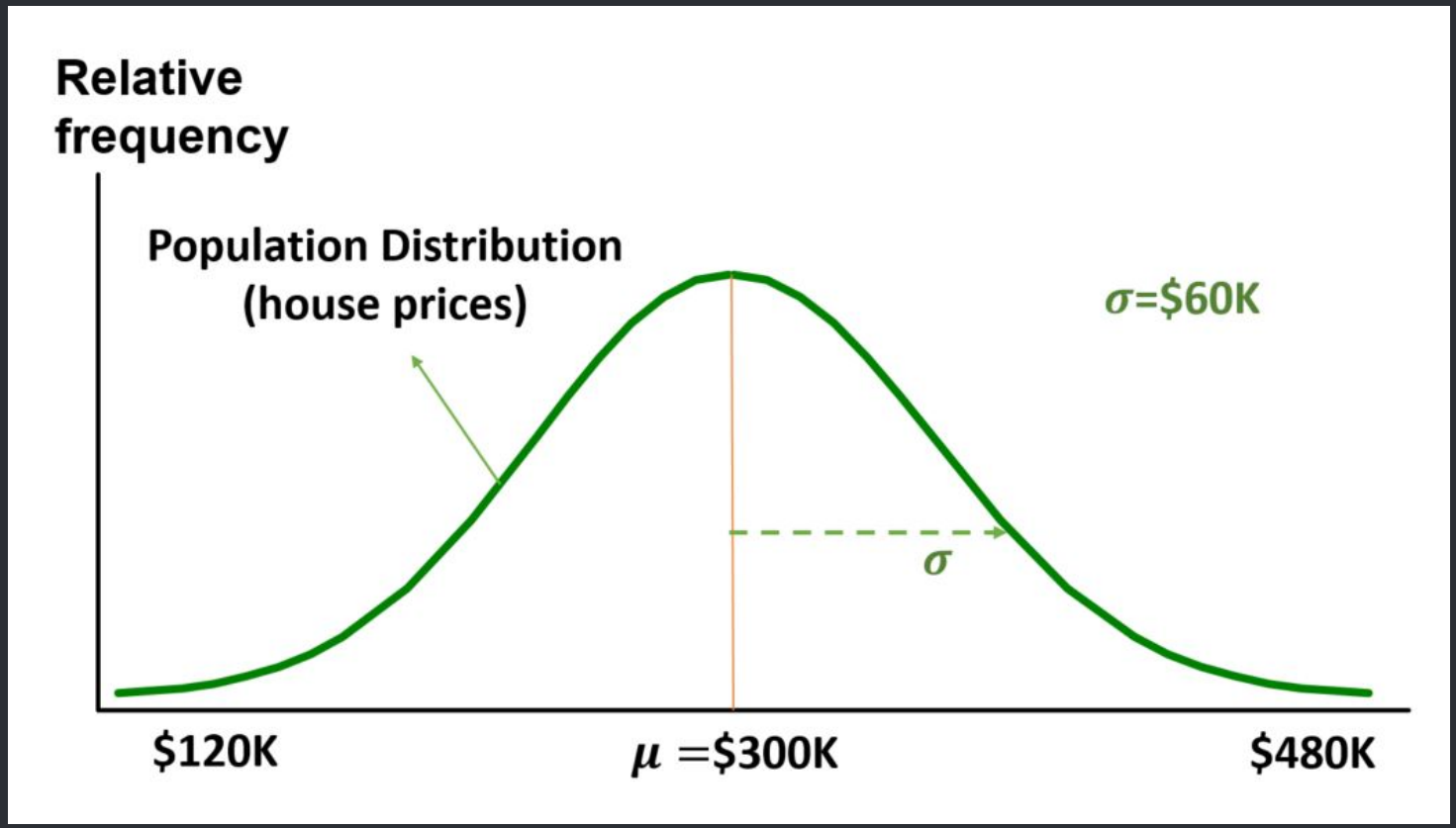
What would you expect if you had 10 000 houses in your survey?

Sampling Distribution

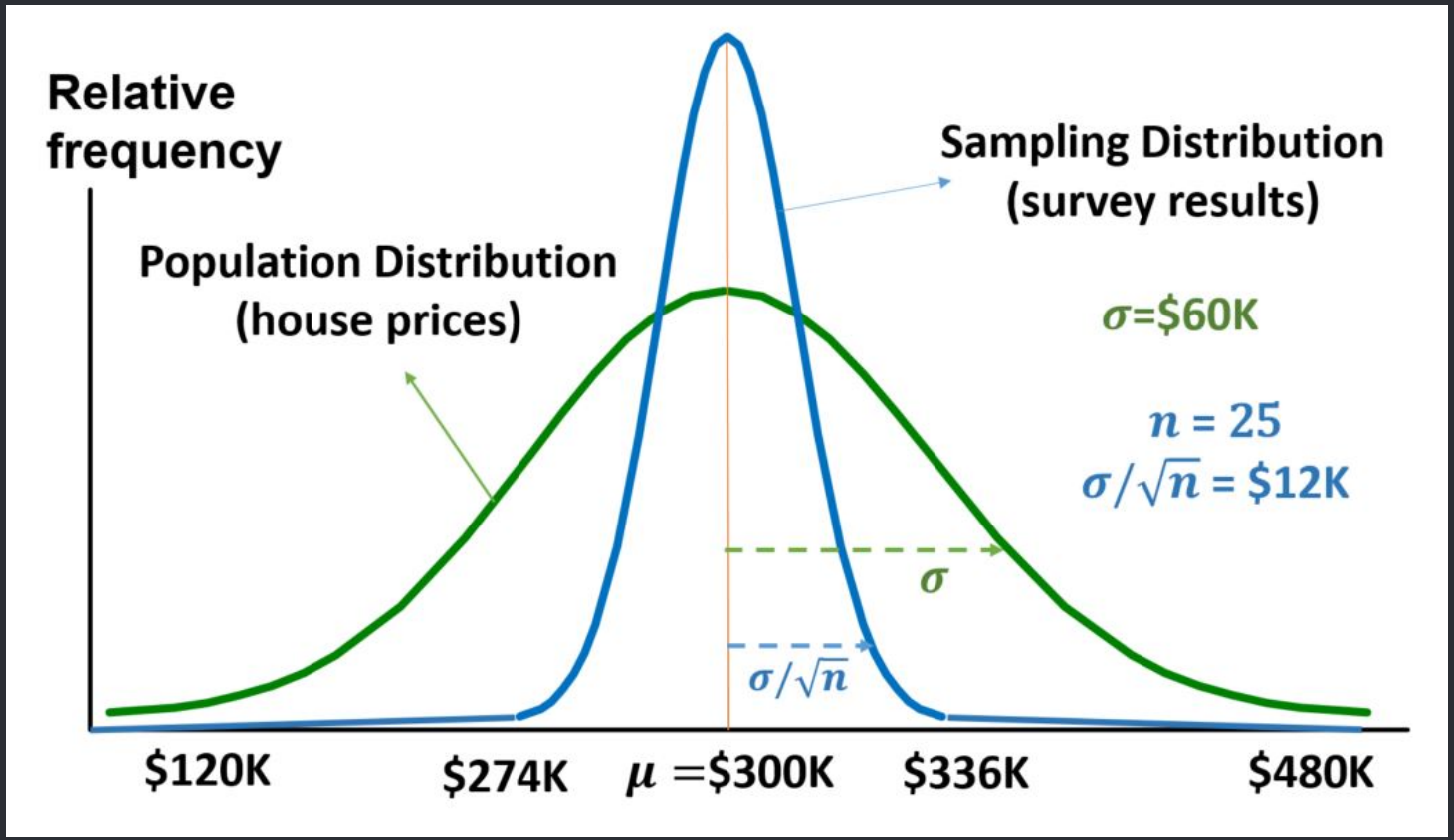
Let's compare sample mean of 5 houses vs 15 houses.

What do you expect to see?

Sampling Distribution



Sampling Distribution



Sampling Distribution

Assume μ =\$300K, σ =\$60K.

	n	$\sigma/\sqrt{n}$	3 std. dev. range (99.7%)
Survey 1	25	\$12K	\$274K \$336K
Survey 2	100	\$6K	\$282K \$318K
Survey 3	3600	\$1K	\$297K ... \$303K

t Distribution

We often do not know population variance and use sample variance instead.

In that case, the sample mean will have a *t* distribution.

Hypothesis Testing

Hypothesis: The average house price in Austin is $1M.
Your survey on 25 houses: Average price is $305*K*.

Questions, questions...

- Would you reject the hypothesis? Why?
- Is it possible that, out of bad luck, you picked the cheapest houses?
- Would you be more comfortable with your conclusion if you had 1000 houses in your survey?
- When should you reject the hypothesis? When not?

P-Value

Your sample mean: \$305K.

$H_0 : \mu = \$1M$ (Null hypothesis)
$H_1 : \mu < \$1M$ (Alternative hypothesis)

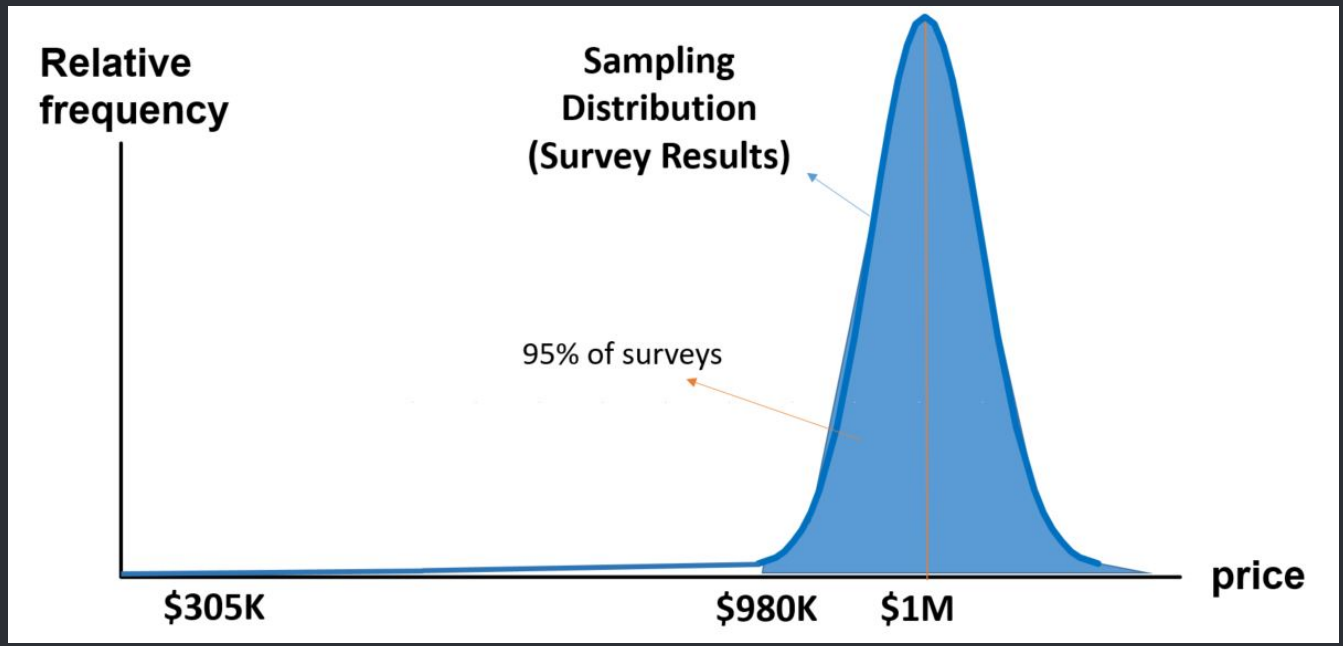
The *P*-value is "the probability of observing such an extreme (\$305K or less) sample statistic given the null hypothesis is true."

- *P*-value $\leq \alpha$, reject the null hypothesis
- *P*-value $> \alpha$, reject the null hypothesis

α is usually chosen as 0.05 prior to sampling.

P-Value

If the null hypothesis were true...



P-value is smaller than 10^{-100}, while $\alpha = 0.05$.

Rather than thinking you are cursed, you simply reject the hypothesis!

Confidence Interval

Content Here

Confidence Interval

Add the following to your R script

```
# Calculate 95% confidence interval (default)
avg_price_ci_95 <- t.test(sample_house_prices)
# Calculate 99% confidence interval
avg_price_ci_99 <- t.test(sample_house_prices, conf.level = 0.99)
# Display results
cat("95% confidence interval is:", avg_price_ci_95$conf.int)
cat("99% confidence interval is:", avg_price_ci_99$conf.int)
```

Enter your results on Learning Catalytics.