THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 3

Lecture 4

STA 371G

Who are these people?

Zack

Kelly

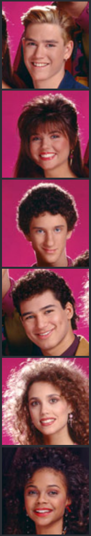
Screech

Slater

Jessie

Lisa

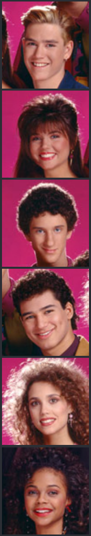
Some events



Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character

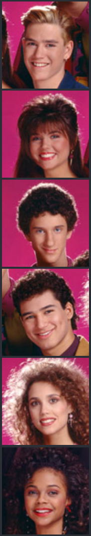
Some events



Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where “lush” means “long curly hair” (mullet included!)

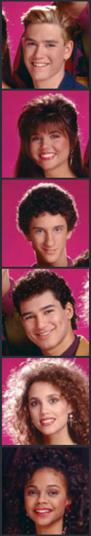
Some events



Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*

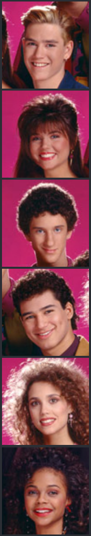
Some events



Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

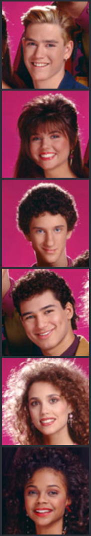
Some events



Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

Some events

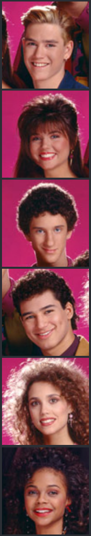


Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where “lush” means “long curly hair” (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

$P(M) =$

Some events

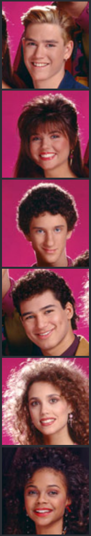


Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

$P(M) = 3/6 \qquad P(L) =$

Some events



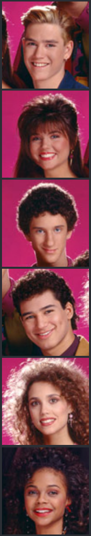
Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

$P(M) = 3/6 \qquad P(L) = 3/6$

$P(M|L) =$

Some events



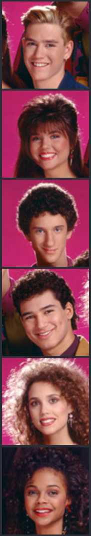
Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

$P(M) = 3/6 \qquad P(L) = 3/6$

$P(M|L) = 1/3 \qquad P(L|M) =$

Some events



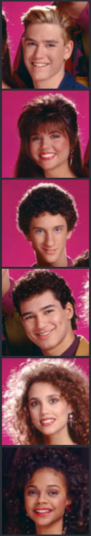
Suppose we pick a *Saved by the Bell* character at random. Each of these are events:

- $M =$ we select a male character
- $L =$ we select someone with lush hair, where "lush" means "long curly hair" (mullet included!)
- $M|L =$ we select a male *from among the lush-haired characters*
- $L|M =$ we select a lush-haired character *from among the male characters*

$P(M) = 3/6 \qquad P(L) = 3/6$

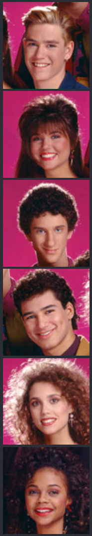
$P(M|L) = 1/3 \qquad P(L|M) = 1/3$

Joint probability



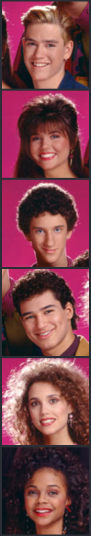
$P(M \text{ and } L) =$ probability of selecting a lush-haired male

Joint probability



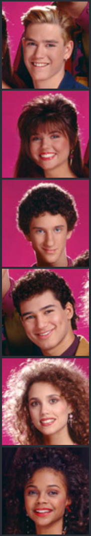
$$\begin{aligned} P(M \text{ and } L) &= \text{probability of selecting a lush-haired male} \\ &= 1/6 \end{aligned}$$

Joint probability



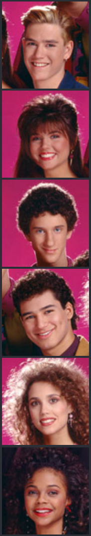
$$
\begin{aligned}
P(M \text{ and } L) &= \text{probability of selecting a lush-haired male} \\
&= 1/6 \\
&= P(M)P(L|M) = \frac{3}{6} \cdot \frac{1}{3} \\
&= P(L)P(M|L) = \frac{3}{6} \cdot \frac{1}{3}
\end{aligned}
$$

Joint probability



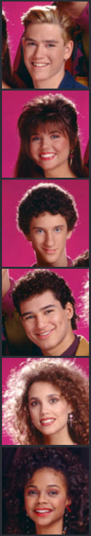
$$
\begin{aligned}
P(M \text{ and } L) &= \text{probability of selecting a lush-haired male} \\
&= 1/6 \\
&= P(M)P(L|M) = \frac{3}{6} \cdot \frac{1}{3} \\
&= P(L)P(M|L) = \frac{3}{6} \cdot \frac{1}{3} \\
&\neq P(M)P(L)
\end{aligned}
$$

One or the other



$P(M \text{ or } L) =$ probability of selecting a lush-haired character, a male character, or both

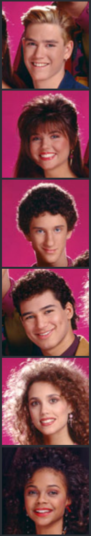
One or the other



$$
\begin{aligned}
P(M \text{ or } L) &= \text{probability of selecting a lush-haired} \\
&\qquad \text{character, a male character, or both} \\
&= 5/6
\end{aligned}
$$

We have to subtract off $P(M \text{ and } L)$ because otherwise we are double-counting Slater!

One or the other



$$\begin{aligned} P(M \text{ or } L) &= \text{probability of selecting a lush-haired} \\ &\qquad \text{character, a male character, or both} \\ &= 5/6 \\ &= P(M) + P(L) - P(M \text{ and } L) \\ &= 3/6 + 3/6 - 1/6 \end{aligned}$$

We have to subtract off $P(M \text{ and } L)$ because otherwise we are double-counting Slater!

Conditional probability

When we say $P(A|B)$, what we mean is: "In a world where we know B has already happened, how likely is it that A also happened?"


HOPE SHE WAS

Now that you've found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

Now that you've found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

It's hard to know how to estimate this directly!

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating})$

Let's come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating})$
- $P(\text{underwear found} \mid \text{partner is not cheating})$

Let’s come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating})$
- $P(\text{underwear found} \mid \text{partner is not cheating})$
- $P(\text{partner is cheating})$ (before we found the underwear!)

Bayes' Rule

Bayes' Rule lets us reverse the conditional probabilities!

$U =$ underwear found, $C =$ partner is cheating

$$P(C|U) = \frac{P(U|C)P(C)}{P(U|C)P(C) + P(U|\overline{C})P(\overline{C})}$$

Think of Bayes' Rule as a way to update our thinking based on new information:

$$
\begin{array}{ll}
P(C) & \longleftarrow \text{Prior probability} \\
P(C|U) & \longleftarrow \text{Posterior probability (includes new information)}
\end{array}
$$

HIV Testing

- HIV testing is important for public health, but HIV tests are not perfect

HIV Testing

- HIV testing is important for public health, but HIV tests are not perfect
- The OraQuick ADVANCE Rapid HIV-1/2 Antibody Test has the following properties:

HIV Testing

- HIV testing is important for public health, but HIV tests are not perfect
- The OraQuick ADVANCE Rapid HIV-1/2 Antibody Test has the following properties:
  - If you **have HIV**, there is a 99.3% chance the test will show a **positive result**

HIV Testing

- HIV testing is important for public health, but HIV tests are not perfect
- The OraQuick ADVANCE Rapid HIV-1/2 Antibody Test has the following properties:
  - If you **have HIV**, there is a 99.3% chance the test will show a **positive result**
  - If you **do not have HIV**, there is a 99.8% chance the test will show a **negative result**

HIV Testing

- HIV testing is important for public health, but HIV tests are not perfect
- The OraQuick ADVANCE Rapid HIV-1/2 Antibody Test has the following properties:
  - If you **have HIV**, there is a 99.3% chance the test will show a **positive result**
  - If you **do not have HIV**, there is a 99.8% chance the test will show a **negative result**
- 0.4% of people in the US are HIV-positive

HIV Testing

We know $P(TP|HP) = 0.993$, but we really want to know $P(HP|TP)$!

HIV Testing

- If you have HIV, there is a 99.3% chance the test will show a positive result

HIV Testing

- If you have HIV, there is a 99.3% chance the test will show a positive result
- If you do not have HIV, there is a 99.8% chance the test will show a negative result

HIV Testing

- If you have HIV, there is a 99.3% chance the test will show a positive result
- If you do not have HIV, there is a 99.8% chance the test will show a negative result
- But if you test positive there is about a 1/3 chance you do NOT have HIV!

HIV Testing

- If you have HIV, there is a 99.3% chance the test will show a positive result
- If you do not have HIV, there is a 99.8% chance the test will show a negative result
- But if you test positive there is about a 1/3 chance you do NOT have HIV!
- This is counterintuitive — it's because of the way we are wired (it even has a name: "base rate fallacy")