

Lecture 9

**STA 371G** 

Predicting the fuel economy (MPG) for different car models of '70s.



Predicting the fuel economy (MPG) for different car models of '70s.



- Cylinders
- Displacement
- Horsepower

- Weight
- Acceleration
- Year (After 1975 or not)

Let's load the data from web and save it to the local directory.

```
> # auto_mpg <- read.csv(file_url_goes_here_in_quotes, header=T)
> # to save this to your local directory, use
> # write csv(auto mpg, "./auto mpg.csv")
```



Let's load the data from web and save it to the local directory.

```
> # auto_mpg <- read.csv(file_url_goes_here_in_quotes, header=T)
> # to save this to your local directory, use
> # write_csv(auto_mpg, "./auto_mpg.csv")
```

And calculate the average MPG.



Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                  HP Weight Acceleration After1975
  <dbl>
            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
                                                                <chr>
                8
                            307
                                  130
                                       3504
     18
                                                      12.0
                                                                   No
2
     15
                8
                            350
                                  165
                                      3693
                                                      11.5
                                                                   No
3
     18
                8
                            318
                                  150
                                      3436
                                                      11.0
                                                                   No
4
     16
                8
                            304
                                  150
                                      3433
                                                      12.0
                                                                   No
     17
                            302
                                  140
                                         3449
                                                      10.5
                                                                   No
```

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                 HP Weight Acceleration After1975
  <dbl>
            <int>
                         <dbl> <int>
                                      <int>
                                                    <dbl>
                                                              <chr>
                                      3504
     18
                8
                           307
                                 130
                                                     12.0
                                                                 No
2
     15
                8
                           350
                                 165 3693
                                                     11.5
                                                                 No
3
     18
                8
                           318
                                 150 3436
                                                     11.0
                                                                 No
4
     16
                8
                           304
                                 150 3433
                                                     12.0
                                                                 No
     17
                           302
                                                     10.5
                                 140
                                        3449
                                                                 No
```

No??? What the... What to do with that?

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                   HP Weight Acceleration After1975
  <dbl>
            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
                                                               <chr>
                                       3504
     18
                8
                            307
                                  130
                                                      12.0
                                                                  No
2
     15
                8
                            350
                                  165
                                      3693
                                                      11.5
                                                                   No
3
     18
                8
                            318
                                  150
                                      3436
                                                      11.0
                                                                   No
4
     16
                8
                            304
                                  150
                                      3433
                                                      12.0
                                                                  No
     17
                            302
                                                      10.5
                                  140
                                        3449
                                                                   No
```

No??? What the... What to do with that? Maybe just omit the "After1975" column?

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                  HP Weight Acceleration After1975
  <dbl>
            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
                                                                <chr>
                8
                            307
                                  130
                                       3504
     18
                                                      12.0
                                                                   No
2
                                      3693
     15
                8
                            350
                                  165
                                                      11.5
                                                                   No
3
     18
                8
                            318
                                  150
                                      3436
                                                      11.0
                                                                   No
4
     16
                8
                            304
                                  150
                                      3433
                                                      12.0
                                                                   No
     17
                            302
                                  140
                                        3449
                                                      10.5
                                                                   No
```

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                 HP Weight Acceleration After1975
  <dbl>
            <int>
                         <dbl> <int>
                                      <int>
                                                    <dbl>
                                                              <chr>
                                      3504
     18
                8
                           307
                                 130
                                                     12.0
                                                                 No
2
     15
                8
                           350
                                 165 3693
                                                     11.5
                                                                 No
3
     18
                8
                           318
                                 150 3436
                                                     11.0
                                                                 No
4
     16
                8
                           304
                                 150 3433
                                                     12.0
                                                                 No
     17
                           302
                                 140
                                                     10.5
                                       3449
                                                                 No
```

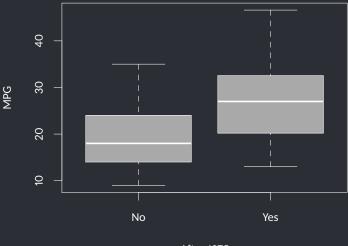
No??? What the... What to do with that?

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                   HP Weight Acceleration After1975
  <dbl>
            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
                                                               <chr>
                                       3504
     18
                8
                            307
                                  130
                                                      12.0
                                                                  No
2
     15
                8
                            350
                                  165
                                      3693
                                                      11.5
                                                                   No
     18
                8
                            318
                                  150
                                      3436
                                                      11.0
                                                                   No
4
     16
                8
                            304
                                  150
                                      3433
                                                      12.0
                                                                  No
     17
                            302
                                                      10.5
                                  140
                                        3449
                                                                   No
```

No??? What the... What to do with that? Maybe just omit the "After1975" column?





After 1975

Can we go ahead and run a regression anyway?

#### Can we go ahead and run a regression anyway?

Can we go ahead and run a regression anyway?

R was able to handle the "After1975" column, which is a categorical variable (or a factor as R calls them).

```
round(summary(model)$coefficients, 2)
          Estimate Std. Error t value Pr(>|t|)
(Intercept)
             42.19
                       2.37
                            17.81
                                     0.00
             -0.58
Cylinders
                       0.36 -1.62
                                    0.11
Displacement
             0.01
                       0.01 0.94
                                    0.35
HP
             -0.02
                      0.01 -1.35
                                    0.18
Weight
             -0.01
                      0.00
                            -8.33
                                    0.00
Acceleration
             0.04
                       0.11 0.32
                                     0.75
After1975Yes
             4.36
                       0.40 10.85
                                     0.00
```

R has created a dummy variable, "After1975Yes."

```
round(summary(model)$coefficients, 2)
           Estimate Std. Error t value Pr(>|t|)
(Intercept)
              42.19
                        2.37
                              17.81
                                       0.00
Cylinders
              -0.58
                        0.36 -1.62
                                       0.11
Displacement
              0.01
                        0.01 0.94
                                       0.35
HP
              -0.02
                        0.01 -1.35
                                       0.18
                        0.00
                              -8.33
                                       0.00
Weight
              -0.01
Acceleration
              0.04
                        0.11 0.32
                                       0.75
After1975Yes
              4.36
                        0.40 10.85
                                       0.00
```

R has created a dummy variable, "After1975Yes." A dummy variable is always 0 or 1, indicating the absence or presence of some categorical effect.

"After1975Yes" is 1 whenever "After1975" is a "Yes," and 0 otherwise.

| MPG    | <br>Acceleration | After1975 | After1975Yes |  |
|--------|------------------|-----------|--------------|--|
|        |                  |           |              |  |
| <br>25 | <br>13.5         | <br>No    |              |  |
|        |                  |           |              |  |
| 33     | 17.5             | No        | 0            |  |
| 28     | 15.5             | Yes       | 1            |  |
| 25     | 16.9             | Yes       | 1            |  |
|        | <br>             |           |              |  |

"After1975Yes" is 1 whenever "After1975" is a "Yes," and 0 otherwise.

| MPG | <br>Acceleration | After1975 | After1975Yes |  |
|-----|------------------|-----------|--------------|--|
|     | <br>             |           |              |  |
| 25  | <br>13.5         | No        | 0            |  |
| 33  | <br>17.5         | No        | 0            |  |
| 28  | 15.5             | Yes       | 1            |  |
| 25  | 16.9             | Yes       | 1            |  |
|     |                  |           |              |  |
|     | <u> </u>         |           |              |  |

Notice that we do not have a "After1975No" variable.

It would cause problems because it would be perfectly correlated with

"After1975Yes."

Our model contains some statistically insignificant variables. Your task is to omit them one by one. What is the  $R^2$  in your final model?



```
> model <- lm(MPG ~ HP + Weight + After1975,</pre>
                   data=auto mpg)
+
> summary(model)$r.squared
[1] 0.7745063
> round(summary(model)$coefficients, 2)
            Estimate Std. Error t value Pr(>|t|)
(Intercept)
              41.71
                         0.78 53.15
                                        0.00
              -0.02 0.01 -2.30 0.02
HP
Weight
            -0.01 0.00 -13.84 0.00
After1975Yes 4.33
                         0.40 10.83 0.00
```

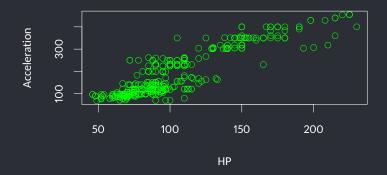
```
> model <- lm(MPG ~ HP + Weight + After1975,</pre>
                  data=auto mpg)
+
> summary(model)$r.squared
[1] 0.7745063
> round(summary(model)$coefficients, 2)
           Estimate Std. Error t value Pr(>|t|)
(Intercept)
                        0.78 53.15
              41.71
                                        0.00
HP
              -0.02 0.01 -2.30 0.02
Weight
             -0.01 0.00 -13.84 0.00
After1975Yes 4.33
                        0.40 10.83 0.00
```

Horsepower seems to be already capturing the information in Cylinders, Displacement and Acceleration.

#### To see the correlation between variables:

```
> mpg numeric = auto mpg[,c(1,2,3,4,5,6)]
> round(cor(mpg numeric),2)
             MPG Cylinders Displacement HP Weight Acceleration
MPG
            1.00
                    -0.78
                               -0.81 -0.78 -0.83
                                                       0.42
Cylinders -0.78
                                0.95 0.84
                                           0.90
                                                      -0.50
                     1.00
Displacement -0.81
                     0.95
                                1.00 0.90 0.93
                                                      -0.54
HP
           -0.78
                    0.84
                                0.90 1.00
                                           0.86
                                                      -0.69
Weight
       -0.83
                    0.90
                                0.93 0.86 1.00
                                                      -0.42
Acceleration 0.42
                               -0.54 -0.69 -0.42
                    -0.50
                                                       1.00
```

```
> plot(auto_mpg$HP, auto_mpg$Displacement,
+ xlab='HP', ylab='Acceleration',col='green', main='')
```



#### Consider this:

- Model A and B have the same HP and Weight.
- Model A was manufactured before 1975, whereas B was manufactured after 1975.
- Our model's prediction for Model A is 21 MPG.
- What is the prediction for Model B?



Our "reference level" is the cars manufactured before 1975.

Our "reference level" is the cars manufactured before 1975.

For the same Weight and HP, our MPG prediction for a car manufactured after 1975 is always exactly 4.33 higher compared to its reference.

Our "reference level" is the cars manufactured before 1975.

For the same Weight and HP, our MPG prediction for a car manufactured after 1975 is always exactly 4.33 higher compared to its reference.

 $\beta$  gives us the increment in our prediction for the cars manufactured after 1975.

Our "reference level" is the cars manufactured before 1975.

For the same Weight and HP, our MPG prediction for a car manufactured after 1975 is always exactly 4.33 higher compared to its reference.

 $\beta$  gives us the increment in our prediction for the cars manufactured after 1975.

There are other coding schemes too, where the reference is chosen differently, therefore  $\beta$  is interpreted differently.

## What if there are more than two categories?

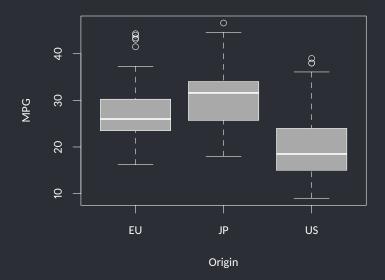
```
> auto mpg all[1:5,]
# A tibble: 5 8
    MPG Cylinders Displacement
                                HP Weight Acceleration After1975 Origin
  <fd>>
            <int>
                         <dbl> <int>
                                      <int>
                                                   <fdh>>
                                                              <chr>
                                                                     <chr>
     18
                8
                           307
                                 130
                                     3504
                                                    12.0
                                                                 No
                                                                        US
  15
                8
                           350
                                 165 3693
                                                    11.5
                                                                 No
                                                                        US
    18
                8
                           318
                                 150 3436
                                                    11.0
                                                                        IIS
                                                                 Nο
4
    16
                8
                           304
                                 150
                                     3433
                                                    12.0
                                                                        US
                                                                 Nο
     17
                8
                           302
                                 140
                                       3449
                                                    10.5
                                                                 No
                                                                        US
> levels(as.factor(auto mpg all$0rigin))
[1] "EU" "JP" "US"
```

## What if there are more than two categories?

```
> auto mpg all[1:5,]
# A tibble: 5 8
   MPG Cylinders Displacement
                                 HP Weight Acceleration After1975 Origin
  <fd>>
            <int>
                        <dbl> <int>
                                      <int>
                                                   <fdh>>
                                                             <chr>
                                                                    <chr>
    18
               8
                          307
                                 130
                                    3504
                                                    12.0
                                                                No
                                                                       US
 15
                8
                          350
                                165 3693
                                                    11.5
                                                                No
                                                                       US
   18
                8
                          318
                                150 3436
                                                    11.0
                                                                       IIS
                                                                Nο
4
    16
               8
                          304
                                150 3433
                                                    12.0
                                                                       US
                                                                Nο
    17
                8
                          302
                                 140
                                      3449
                                                    10.5
                                                                No
                                                                       US
> levels(as.factor(auto mpg all$0rigin))
[1] "EU" "JP" "US"
```

Let's first see if "Origin" makes a difference.

```
> boxplot(MPG ~ Origin, data=auto_mpg_all, ylab="MPG",
+ xlab="Origin", col='darkgray')
```



```
> omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,</pre>
                  data=auto mpg all)
> round(summary(omodel)$coefficients,3)
           Estimate Std. Error t value Pr(>|t|)
             40.182
                       0.874 45.961
                                      0.000
(Intercept)
                       0.010 -2.837
HP
             -0.028
                                      0.005
Weight
       -0.005
                       0.000 -10.815
                                      0.000
After1975Yes 4.334
                       0.393 11.033
                                      0.000
OriginJP
         1.001
                       0.612 1.635
                                      0.103
OriginUS
        -1.593
                       0.562 -2.834
                                      0.005
```

```
> omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,</pre>
                  data=auto mpg all)
 round(summary(omodel)$coefficients,3)
           Estimate Std. Error t value Pr(>|t|)
             40.182
                        0.874 45.961
                                       0.000
(Intercept)
                        0.010 - 2.837
HP
             -0.028
                                       0.005
                        0.000 -10.815
Weight
       -0.005
                                       0.000
After1975Yes 4.334
                        0.393 11.033
                                       0.000
<u>OriginJP</u>
          1.001
                        0.612 1.635
                                       0.103
OriginUS
         -1.593
                        0.562 -2.834
                                       0.005
```

For the origin variable, R has chosen "EU" as the base, created a dummy variable for JP and US each.

While dealing with categorical variables, we look at the significance of the categorical variable as a whole.

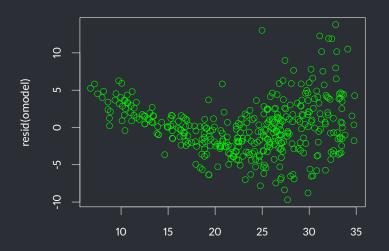
Unless all the dummy variables are insignificant, we do not omit the column of that categorical variable.

### **Assumptions**



What are the issues with this model?

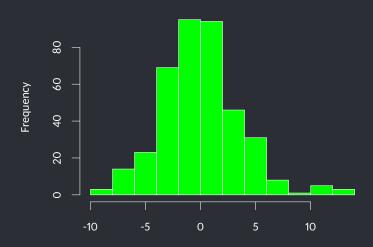
> plot(predict. lm(omodel), resid(omodel), col='green', main='')



### **Assumptions**

### What about normality?

> hist(resid(omodel), col='green', main='')



### **Assumptions**

### What about normality?

> qqnorm(resid(omodel), col='green', main='')

