

Lecture 9

**STA 371G** 

Predicting the fuel economy (MPG) for different car models of '70s.



Predicting the fuel economy (MPG) for different car models of '70s.



- Cylinders
- Displacement
- Horsepower

- Weight
- Acceleration
- Year (After 1975 or not)

Let's load the data from web and save it to the local directory.

```
> # auto_mpg <- read.csv(file_url_goes_here_in_quotes, header=T)
> # to save this to your local directory, use
> # write csv(auto mpg, "./auto mpg.csv")
```



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> # to save this to your local directory, use
> # write csv(auto mpg, "./auto mpg.csv")
```

And calculate the average MPG.



Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
# A tibble: 5 7
    MPG Cylinders Displacement
                                  HP Weight Acceleration After1975
  <dbl>
            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
                                                                <chr>
                8
                            307
                                  130
                                       3504
     18
                                                      12.0
                                                                   No
2
     15
                8
                            350
                                  165
                                      3693
                                                      11.5
                                                                   No
3
     18
                8
                            318
                                  150
                                      3436
                                                      11.0
                                                                   No
4
     16
                8
                            304
                                  150
                                      3433
                                                      12.0
                                                                   No
     17
                            302
                                  140
                                         3449
                                                      10.5
                                                                   No
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                                       3449
                                                                 No
```

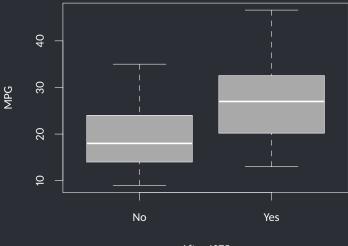
No??? What the... What to do with that?

Let's display the first 5 rows (and all columns).

```
> auto mpg[1:5,]
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    MPG Cylinders Displacement
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            <int>
                          <dbl> <int>
                                       <int>
                                                     <dbl>
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                                                      12.0
                                                                  No
     17
                            302
                                                      10.5
                                  140
                                        3449
                                                                  No
```

No??? What the... What to do with that? Maybe just omit the "After1975" column?





After 1975

How can we incorporate the "After1975" variable into a regression model?



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Create a dummy variable that maps a "Yes" to 1, and "No" to 0.



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Create a dummy variable that maps a "Yes" to 1, and "No" to 0.

> auto mpg\$LateModel <- ifelse(auto mpg\$After1975 == "Yes", 1, 0)</pre>



How can we incorporate the "After1975" variable into a regression model?

Create a dummy variable that maps a "Yes" to 1, and "No" to 0.

```
> auto_mpg$LateModel <- ifelse(auto_mpg$After1975 == "Yes", 1, 0)</pre>
```

Now run a regression model using the predictors Cylinders, Displacement, HP, Weight, Acceleration and LateModel. What is your  $\mathbb{R}^2$ ?



Let's see how R handles it.

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R was able to handle the "After1975" column, which is a categorical variable (or a factor as R calls them).

```
round(summary(model)$coefficients, 2)
           Estimate Std. Error t value Pr(>|t|)
(Intercept)
             42.19
                             17.81
                                     0.00
                       2.37
Cylinders
             -0.58
                       0.36 -1.62
                                     0.11
Displacement
            0.01
                       0.01 0.94
                                     0.35
ΗP
             -0.02
                       0.01 -1.35
                                     0.18
Weight
             -0.01
                       0.00
                                     0.00
                             -8.33
Acceleration
              0.04
                       0.11 0.32
                                     0.75
After1975Yes
              4.36
                       0.40
                            10.85
                                     0.00
```

R has created a dummy variable, "After1975Yes."

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round(summary(model)$coefficients, 2)
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                             17.81
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                       0.00
                                      0.00
                              -8.33
Acceleration
              0.04
                       0.11
                              0.32
                                      0.75
After1975Yes
              4.36
                       0.40
                             10.85
                                      0.00
```

R has created a dummy variable, "After1975Yes."

A dummy variable is always 0 or 1, indicating the absence or presence of some categorical effect.

"After1975Yes" is 1 whenever "After1975" is a "Yes," and 0 otherwise.

MPG	 Acceleration	After1975	After1975Yes	
 25	 13.5	 No		
33	17.5	No	0	
28	15.5	Yes	1	
25	16.9	Yes	1	

"After1975Yes" is 1 whenever "After1975" is a "Yes," and 0 otherwise.

MPG	 Acceleration	After1975	After1975Yes	
25	13.5	No	0	
33	17.5	No	0	
28	15.5	Yes	1	
25	16.9	Yes	1	
	 	•••		

Notice that we do not have a "After1975No" variable.

It would cause problems because it would be perfectly correlated with

"After1975Yes."

Our model contains some statistically insignificant variables. Your task is to omit them one by one. What is the  $R^2$  in your final model?



```
> model <- lm(MPG ~ HP + Weight + After1975,</pre>
                   data=auto mpg)
+
> summary(model)$r.squared
[1] 0.7745063
> round(summary(model)$coefficients, 2)
            Estimate Std. Error t value Pr(>|t|)
(Intercept)
              41.71
                         0.78 53.15
                                        0.00
              -0.02 0.01 -2.30 0.02
HP
Weight
            -0.01 0.00 -13.84 0.00
After1975Yes 4.33
                         0.40 10.83 0.00
```

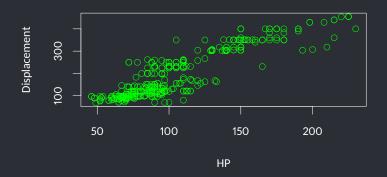
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              41.71
                                        0.00
HP
              -0.02 0.01 -2.30 0.02
Weight
              -0.01 0.00 -13.84 0.00
After1975Yes 4.33
                         0.40 10.83 0.00
```

Horsepower seems to be already capturing the information in Cylinders, Displacement and Acceleration.

#### To see the correlation between variables:

```
> mpg numeric = auto mpg[,c(1,2,3,4,5,6)]
> round(cor(mpg numeric),2)
             MPG Cylinders Displacement HP Weight Acceleration
MPG
            1.00
                    -0.78
                               -0.81 -0.78 -0.83
                                                       0.42
Cylinders -0.78
                                0.95 0.84
                                           0.90
                                                      -0.50
                     1.00
Displacement -0.81
                     0.95
                                1.00 0.90 0.93
                                                      -0.54
HP
           -0.78
                    0.84
                                0.90 1.00
                                           0.86
                                                      -0.69
Weight
       -0.83
                    0.90
                                0.93 0.86 1.00
                                                      -0.42
Acceleration 0.42
                               -0.54 -0.69 -0.42
                    -0.50
                                                       1.00
```

```
> plot(auto_mpg$HP, auto_mpg$Displacement,
+ xlab='HP', ylab='Displacement',col='green', main='')
```



#### Consider this:

- Model A and B have the same HP and Weight.
- Model A was manufactured before 1975, whereas B was manufactured after 1975.
- Our model's prediction for Model A is 21 MPG.
- What is the prediction for Model B?



Our "reference level" is the cars manufactured before 1975.

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For the same Weight and HP, our MPG prediction for a car manufactured after 1975 is always exactly 4.33 higher compared to its reference.

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 $\beta$  gives us the increment in our prediction for the cars manufactured after 1975.

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For the same Weight and HP, our MPG prediction for a car manufactured after 1975 is always exactly 4.33 higher compared to its reference.

 $\beta$  gives us the increment in our prediction for the cars manufactured after 1975.

There are other coding schemes too, where the reference is chosen differently, therefore  $\beta$  is interpreted differently.

#### What if there are more than two categories?

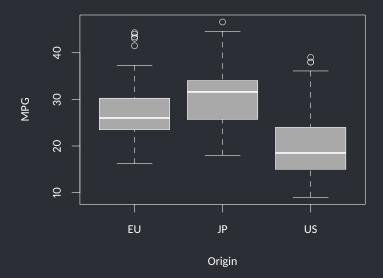
```
> auto mpg all[1:5,]
# A tibble: 5 8
    MPG Cylinders Displacement
                                  HP Weight Acceleration After1975 Origin
  <fd>>
            <int>
                         <dbl> <int>
                                      <int>
                                                    <fdh>>
                                                              <chr>
                                                                     <chr>
     18
                8
                           307
                                 130
                                     3504
                                                     12.0
                                                                 No
                                                                        US
  15
                8
                           350
                                 165 3693
                                                     11.5
                                                                 No
                                                                        US
    18
                8
                           318
                                 150 3436
                                                     11.0
                                                                        IIS
                                                                 Nο
4
    16
                8
                           304
                                 150
                                     3433
                                                     12.0
                                                                        US
                                                                 Nο
     17
                8
                           302
                                 140
                                       3449
                                                     10.5
                                                                 No
                                                                        US
> levels(as.factor(auto mpg all$0rigin))
[1] "EU" "JP" "US"
```

#### What if there are more than two categories?

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> auto mpg all[1:5,]
# A tibble: 5 8
   MPG Cylinders Displacement
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> levels(as.factor(auto mpg all$0rigin))
[1] "EU" "JP" "US"
```

Let's first see if "Origin" makes a difference.

```
> boxplot(MPG ~ Origin, data=auto_mpg_all, ylab="MPG",
+ xlab="Origin", col='darkgray')
```



```
> omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,</pre>
                  data=auto mpg all)
> round(summary(omodel)$coefficients,3)
           Estimate Std. Error t value Pr(>|t|)
            40.182
                       0.874 45.961
                                      0.000
(Intercept)
                       0.010 -2.837
HP
             -0.028
                                      0.005
Weight
       -0.005
                       0.000 -10.815
                                      0.000
After1975Yes 4.334
                       0.393 11.033
                                      0.000
OriginJP
         1.001
                       0.612 1.635
                                      0.103
OriginUS
        -1.593
                       0.562 -2.834
                                      0.005
```

```
> omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,</pre>
                  data=auto mpg all)
 round(summary(omodel)$coefficients,3)
           Estimate Std. Error t value Pr(>|t|)
             40.182
                        0.874 45.961
                                       0.000
(Intercept)
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HP
             -0.028
                                       0.005
                        0.000 -10.815
Weight
       -0.005
                                       0.000
After1975Yes 4.334
                        0.393 11.033
                                       0.000
<u>OriginJP</u>
          1.001
                        0.612 1.635
                                       0.103
OriginUS
         -1.593
                        0.562 -2.834
                                       0.005
```

For the origin variable, R has chosen "EU" as the base, created a dummy variable for JP and US each.

While dealing with categorical variables, we look at the significance of the categorical variable as a whole.

Unless all the dummy variables are insignificant, we do not omit the column of that categorical variable.

# Categorical Variables with Numeric Representations

In the original dataset, the origin was represented as 1 for U.S., 2 for EU and 3 for JP.

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Or, assume that we have a column for the "U.S. News Brand Ranking."

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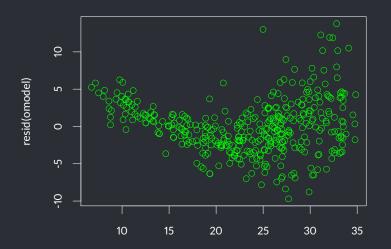
Or, assume that we have a column for the "U.S. News Brand Ranking."

They are still categorical variables and should be treated as such.

#### **Assumptions**

What are the issues with this model?

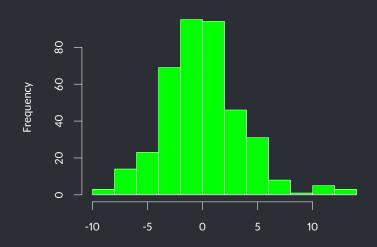
> plot(predict. lm(omodel), resid(omodel), col='green', main='')



#### **Assumptions**

What about normality?

> hist(resid(omodel), col='green', main='')



#### **Assumptions**

#### What about normality?

> qqnorm(resid(omodel), col='green', main='')

