THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 3

Lecture 4

STA 371G

- Starting tonight: Weekly R help session in the ModLab, 6:30-7:30 PM

Who are these people?

Zack

Kelly

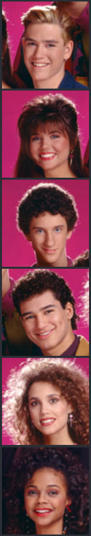
Screech

Slater

Jessie

Lisa

Some events



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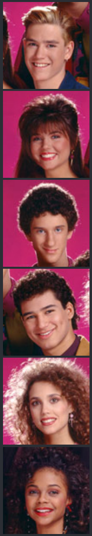

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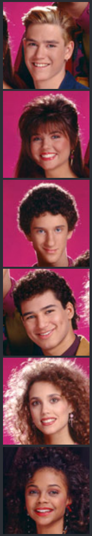

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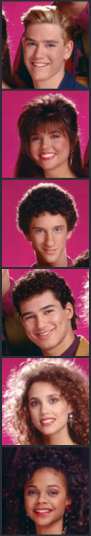

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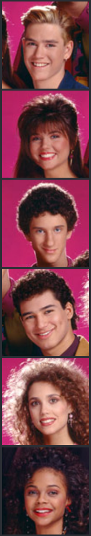

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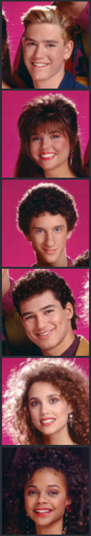

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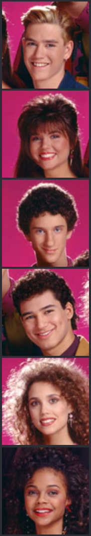
$P(M) =$

Some events



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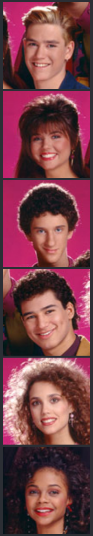
$P(M) = 3/6$ $\qquad$ $P(L) =$

Some events



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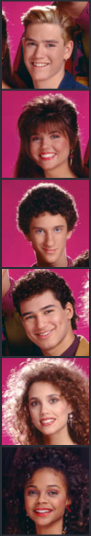

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$P(M|L) = 1/3$ $\quad P(L|M) =$

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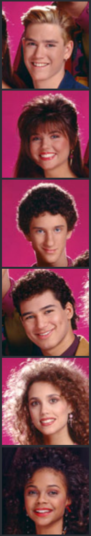

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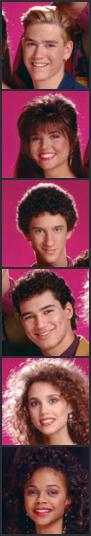
$P(M|L) = 1/3 \qquad P(L|M) = 1/3$

Joint probability



$P(M \text{ and } L) =$ probability of selecting a lush-haired male

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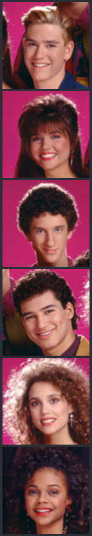

$$P(M \text{ and } L) = \text{probability of selecting a lush-haired male}$$
$$= 1/6$$

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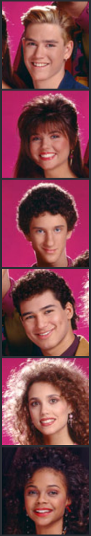
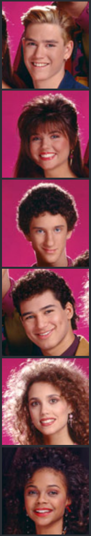


$$
\begin{aligned}
P(M \text{ and } L) &= \text{probability of selecting a lush-haired male} \\
&= 1/6 \\
&= P(M)P(L|M) = \frac{3}{6} \cdot \frac{1}{3} \\
&= P(L)P(M|L) = \frac{3}{6} \cdot \frac{1}{3}
\end{aligned}
$$

Joint probability



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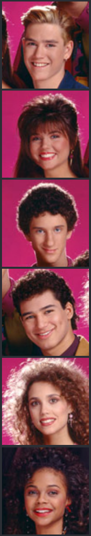
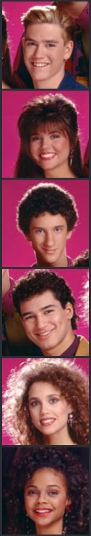
\begin{aligned}
P(M \text{ and } L) &= \text{probability of selecting a lush-haired male} \\
&= 1/6 \\
&= P(M)P(L|M) = \frac{3}{6} \cdot \frac{1}{3} \\
&= P(L)P(M|L) = \frac{3}{6} \cdot \frac{1}{3} \\
&\neq P(M)P(L)
\end{aligned}
$$

One or the other



$$P(M \text{ or } L) = \text{probability of selecting a lush-haired character, a male character, or both}$$

One or the other



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\begin{aligned}
P(M \text{ or } L) &= \text{probability of selecting a lush-haired} \\
&\qquad \text{character, a male character, or both} \\
&= 5/6
\end{aligned}
$$

We have to subtract off $P(M \text{ and } L)$ because otherwise we are double-counting Slater!

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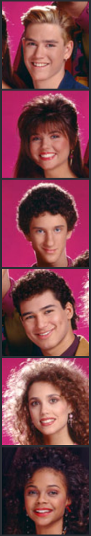

$$\begin{aligned} P(M \text{ or } L) &= \text{probability of selecting a lush-haired} \\ &\qquad \text{character, a male character, or both} \\ &= 5/6 \\ &= P(M) + P(L) - P(M \text{ and } L) \\ &= 3/6 + 3/6 - 1/6 \end{aligned}$$

We have to subtract off $P(M \text{ and } L)$ because otherwise we are double-counting Slater!

Conditional probability

When we say $P(A|B)$, what we mean is: “In a world where we know B has already happened, how likely is it that A also happened?”


HOPE SHE WAS

Now that you’ve found a strange pair of underwear in your dresser, what is the probability that your partner is cheating on you?

$$P(\text{partner is cheating} \mid \text{underwear found})$$

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It's hard to know how to estimate this directly!

Let’s come up with estimates for the following:

- $P(\text{underwear found} \mid \text{partner is cheating})$

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- $P(\text{underwear found} \mid \text{partner is cheating})$
- $P(\text{underwear found} \mid \text{partner is not cheating})$
- $P(\text{partner is cheating})$ (before we found the underwear!)

Bayes' Rule

Bayes' Rule lets us reverse the conditional probabilities!

$U =$ underwear found, $C =$ partner is cheating

$$P(C|U) = \frac{P(U|C)P(C)}{P(U|C)P(C) + P(U|\overline{C})P(\overline{C})}$$

Think of Bayes' Rule as a way to update our thinking based on new information:

$$\begin{aligned} &P(C) &&\longleftarrow \text{Prior probability} \\ &P(C|U) &&\longleftarrow \text{Posterior probability (includes new information)} \end{aligned}$$

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- 0.4% of people in the US are HIV-positive

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We know $P(TP|HP) = 0.993$, but we really want to know $P(HP|TP)$!

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- If you do not have HIV, there is a 99.8% chance the test will show a negative result
- But if you test positive there is about a 1/3 chance you do NOT have HIV!
- This is counterintuitive — it's because of the way we are wired (it even has a name: "base rate fallacy")