THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Multiple Regression 2

Lecture 8

STA 371G

Predicting House prices in the Greater Boston Area

Median house price for each census tract, along with other data.

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Median house price for each census tract, along with other data.
The final model:

```
> model <- lm(MEDV ~ CRIME+ZONE+NOX+ROOM+DIST
+                    +RADIAL+TAX+PTRATIO+LSTAT, data=boston)
```

- MEDV: Median Price (response)
- CRIME: Per capita crime rate
- ZONE: Proportion of large lots
- NOX: Nitrogen Oxide concentration
- DIST: Distance to employment centers
- ROOM: Average # of rooms
- RADIAL: Accessibility to highways
- TAX: Tax rate (per $10K)
- PTRATIO: Pupil-to-teacher ratio
- LSTAT: Proportion of “lower status”

Overall Null Hypothesis

Is our model useful? Check the R-squared:

```
> summary(model)$r.squared

[1] 0.7282911
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or

$H_0 : R^2 = 0$

$H_1 : R^2 > 0$

Overall Null Hypothesis

Check the P-value for the F-statistic in the summary

```
Residual standard error: 96.75 on 496 degrees of freedom
Multiple R-squared:  0.7283,    Adjusted R-squared:  0.7234
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So we can reject the overall null hypothesis!
R-squared was already too big to suspect that it is zero and we already knew some predictors are statistically significant.

How good are our predictions?

Let's plot the residuals, i.e., discrepancies between the predictions and the data.

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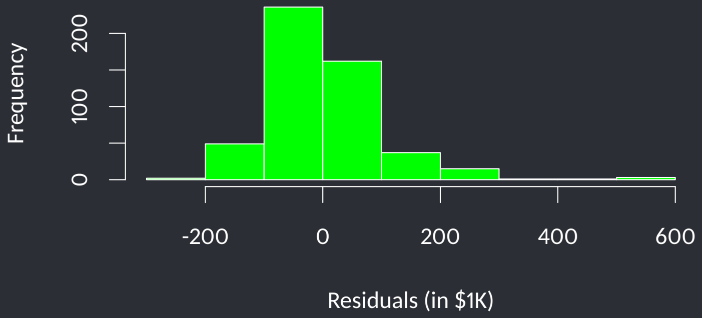
```
> hist(model$residuals, col='green',
+   main='', xlab='Residuals (in $1K)', ylab='Frequency')
```



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By the 2 standard deviation rule, we could estimate that 95% of the time residuals are in [-$192K, $192K] range.

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Again: regression assumptions

Remember the big four:

1. The residuals are independent.
2. Y is a linear function of Xs (except for the errors).
3. The residuals are normally distributed.
4. The variance of Y is the same for any value of Xs (“homoscedasticity”).

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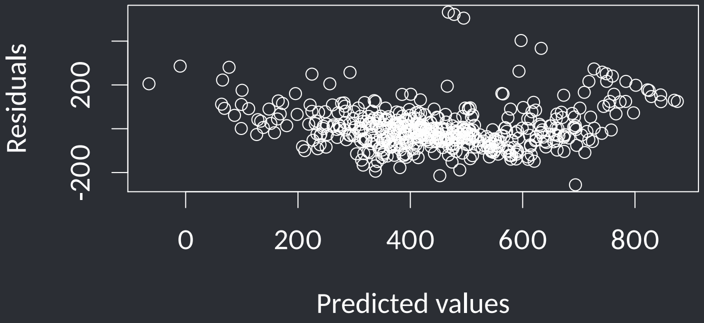
Assumption 2: Linearity

Plot the residuals vs the predicted Y-values and ensure there is no trend:

```
> plot(predict.lm(model), resid(model),
+   xlab='Predicted values', ylab='Residuals')
```



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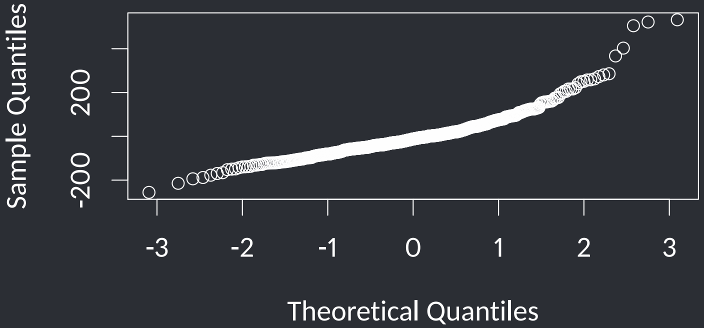
Assumption 3: Normally distributed residuals

Ensure that the Q-Q plot shows a (roughly) straight line:

```
> qqnorm(resid(model), main='')
```



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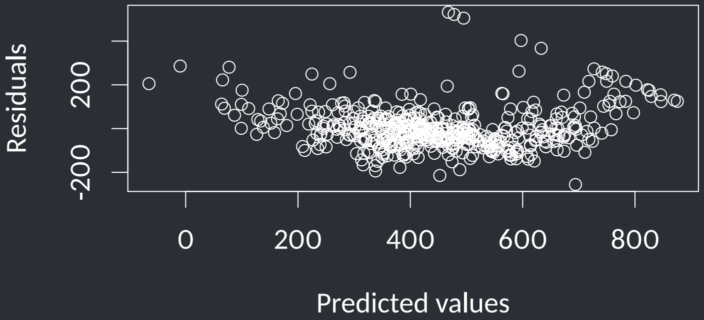
Assumption 4: The variance of Y is the same across

Look for a (roughly) constant vertical "thickness":

```
> plot(predict.lm(model), resid(model),
+   xlab='Predicted values', ylab='Residuals')
```



We have a model. Then what?

Let's make some predictions.

Model Coefficients

Regression model estimates the coefficients of the predictors.

```
> round(summary(model)$coefficients,2)

            Estimate Std. Error t value Pr(>|t|)
(Intercept)   840.07      99.00    8.49        0
CRIME          -2.57       0.66   -3.87        0
ZONE            0.92       0.28    3.34        0
NOX          -346.93      71.81   -4.83        0
ROOM           74.24       8.26    8.99        0
DIST          -31.05       3.78   -8.20        0
RADIAL          6.00       1.29    4.66        0
TAX            -0.27       0.07   -3.87        0
PTRATIO       -19.28       2.63   -7.34        0
LSTAT         -11.07       0.96  -11.56        0
```

Model Coefficients

Let's estimate the median house price in a particular district:

j	Predictor	β_j	X_j	$\beta_j X_j$
0	Intercept	840.07	1	840.07
1	CRIME	-2.57	0.03	-0.0771
2	ZONE	0.92	10	9.2
3	NOX	-346.93	0.5	-173.465
4	ROOM	74.24	4	296.96
5	DIST	-31.05	5	-155.25
6	RADIAL	6	1	6
7	TAX	-0.27	300	-81
8	PTRATIO	-19.28	15	-385.6
9	LSTAT	-11.07	10	-110.7
Price	Estimate	($1000)		342.538

Model Coefficients

Let R do it for us!

```
> predict.lm(model, list(CRIME=0.03, ZONE=10,
+                        NOX=0.5, ROOM=4,
+                        DIST=5,  RADIAL=1,
+                        TAX=300, PTRATIO=15,
+                        LSTAT=10))

       1
343.9552
```

Model Coefficients

Assume that there are 420 students and 28 teachers in the disctrict ($\text{PTRATIO} = 420/28 = 15$).

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       1
363.2349
```

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Nothing is free. To be able to compansate the new hires, the ISD decides to add $50 more on your tax bill for every $10K of your house price.

So, the tax rate increases to 350 per $10K. How would this affect the median house price?

Confidence intervals

We all know our predictions are not exactly right.

Can we come up with some confidence intervals on our predictions?

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Remember the two kinds of intervals:

Confidence	Predicting the mean value of Y for a particular set of X values.	Among all the districts whose predictors are as above, what is the mean value of median house price?
Prediction	Predicting Y for a single new case.	If Springfield has the predictors above, what is the median house price in Springfield?

Confidence intervals

```
> predict.lm(model, list(CRIME=0.03, ZONE=10,
+                        NOX=0.5, ROOM=4,
+                        DIST=5,  RADIAL=1,
+                        TAX=350, PTRATIO=14,
+                        LSTAT=10),
+                        interval = 'confidence')

       fit      lwr      upr
1 349.9684 301.9485 397.9883
```

We can also put a confidence interval on a coefficient to estimate the plausible range of its effect.

```
> confint(model)

                   2.5 %       97.5 %
(Intercept)  645.5520530 1034.5782470
CRIME         -3.8703245   -1.2618439
ZONE           0.3792933    1.4647029
NOX         -488.0175640 -205.8337804
ROOM          58.0099148   90.4751248
DIST         -38.4860994  -23.6129585
RADIAL         3.4693548    8.5311305
TAX           -0.4000457   -0.1306157
PTRATIO      -24.4415728  -14.1179304
LSTAT        -12.9529546   -9.1905075
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```

Reducing the parent/teacher ratio (PTRATIO) by one could increase the median house price from $14K to $24K!