THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

# Dummy Variables

## Lecture 9

STA 371G

Let’s predict fuel economy (miles per gallon) for different car models of the 70s.

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- Cylinders
- Displacement
- Horsepower
- Weight
- Acceleration
- Year (After 1975 or not)

# Exploring the data

Let's display the first 5 rows (and all columns).

```
auto_mpg[1:5,]

# A tibble: 5 <U+00D7> 7
    MPG Cylinders Displacement    HP Weight Acceleration After1975
  <dbl>     <int>        <dbl> <int>  <int>        <dbl>     <chr>
1    18         8          307   130   3504         12.0        No
2    15         8          350   165   3693         11.5        No
3    18         8          318   150   3436         11.0        No
4    16         8          304   150   3433         12.0        No
5    17         8          302   140   3449         10.5        No
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```
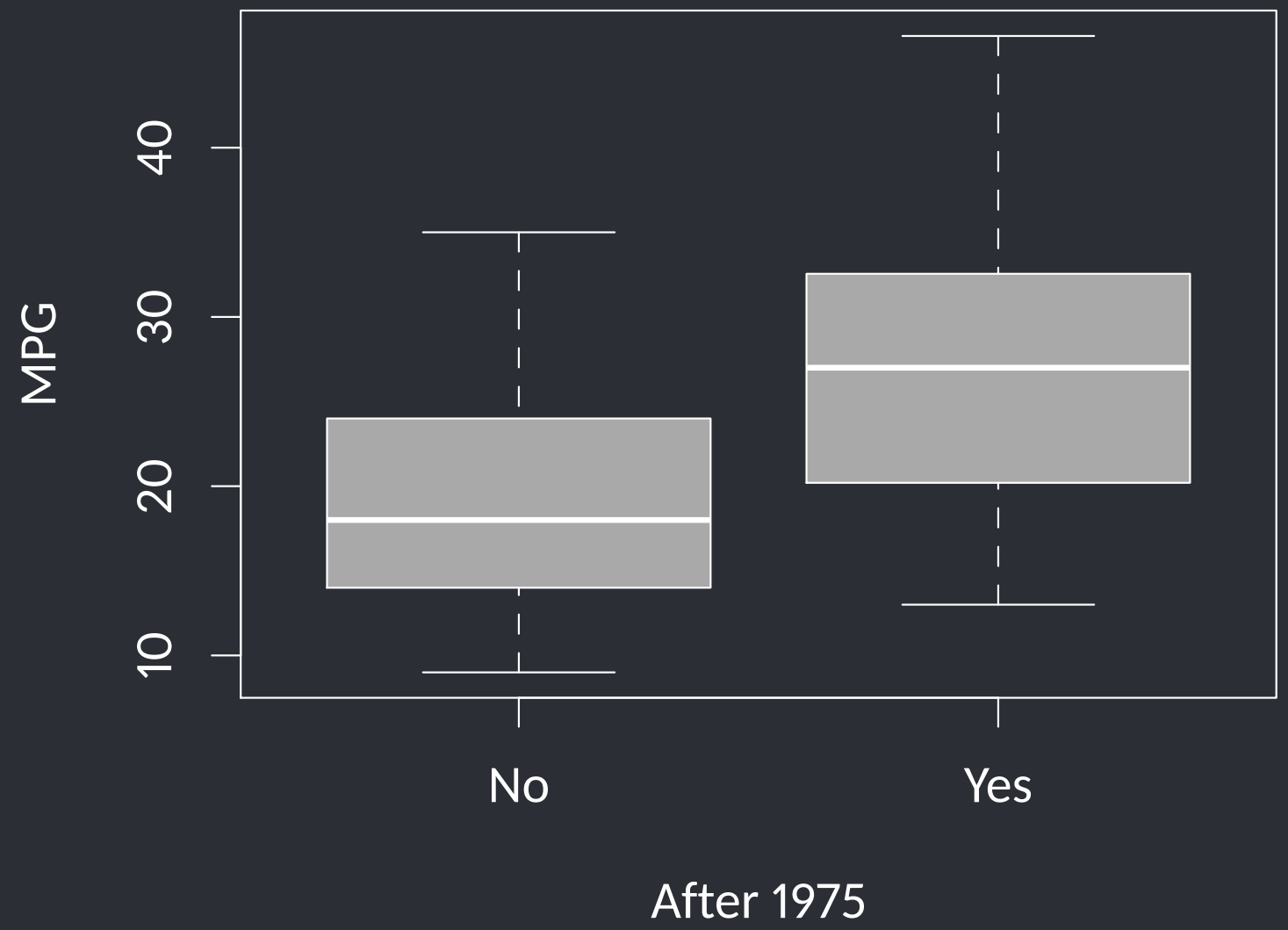

How do we handle the Yes/No data in the "After1975" column?

```
boxplot(MPG ~ After1975, data=auto_mpg, ylab="MPG",
                xlab="After 1975", col='darkgray')
```



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```
auto_mpg$LateModel <-
  ifelse(auto_mpg$After1975 == "Yes", 1, 0)
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```
auto_mpg$LateModel <-
  ifelse(auto_mpg$After1975 == "Yes", 1, 0)
```

Now let’s a regression model using the predictors Cylinders, Displacement, HP, Weight, Acceleration and LateModel.

# Regression with categorical variables

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```
model <- lm(MPG ~ Cylinders + Displacement + HP
            + Weight + Acceleration + After1975,
            data=auto_mpg)
summary(model)$r.squared

[1] 0.776176
```

# Regression with categorical variables

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```

R was able to handle the “After1975” column, which is a categorical variable (or a factor as R calls them).

# Dummy variables

```
round(summary(model)$coefficients, 2)
```

| | Estimate | Std. Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| (Intercept) | 42.19 | 2.37 | 17.81 | 0.00 |
| Cylinders | -0.58 | 0.36 | -1.62 | 0.11 |
| Displacement | 0.01 | 0.01 | 0.94 | 0.35 |
| HP | -0.02 | 0.01 | -1.35 | 0.18 |
| Weight | -0.01 | 0.00 | -8.33 | 0.00 |
| Acceleration | 0.04 | 0.11 | 0.32 | 0.75 |
| After1975Yes | 4.36 | 0.40 | 10.85 | 0.00 |

R has created a dummy variable "After1975Yes."

# Dummy variables

"After1975Yes" is 1 whenever "After1975" is "Yes," and 0 otherwise:

| MPG | ... | Acceleration | After1975 | After1975Yes |
|---|---|---|---|---|
| ... | ... | ... | ... | ... |
| 25 | ... | 13.5 | No | 0 |
| 33 | ... | 17.5 | No | 0 |
| 28 | ... | 15.5 | Yes | 1 |
| 25 | ... | 16.9 | Yes | 1 |
| ... | ... | ... | ... | ... |

# Dummy variables

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Notice that we do not have a "After1975No" variable.

# Dummy variables

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|---|---|---|---|---|
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| 25 | ... | 13.5 | No | 0 |
| 33 | ... | 17.5 | No | 0 |
| 28 | ... | 15.5 | Yes | 1 |
| 25 | ... | 16.9 | Yes | 1 |
| ... | ... | ... | ... | ... |

Notice that we do not have a "After1975No" variable.
It would cause problems because it would be perfectly correlated with "After1975Yes."

# Regression with categorical variables

Let's simplify our model by omitting statistically insignificant variables one by one. (Make sure to re-run the model after omitting each variable, starting with the least signfiicant.)
What is the $R^2$ in your final model?

# Regression with categorical variables

```
model <- lm(MPG ~  HP + Weight + After1975,
                   data=auto_mpg)
summary(model)$r.squared

[1] 0.7745063

round(summary(model)$coefficients, 2)

             Estimate Std. Error t value Pr(>|t|)
(Intercept)     41.71       0.78   53.15     0.00
HP              -0.02       0.01   -2.30     0.02
Weight          -0.01       0.00  -13.84     0.00
After1975Yes     4.33       0.40   10.83     0.00
```

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```

Is Horsepower capturing the information in Cylinders, Displacement and Acceleration?

# Regression with categorical variables

Let's look at the correlations between variables:

```
cor(auto_mpg[,c(1,2,3,4,5,6)])
```

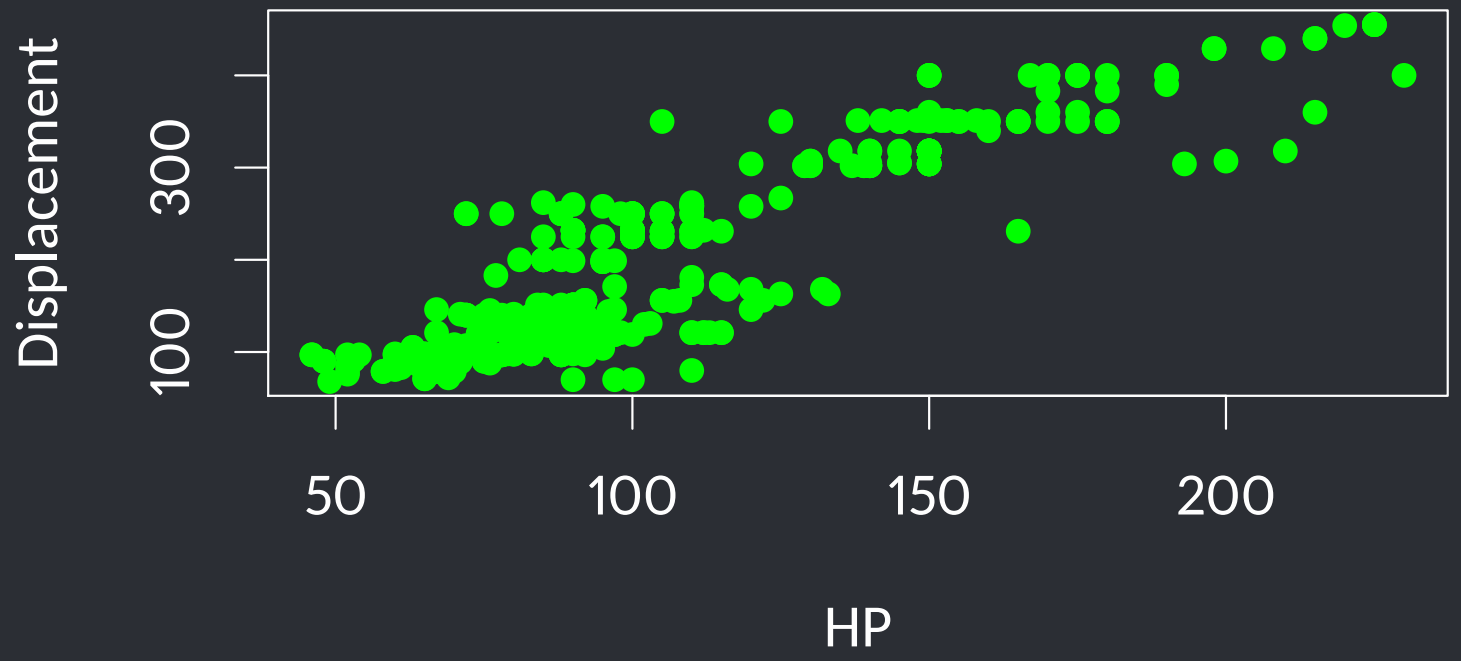
```
                MPG Cylinders Displacement    HP Weight Acceleration
MPG            1.00     -0.78         -0.81 -0.78  -0.83         0.42
Cylinders     -0.78      1.00          0.95  0.84   0.90        -0.50
Displacement  -0.81      0.95          1.00  0.90   0.93        -0.54
HP            -0.78      0.84          0.90  1.00   0.86        -0.69
Weight        -0.83      0.90          0.93  0.86   1.00        -0.42
Acceleration   0.42     -0.50         -0.54 -0.69  -0.42         1.00
```

We have multicollinearity between HP, Cylinders, Displacement, and Acceleration — all are highly correlated so it only makes sense to have one of these in the model.

# Regression with categorical variables

The information in Displacement is already mostly captured by HP:

```
plot(auto_mpg$HP, auto_mpg$Displacement, pch=16,
  xlab='HP', ylab='Displacement',col='green', main='')
```



# Interpretation of the $\hat{\beta}$ of the dummy variable

Our regression equation is:

$$\widehat{\text{Price}} = 41.71 - 0.02 \cdot \text{HP} - 0.01 \cdot \text{Weight} + 4.33 \cdot \text{After1975Yes}.$$

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Let's interpret the coefficient 4.33. Consider this:

- Model A and B have the same HP and Weight.
- Model A was manufactured before 1975, whereas B was manufactured after 1975.
- We predict Model B will have a MPG that is 4.33 higher than Model A.

# Interpretation of the $\hat{\beta}$ of the dummy variable

R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975.

# Interpretation of the $\hat{\beta}$ of the dummy variable

R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975. If we created

a dummy variable After1975No that is 1 for cars manufactured *before* 1975, what would the regression look like?

# What if there are more than two categories?

The Origin variable represents the country of manufacture.

```
auto_mpg_all[1:5,]

# A tibble: 5 <U+00D7> 8
    MPG Cylinders Displacement    HP Weight Acceleration After1975 Origi
  <dbl>     <int>        <dbl> <int>  <int>        <dbl>     <chr>  <chr
1    18         8          307   130   3504           12        No     U
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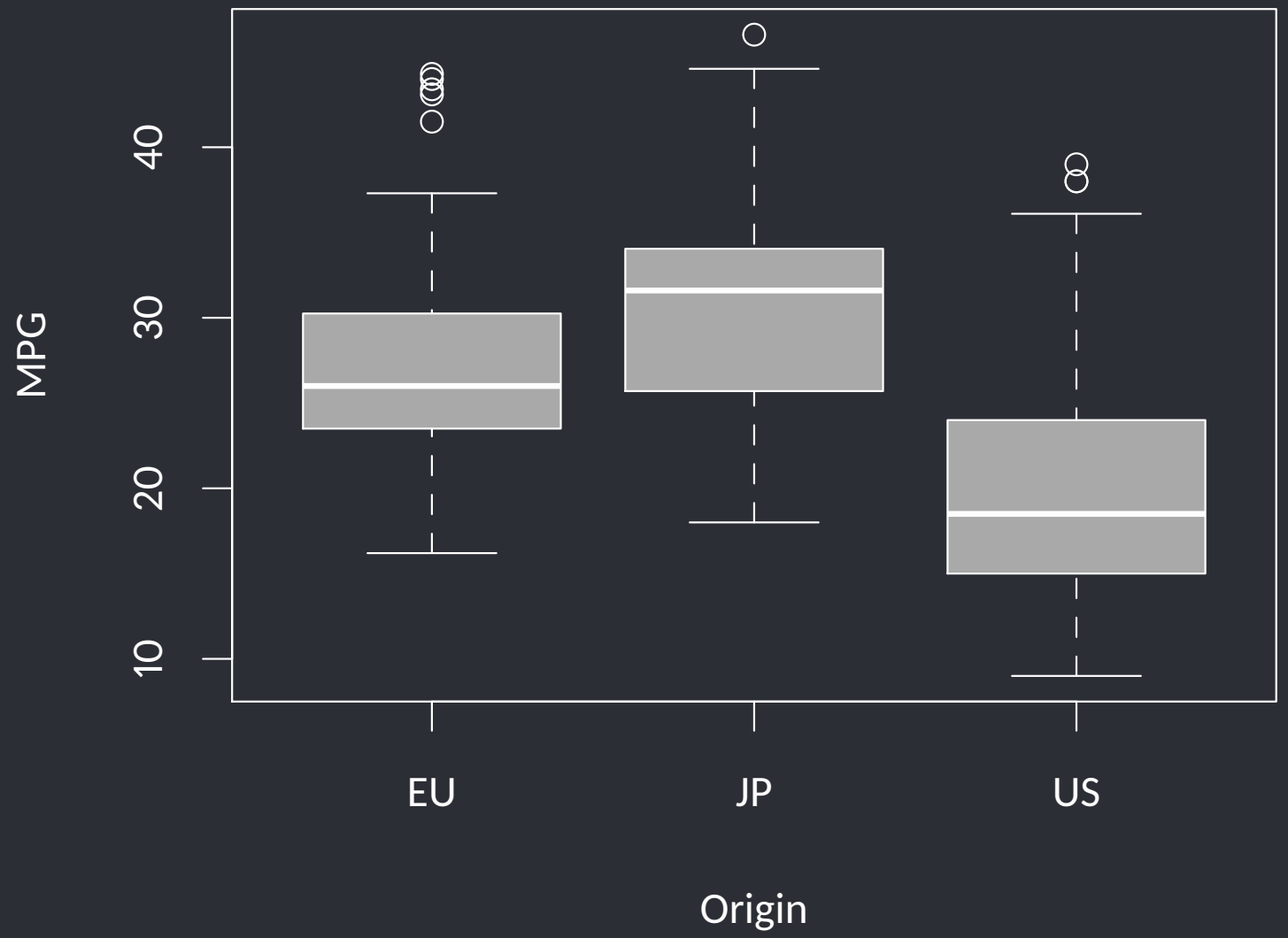
levels(as.factor(auto_mpg_all$Origin))

[1] "EU" "JP" "US"
```

```
boxplot(MPG ~ Origin, data=auto_mpg_all, ylab="MPG",
                xlab="Origin", col='darkgray')
```



# Regression with categorical variables

```
omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,
             data=auto_mpg_all)
round(summary(omodel)$coefficients,3)
```

```
             Estimate Std. Error t value Pr(>|t|)
(Intercept)    40.182       0.87    46.0    0.000
HP             -0.028       0.01    -2.8    0.005
Weight         -0.005       0.00   -10.8    0.000
After1975Yes    4.334       0.39    11.0    0.000
OriginJP        1.001       0.61     1.6    0.103
OriginUS       -1.593       0.56    -2.8    0.005
```

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OriginJP        1.001       0.61     1.6    0.103
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```

For Origin, R has chosen EU as the reference level and create dummy variables for both JP and US.

# A warning about categorical variables with numeric representations

In the original dataset, the origin was represented as 1 for U.S., 2 for EU and 3 for JP.

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In the original dataset, the origin was represented as 1 for U.S., 2 for EU and 3 for JP. We would NOT want to just put these numbers in the regression as numbers,

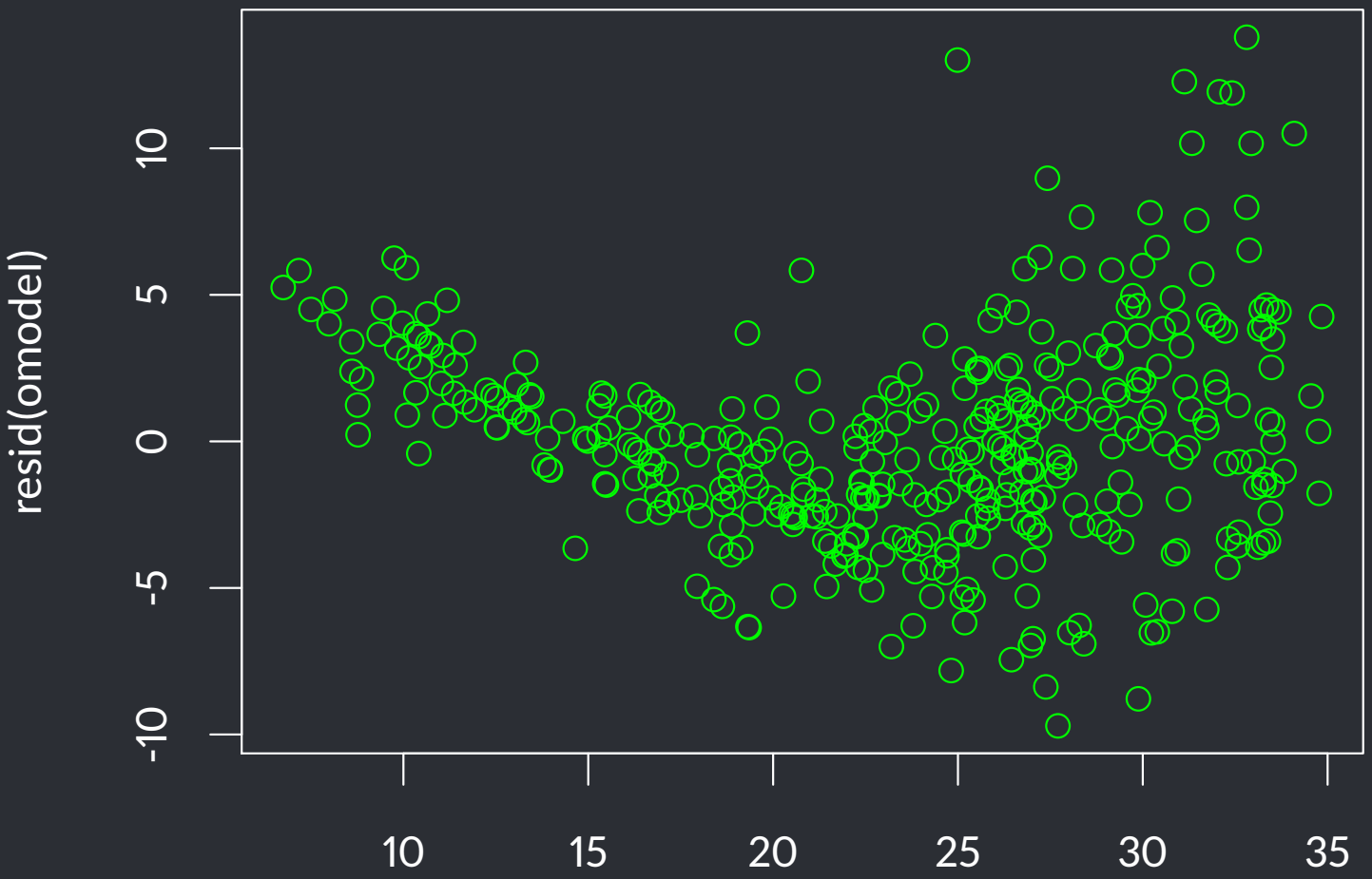
because then regression would treat this as if it were a quantitative variable! Even

though the representation in the file is numeric, it is still a categorical variable and should be treated as such.

# Assumptions

What are the issues with this model?

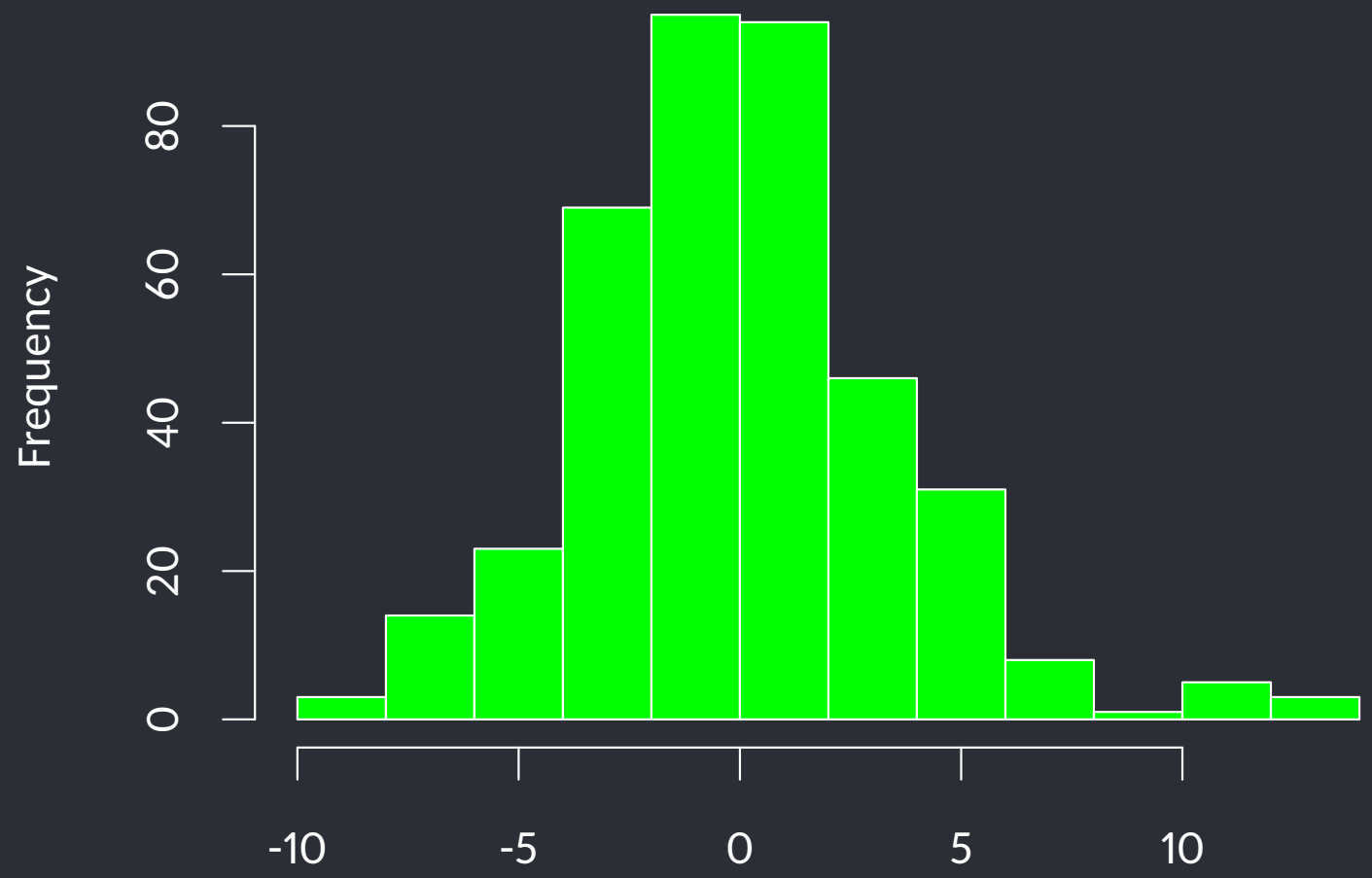
```
plot(predict.lm(omodel), resid(omodel), col='green', main='')
```



# Assumptions

What about normality?

```
hist(resid(omodel), col='green', main='')
```



# Assumptions

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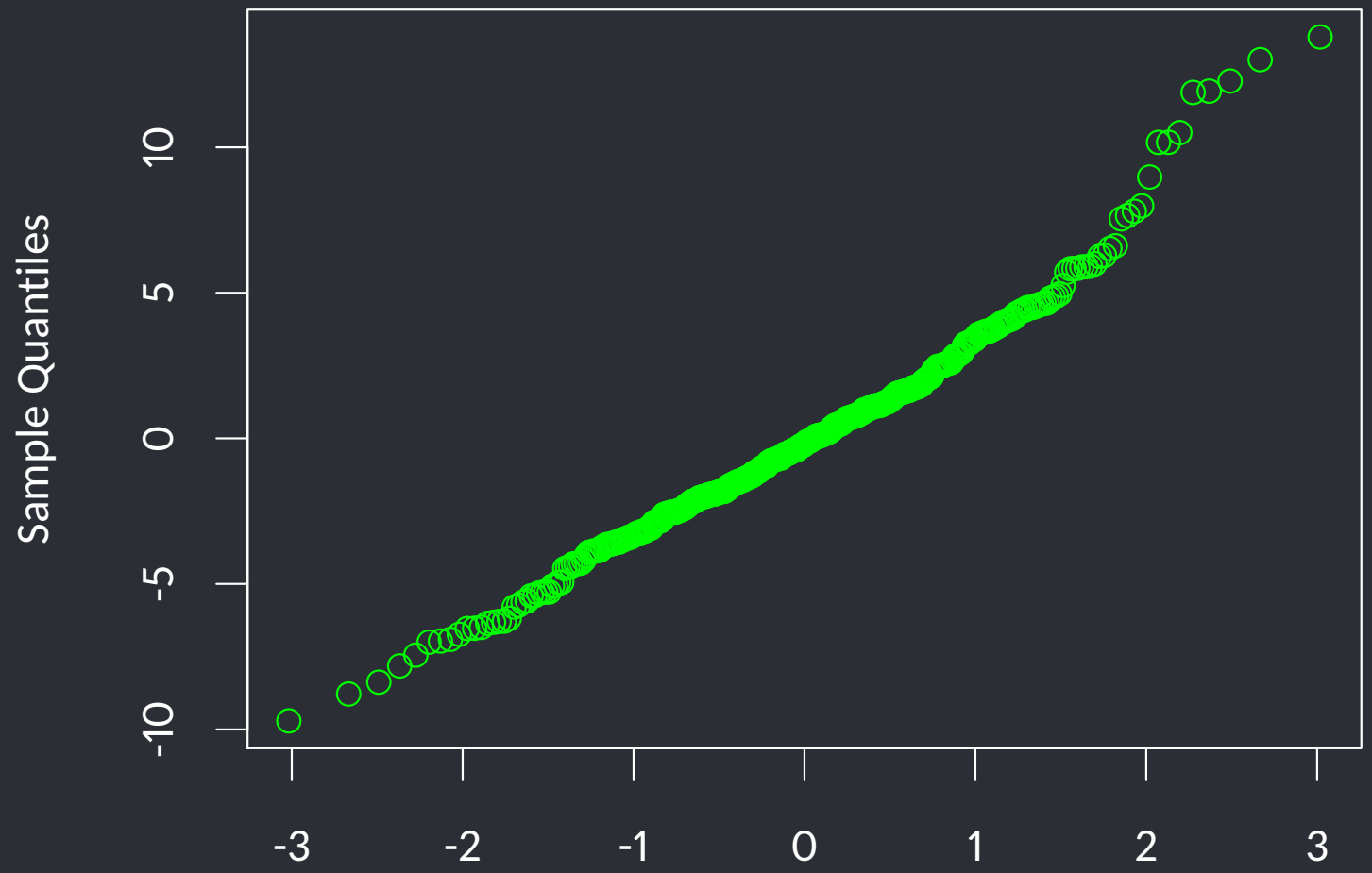
```
qqnorm(resid(omodel), col='green', main='')
```



# Testing the significance of a categorical variable

While dealing with categorical variables, we want to look at the significance of the categorical variable as a whole, rather than looking at *p*-values of individual dummy variables.

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While dealing with categorical variables, we want to look at the significance of the categorical variable as a whole, rather than looking at $p$-values of individual dummy variables.

We want to test the null hypothesis

$$H_0 : \beta_{\text{US}} = \beta_{\text{EU}} = 0.$$

# Testing the significance of a categorical variable

To do this, we look at the ANOVA table; the $p$-value on the Origin line ($2.4 \times 10^{-5}$) is the $p$-value for the compound null hypothesis $H_0 : \beta_{US} = \beta_{EU} = 0$.

```
anova(omodel)

Analysis of Variance Table

Response: MPG
           Df Sum Sq Mean Sq F value  Pr(>F)
HP          1  14433   14433  1096.0 < 2e-16 ***
Weight      1   2392    2392   181.6 < 2e-16 ***
After1975   1   1623    1623   123.2 < 2e-16 ***
Origin      2    288     144    10.9 2.4e-05 ***
Residuals 386   5083      13
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```