



THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Dummy Variables

Lecture 9

STA 371G

Let's predict fuel economy (miles per gallon) for different car models of the 70s.



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- Cylinders
- Displacement
- Horsepower
- Weight
- Acceleration
- Year (After 1975 or not)

Exploring the data

Let's display the first 5 rows (and all columns).

```
auto_mpg[1:5,]
```

```
# A tibble: 5 <U+00D7> 7
```

	MPG	Cylinders	Displacement	HP	Weight	Acceleration	After1975
	<dbl>	<int>	<dbl>	<int>	<int>	<dbl>	<chr>
1	18	8	307	130	3504	12.0	No
2	15	8	350	165	3693	11.5	No
3	18	8	318	150	3436	11.0	No
4	16	8	304	150	3433	12.0	No
5	17	8	302	140	3449	10.5	No

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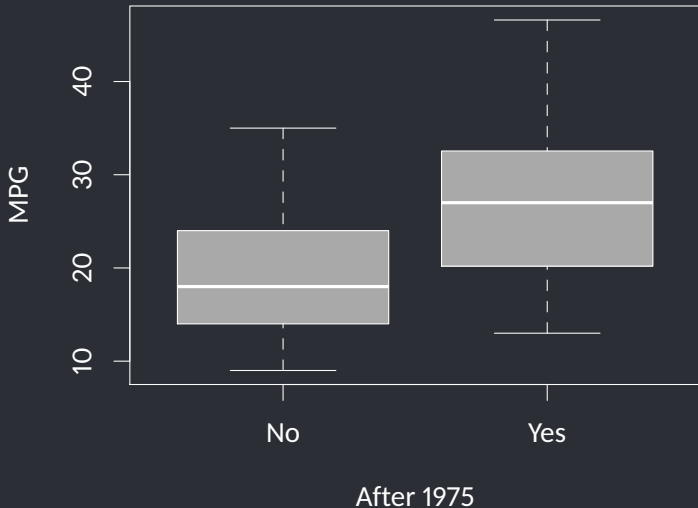
```
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How do we handle the Yes/No data in the "After1975" column?

```
boxplot(MPG ~ After1975, data=auto_mpg, ylab="MPG",  
        xlab="After 1975", col='darkgray')
```



Exploring the data

To incorporate the “After1975” variable into a regression model, create a **dummy variable** that maps a “Yes” to 1, and “No” to 0.



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auto_mpg$LateModel <-  
  ifelse(auto_mpg$After1975 == "Yes", 1, 0)
```



Exploring the data

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```
auto_mpg$LateModel <-  
  ifelse(auto_mpg$After1975 == "Yes", 1, 0)
```

Now let's a regression model using the predictors Cylinders, Displacement, HP, Weight, Acceleration and LateModel.



Regression with categorical variables

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```
model <- lm(MPG ~ Cylinders + Displacement + HP  
            + Weight + Acceleration + After1975,  
            data=auto_mpg)  
summary(model)$r.squared  
  
[1] 0.776176
```

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R was able to handle the “After1975” column, which is a **categorical variable** (or a **factor** as R calls them).

Dummy variables

```
round(summary(model)$coefficients, 2)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	42.19	2.37	17.81	0.00
Cylinders	-0.58	0.36	-1.62	0.11
Displacement	0.01	0.01	0.94	0.35
HP	-0.02	0.01	-1.35	0.18
Weight	-0.01	0.00	-8.33	0.00
Acceleration	0.04	0.11	0.32	0.75
After1975Yes	4.36	0.40	10.85	0.00

R has created a **dummy variable** "After1975Yes."

Dummy variables

“After1975Yes” is 1 whenever “After1975” is “Yes,” and 0 otherwise:

MPG	...	Acceleration	After1975	After1975Yes
...
25	...	13.5	No	0
33	...	17.5	No	0
28	...	15.5	Yes	1
25	...	16.9	Yes	1
...

Dummy variables

“After1975Yes” is 1 whenever “After1975” is “Yes,” and 0 otherwise:

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25	...	13.5	No	0
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...

Notice that we do not have a “After1975No” variable.

Dummy variables

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25	...	13.5	No	0
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28	...	15.5	Yes	1
25	...	16.9	Yes	1
...

Notice that we do not have a “After1975No” variable.

It would cause problems because it would be perfectly correlated with “After1975Yes.”

Regression with categorical variables

Let's simplify our model by omitting statistically insignificant variables one by one. (Make sure to re-run the model after omitting each variable, starting with the least significant.)

What is the R^2 in your final model?



Regression with categorical variables

```
model <- lm(MPG ~ HP + Weight + After1975,  
            data=auto_mpg)  
summary(model)$r.squared
```

```
[1] 0.7745063
```

```
round(summary(model)$coefficients, 2)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	41.71	0.78	53.15	0.00
HP	-0.02	0.01	-2.30	0.02
Weight	-0.01	0.00	-13.84	0.00
After1975Yes	4.33	0.40	10.83	0.00

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Is Horsepower capturing the information in Cylinders, Displacement and Acceleration?

Regression with categorical variables

Let's look at the correlations between variables:

```
cor(auto_mpg[,c(1,2,3,4,5,6)])
```

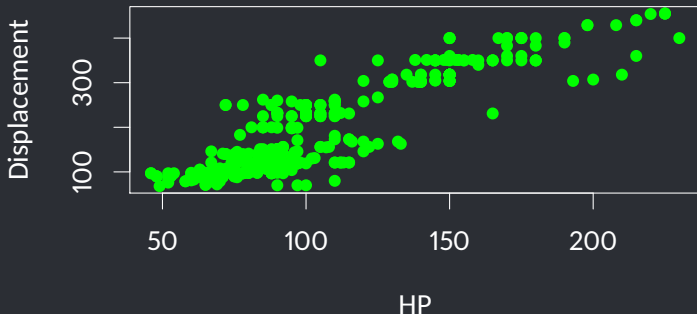
	MPG	Cylinders	Displacement	HP	Weight	Acceleration
MPG	1.00	-0.78	-0.81	-0.78	-0.83	0.42
Cylinders	-0.78	1.00	0.95	0.84	0.90	-0.50
Displacement	-0.81	0.95	1.00	0.90	0.93	-0.54
HP	-0.78	0.84	0.90	1.00	0.86	-0.69
Weight	-0.83	0.90	0.93	0.86	1.00	-0.42
Acceleration	0.42	-0.50	-0.54	-0.69	-0.42	1.00

We have **multicollinearity** between HP, Cylinders, Displacement, and Acceleration — all are highly correlated so it only makes sense to have one of these in the model.

Regression with categorical variables

The information in Displacement is already mostly captured by HP:

```
plot(auto_mpg$HP, auto_mpg$Displacement, pch=16,  
      xlab='HP', ylab='Displacement', col='green', main='')
```



Interpretation of the $\hat{\beta}$ of the dummy variable

Our regression equation is:

$$\widehat{\text{Price}} = 41.71 - 0.02 \cdot \text{HP} - 0.01 \cdot \text{Weight} + 4.33 \cdot \text{After1975Yes}.$$



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Let's interpret the coefficient 4.33. Consider this:

- Model A and B have the same HP and Weight.
- Model A was manufactured before 1975, whereas B was manufactured after 1975.
- We predict Model B will have a MPG that is 4.33 higher than Model A.



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R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975.

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R has assigned “Yes” to 1 and “No” to 0 in our dummy variable, so the “reference level” is cars manufactured before 1975. If we created a dummy variable `After1975No` that is 1 for cars manufactured *before* 1975, what would the regression look like?

What if there are more than two categories?

The Origin variable represents the country of manufacture.

```
auto_mpg_all[1:5,]
```

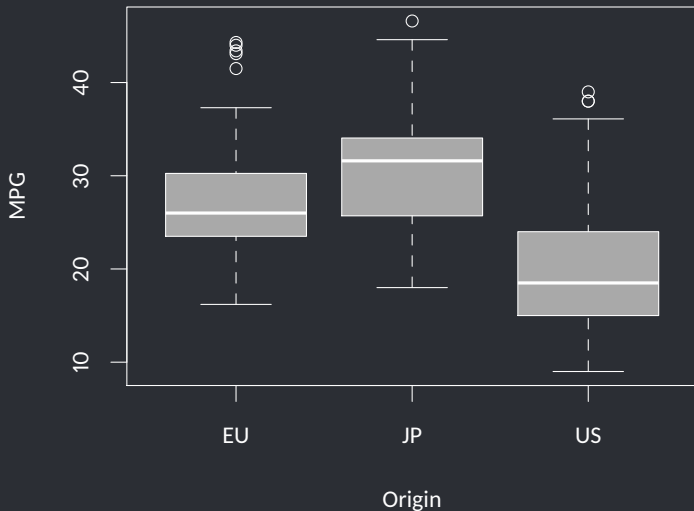
```
# A tibble: 5 × 8
```

	MPG	Cylinders	Displacement	HP	Weight	Acceleration	After1975	Origin
	<dbl>	<int>	<dbl>	<int>	<int>	<dbl>	<chr>	<chr>
1	18	8	307	130	3504	12	No	U
2	15	8	350	165	3693	12	No	U
3	18	8	318	150	3436	11	No	U
4	16	8	304	150	3433	12	No	U
5	17	8	302	140	3449	10	No	U

```
levels(as.factor(auto_mpg_all$Origin))
```

```
[1] "EU" "JP" "US"
```

```
boxplot(MPG ~ Origin, data=auto_mpg_all, ylab="MPG",  
        xlab="Origin", col='darkgray')
```



Regression with categorical variables

```
omodel <- lm(MPG ~ HP + Weight + After1975 + Origin,  
             data=auto_mpg_all)  
round(summary(omodel)$coefficients,3)
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	40.182	0.87	46.0	0.000
HP	-0.028	0.01	-2.8	0.005
Weight	-0.005	0.00	-10.8	0.000
After1975Yes	4.334	0.39	11.0	0.000
OriginJP	1.001	0.61	1.6	0.103
OriginUS	-1.593	0.56	-2.8	0.005

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OriginJP	1.001	0.61	1.6	0.103
OriginUS	-1.593	0.56	-2.8	0.005

For Origin, R has chosen EU as the reference level and create dummy variables for both JP and US.

A warning about categorical variables with numeric representations

In the original dataset, the origin was represented as 1 for U.S., 2 for EU and 3 for JP.

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because then regression would treat this as if it were a quantitative variable! Even

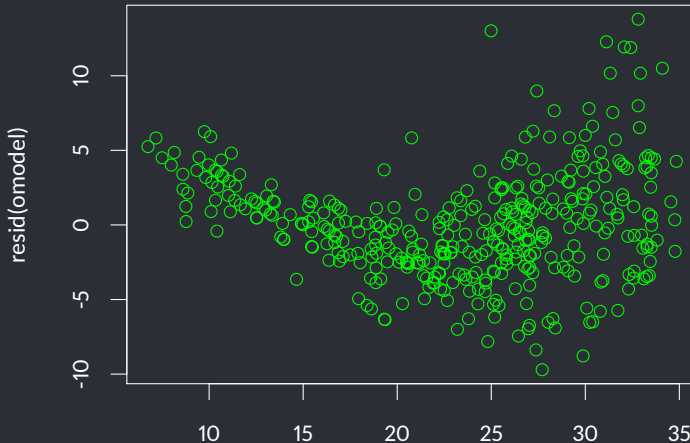
though the representation in the file is numeric, it is still a categorical variable and should be treated as such.

Assumptions

What are the issues with this model?



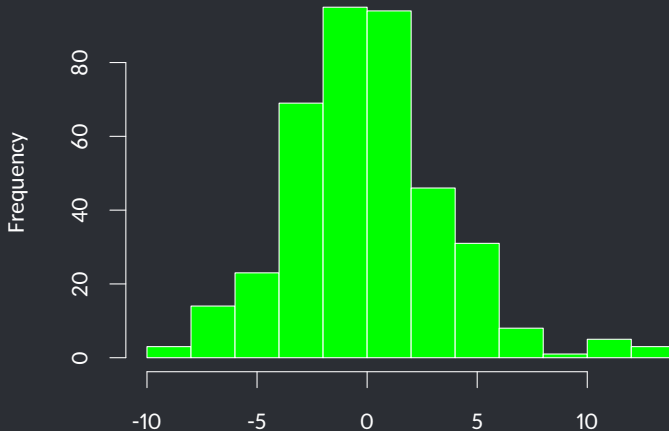
```
plot(predict.lm(omodel), resid(omodel), col='green', main='')
```



Assumptions

What about normality?

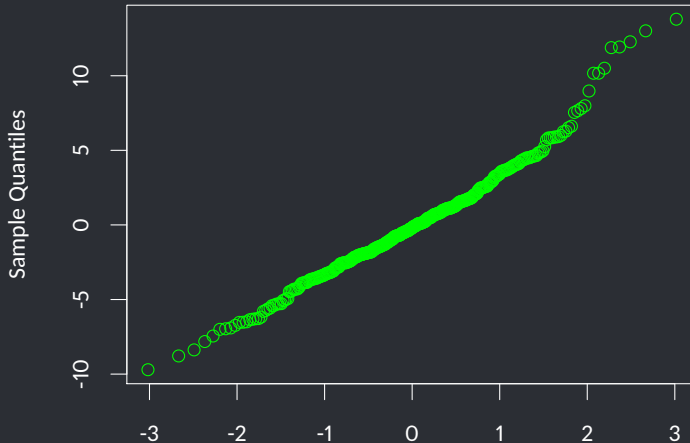
```
hist(resid(omodel), col='green', main='')
```



Assumptions

What about normality?

```
qqnorm(resid(omodel), col='green', main='')
```



Statistical significance of a categorical variable

While dealing with categorical variables, we want to look at the significance of the categorical variable as a whole, rather than looking at p -values of individual dummy variables.

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While dealing with categorical variables, we want to look at the significance of the categorical variable as a whole, rather than looking at p -values of individual dummy variables.

We want to test the **compound null hypothesis**

$$H_0 : \beta_{US} = \beta_{EU} = 0.$$

Statistical significance of a categorical variable

To do this, we look at the ANOVA table; the p -value on the Origin line (2.4×10^{-5}) is the p -value for the compound null hypothesis $H_0 : \beta_{US} = \beta_{EU} = 0$.

```
anova(omodel)
```

Analysis of Variance Table

Response: MPG

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
HP	1	14433	14433	1096.0	< 2e-16	***
Weight	1	2392	2392	181.6	< 2e-16	***
After1975	1	1623	1623	123.2	< 2e-16	***
Origin	2	288	144	10.9	2.4e-05	***
Residuals	386	5083	13			

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Practical significance of a categorical variable

Since $p < .05$, we can conclude that Origin is a statistically significant predictor of MPG. But is it a *practically* significant predictor?

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To do this, compare R^2 values, or standard error of residuals:

Model	R^2	Residual standard error
Without Origin in model	0.77	3.72
With Origin in model	0.79	3.63

We have to decide if the increased precision is worth the extra complexity in the model.