THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 1

Lecture 2

STA 371G

Reading assignments

- Sign up at `perusall.com`

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- First assignment is due **January 31 at 11:55 PM**

Probability Theory

The Concept of Probability

What do each of these have in common?

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- S&P500 index at the and of January
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Yet, we can model them using probability theory and study the values they might take, associated probabilities etc.

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- Random variable $\longrightarrow$

$$Y : \begin{cases} 1, & \text{if outcome is odd number,} \\ 2, & \text{if outcome is even number} \end{cases}$$

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- X : Number of stocks on NYSE whose price change today (discrete)
- Y : Average price change of the stocks on NYSE (continuous)

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- X : S&P500 at the end of 2017, $P(X > 2270) = 0.85$
- Y : Lifetime of your MacBook, $P(Y > 15 \text{ years}) = 0.05$

Probability Distributions

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Continuous random variable $\longrightarrow$ Probability Density Function (p.d.f.)

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Discrete Random Variables

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$$f(x) = \begin{cases} \frac{1}{n} & x = 1, \ldots, n, \\ 0 & \text{otherwise.} \end{cases}$$

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- This is an example of a Discrete Uniform Distribution.

Probability Distributions

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Y : Lifetime of your MacBook (in years)

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What is $P(Y = 5) =?$ or $P(Y = 5.5) =?$ or $P(Y = 5.551234123) =?$

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They are all 0.

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Warning!

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$$P(5 < Y < 7) =$$

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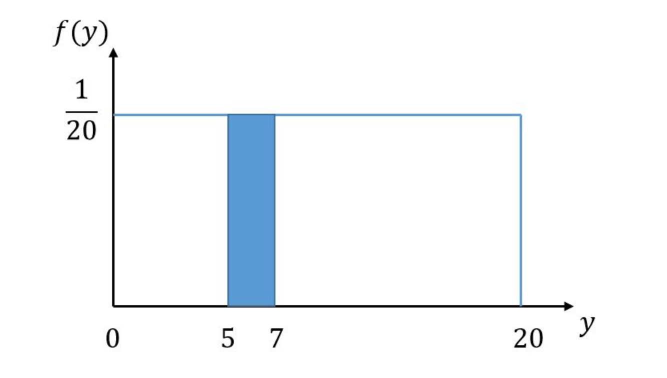
In general, $P(a \leq Y \leq b) = \int_a^b f(y)dy$.

Probability Distributions



Mean, Variance and Standard Deviation

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Mean or Expected Value of a random variable X is a measure of the center of its probability distribution. It is a weighted average of all possible values X can take, where the weights are the corresponding probabilities.

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Discrete random variable X

$$\mu_X = E[X] = \sum_x x f(x) = \sum_x x P(X = x)$$

Continuous random variable Y

$$\mu_Y = E[Y] = \int_y y f(y)\, dy$$

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$$\sigma_Y^2 = Var(Y) = E[(Y - \mu_Y)^2] = \int_y (y - \mu_y)^2 f(y) dy$$

Law of Large Numbers

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For a random variable X, the average of $X_1, X_2, \ldots, X_n$ gets very close to the expected value of X ($E[X]$) for large n.

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A die is rolled $n = 4$ times: $x_1 = 4$, $x_2 = 6$, $x_3 = 1$, $x_4 = 1$. The average is

$$\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{4 + 6 + 1 + 1}{4} = 3$$

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$$\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{4 + 6 + 1 + 1}{4} = 3$$

For large n, the average will be around 3.5; because $E[X] = 3.5$.

Law of Large Numbers

R Exercise

Go to R Studio...

```r
# Simulate rolling a die 1 time
sample(c(1, 2, 3, 4, 5, 6), 1, replace=T)
# Simulate rolling a die 4 times
sample(c(1, 2, 3, 4, 5, 6), 4, replace=T)
# Take the average
mean(sample(c(1, 2, 3, 4, 5, 6), 4, replace=T))
# Let's increase the number of dice
mean(sample(c(1, 2, 3, 4, 5, 6), 10, replace=T))
```

Normal Distribution a.k.a. the Bell Curve

A very common continuous probability distribution.

The mean and variance together uniquely define the distribution:

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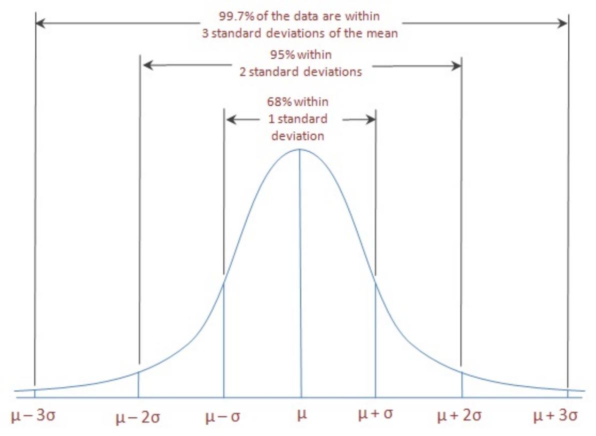

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```
# Simulating a zip code with n=3 houses
runif(3, min=100, max=400)
# Repeat this for k=5 zip codes.
house_prices <- replicate(5, runif(3, min=100, max=400))
# Find the average house price in each zip code
avg_house_prices <- colMeans(house_prices)
# Make a histogram of the average house price in your zip codes
hist(avg_house_prices)
# Increase n and k and try again!
```