



THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Simple Regression 1

Lecture 5

STA 371G



National Longitudinal Study of Adolescent to Adult Health

Nationally representative sample of US students in grades 7-12 were surveyed in the 1994-95 school year

(<http://www.cpc.unc.edu/projects/addhealth>)

Students were followed up on with subsequent in-home interviews four times (most recently 2008)

This is an **awesome** data set, with data on:

- family
- relationships
- health
- military service
- religion
- sex and STDs
- economics
- education
- personality
- criminality
- tobacco
- drugs
- alcohol
- pregnancy
- sleep
- daily activities

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Do people that start drinking younger tend to drink more (or less) when they become adults? We want to know:

- What is our best **prediction** alcohol consumption if we know at what age had their first drink?
- How good is that prediction?
- What is the **relationship** between alcohol consumption and age of first drink?

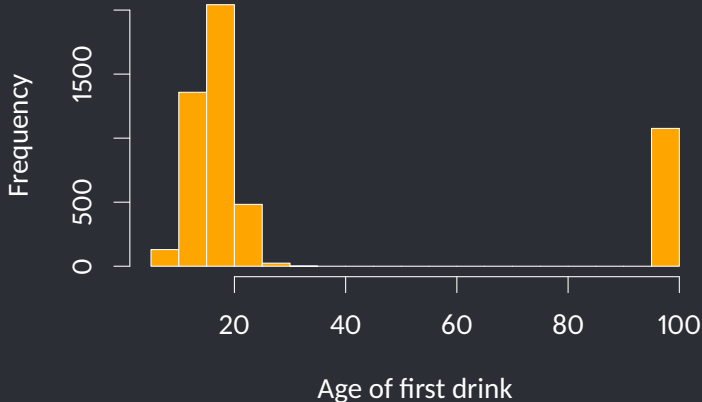
Age of first drink

Predictor variable

Number of drinks consumed as adult

Response variable

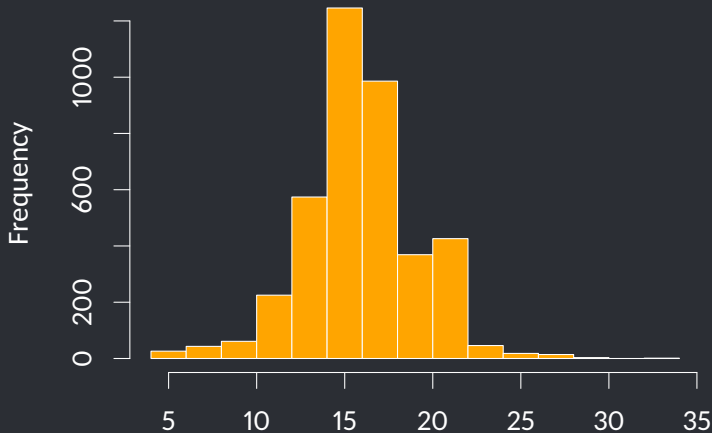

```
> hist(addhealth_public4$h4to34,  
+      main='', xlab='Age of first drink',  
+      col='orange')
```



Let's examine our variables

<i>If Q.33 = 1, ask Q.34, else skip to Q.63.</i>			
H4TO34		Num	34. How old were you when you first had an alcoholic drink? By drink, we mean a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink, not just sips or tastes from someone else's drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
56	0.4%	5	5 years
30	0.2%	6	6 years
21	0.1%	7	7 years
71	0.5%	8	8 years
52	0.3%	9	9 years
12014	76.5%	10-31	NOTE: Range of values omitted from display
1	0.0%	32	32 years
2	0.0%	33	33 years
21	0.1%	96	refused
3322	21.2%	97	legitimate skip
111	0.7%	98	don't know

```
> age <- addhealth_public4$h4to34  
> age[age >= 96] <- NA  
> hist(age, main='', xlab='', col='orange')
```

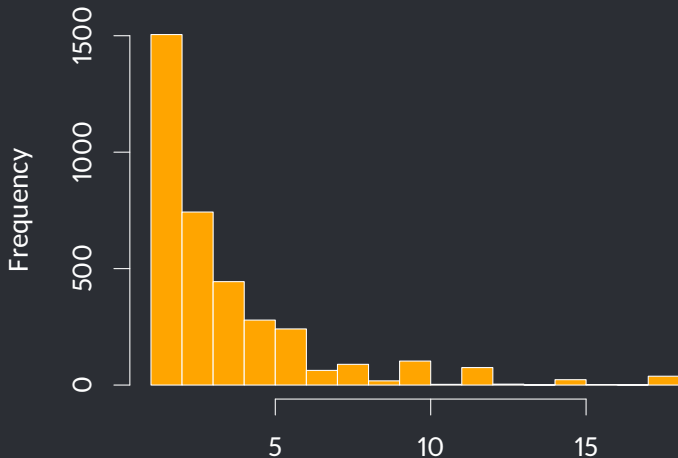


Let's examine our variables

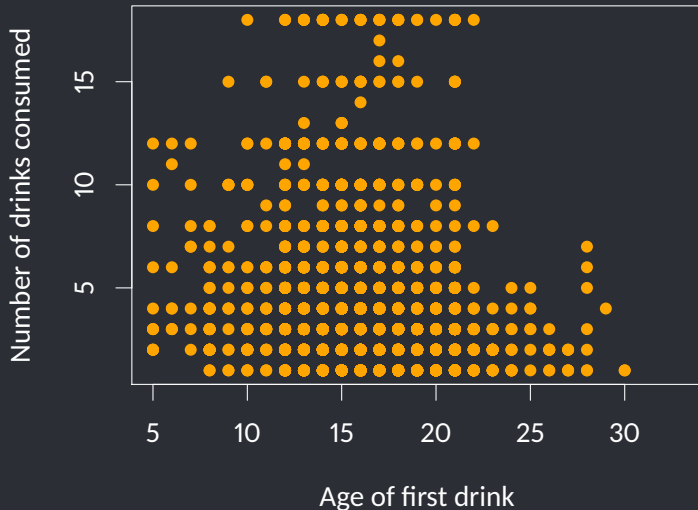
If Q.35 not equal 0, ask Q.36, else if Q.35 = 0, then skip to Q.43.

H4TO36		Num	36. Think of all the times you have had a drink during the past 12 months. How many drinks did you usually have each time? A 'drink' is a glass of wine, a can or bottle of beer, a wine cooler, a shot glass of liquor, or a mixed drink. NOTE: Smallest 5 and largest 5 values are displayed.
Frequency	Percent	Value	Label
1651	10.5%	1	1 drink
3051	19.4%	2	2 drinks
2274	14.5%	3	3 drinks
1343	8.6%	4	4 drinks
891	5.7%	5	5 drinks
1815	11.6%	6-16	NOTE: Range of values omitted from display
4	0.0%	17	17 drinks
108	0.7%	18	18 drinks
27	0.2%	96	refused
4427	28.2%	97	legitimate skip
110	0.7%	98	don't know

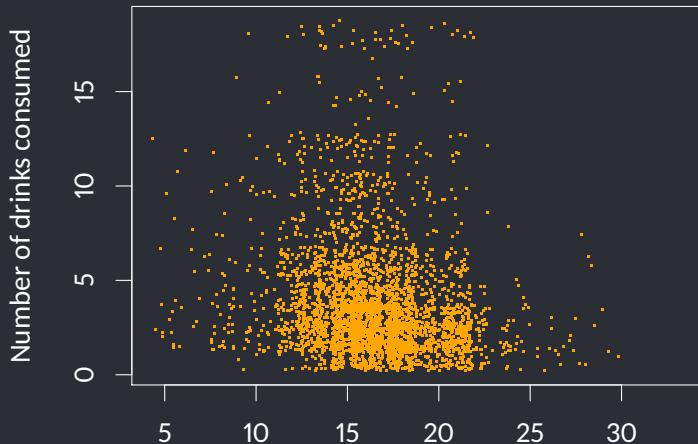
```
> num.drinks <- addhealth_public4$h4to36  
> num.drinks[num.drinks >= 96] <- NA  
> hist(num.drinks, main='', xlab='How many drinks',  
+       col='orange')
```



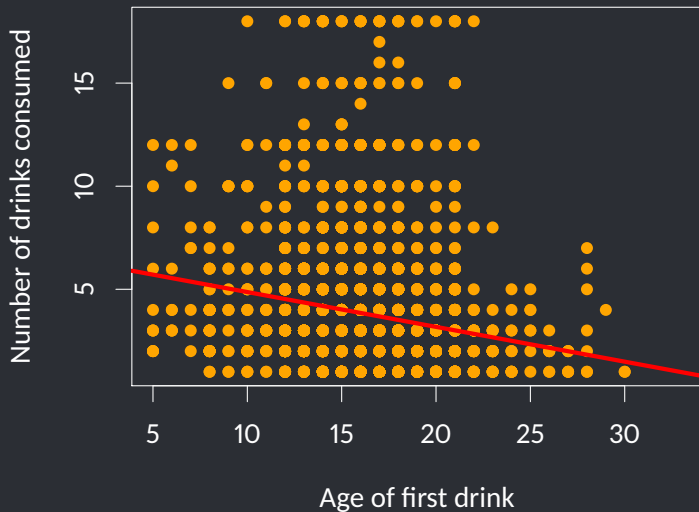
```
> plot(num.drinks ~ age, pch=16, col='orange',  
+      xlab='Age of first drink',  
+      ylab='Number of drinks consumed')
```



```
> plot(jitter(num.drinks, 4) ~ jitter(age, 4),  
+      pch=46, col='orange',  
+      xlab='Age of first drink',  
+      ylab='Number of drinks consumed')
```



The regression line is the line of “best fit” through this plot:



What is linear regression doing?

We model each case (x_i = age for i th person, y_i = number of drinks for i th person) as a linear relationship plus some error:

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$

β_0 and β_1 are the intercept and slope, respectively.

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β_0 and β_1 are the intercept and slope, respectively.

We find estimates for β_0 and β_1 in our sample that *minimize* the errors:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

This is the regression (best fit) line.

```
> model <- lm(num.drinks ~ age)
> summary(model)
```

Call:

```
lm(formula = num.drinks ~ age)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-4.2035	-1.8528	-0.8528	0.8095	15.1602

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	6.55417	0.26532	24.70	<2e-16	***
age	-0.16883	0.01588	-10.63	<2e-16	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.963 on 3600 degrees of freedom
(2902 observations deleted due to missingness)

Multiple R-squared: 0.03044, Adjusted R-squared: 0.03017

F-statistic: 113 on 1 and 3600 DF, p-value: < 2.2e-16

This translates to a regression line of:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot \text{age}$$



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Predict number of drinks for age = 21:

$$\widehat{\text{num drinks}} = 6.55 - 0.17 \cdot 21 = 3.01$$

Or we can use R to do the work for us:

```
> predict.lm(model, list(age=21))
```



How good are our predictions?

R^2 quantifies how closely the model fits the data.

- $R^2 = \text{cor}(X, Y)^2$, i.e., the squared correlation between X and Y .

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- $R^2 = \text{cor}(Y, \hat{Y})^2$, i.e., the squared correlation between the actual and predicted values of Y .



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Call:

lm(formula = num.drinks ~ age)

Residuals:

Min	1Q	Median	3Q	Max
-4.204	-1.853	-0.853	0.810	15.160

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	6.5542	0.2653	24.7	<2e-16 ***
age	-0.1688	0.0159	-10.6	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3 on 3600 degrees of freedom
(2902 observations deleted due to missingness)

Multiple R-squared: 0.0304, Adjusted R-squared: 0.0302

F-statistic: 113 on 1 and 3600 DF, p-value: <2e-16

In our regression, $R^2 = 0.03$, so $r = \sqrt{0.03} = 0.17$.

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- **Statistical significance:** Can we reject the null hypothesis that the correlation between X and Y in the *population* is not zero?

In our regression, $R^2 = 0.03$, so $r = \sqrt{0.03} = 0.17$.

Is this “significant?”

- **Statistical significance:** Can we reject the null hypothesis that the correlation between X and Y in the *population* is not zero?
- **Practical significance:** Is the correlation in our sample large enough to be meaningful?

The overall null hypothesis for a regression model

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The following are equivalent ways to express the overall null hypothesis:

- $R^2 = 0$ (in the population)
- $\text{cor}(X, Y) = 0$ (in the population)
- $\beta_1 = 0$
- The model has no predictive power
- Predictions from this model are no better than predicting \bar{Y} for every case

Two ways to test the overall null hypothesis

- The F -test (tests $H_0 : R^2 = 0$ in the population)
- The t -test for the *slope* (β_1) coefficient (tests $H_0 : \beta_1 = 0$)

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- The t -test for the *slope* (β_1) coefficient (tests $H_0 : \beta_1 = 0$)

Both of these methods are equivalent; the p -values will be exactly the same!



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- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.

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- There is a **statistically significant** relationship between the age someone starts drinking and how much they drink as an adult.
- Or: People that start drinking earlier in life consume **significantly more** alcohol when they drink as adults.
- Each additional year you wait to start drinking is associated with consuming 0.17 fewer drinks as an adult.
- Is this relationship **practically significant**?

Put a confidence interval on it

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- We can use a confidence interval to give a range of plausible values for what this effect size is in the population

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$$\text{estimate} \pm (\text{critical value})(\text{standard error}).$$

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Recall that the critical value for a 95% confidence interval is the cutoff value that cuts off 95% of the area in the middle of the distribution; the sampling distribution of $\hat{\beta}_1$ is a t -distribution.

```
> n <- nobs(model)
> qt(0.975, n-2)

[1] 1.960623
```

Put a confidence interval on it

R will also calculate confidence intervals for us:

```
> confint(model)
```

	2.5 %	97.5 %
(Intercept)	6.0339847	7.0743549
age	-0.1999713	-0.1376959

Put a confidence interval on it

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```
> confint(model)
```

	2.5 %	97.5 %
(Intercept)	6.0339847	7.0743549
age	-0.1999713	-0.1376959

In other words, we are 95% confident that the effect of each additional year's delay in starting to drink is between 0.14 and 0.2.

Put a confidence interval on it, part 2

We can also put a confidence interval on a prediction!

Two kinds of intervals:

Confidence	Predicting the mean value of Y for a particular X .	Among all people that start drinking at age 21, how many drinks do have on average as adults?
Prediction	Predicting Y for a single new case.	If Bob started drinking at age 21, how many drinks do we think will have as an adult?


```
> predict.lm(model, list(age=21),  
+   interval='confidence')
```

	fit	lwr	upr
1	3.008664	2.83616	3.181167

```
> predict.lm(model, list(age=21),  
+   interval='prediction')
```

	fit	lwr	upr
1	3.008664	-2.802894	8.820221

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Why is the prediction interval wider?