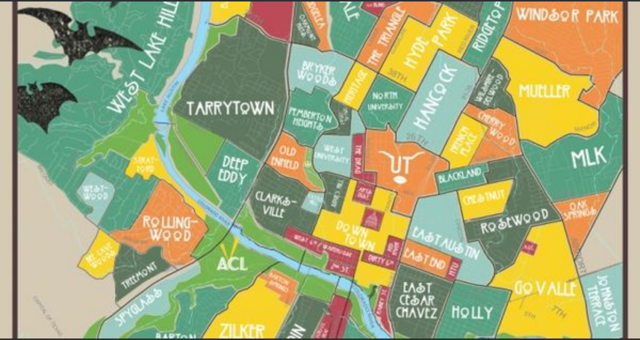
THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 2

Lecture 3

STA 371G

How would you figure out the average house price in Austin?



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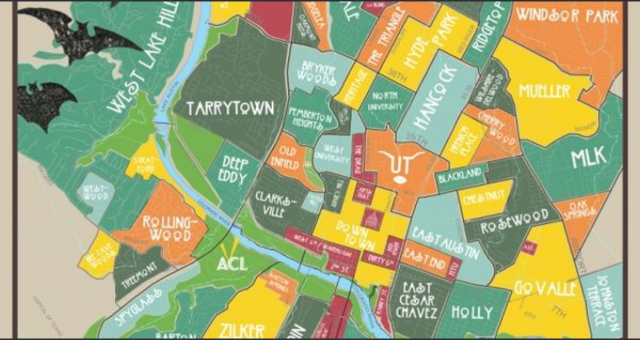

Look up each house price?

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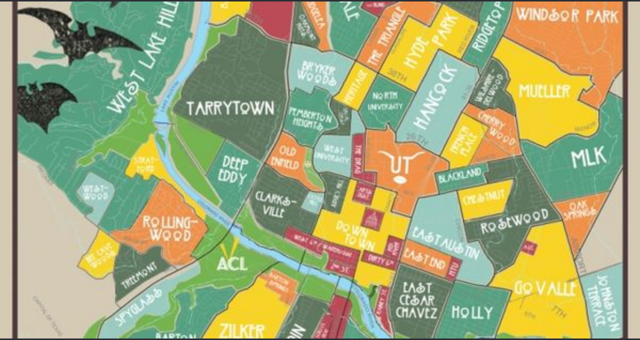

Look up each house price?

There are 360,000 houses in Austin — is there a better way?

Sample vs Population

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Just like making polls to predict election results!

Sample vs Population

	Population	**Sample**
Members	all houses	houses you selected
Mean	population mean (μ)	sample mean ($\hat{\mu}$)
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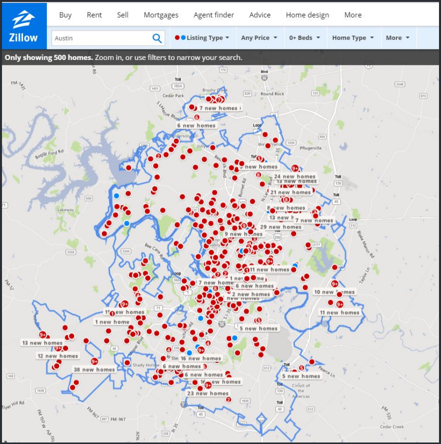
We will estimate a population parameter (population mean) based on a sample statistic (sample mean).

Collecting a sample

- Go to `zillow.com` and search for Austin, TX.
- Click “More Map.”
- Select 15 houses (try to get houses from all over town in a representative way), noting their prices in an R script.
- Do not discard any price, use the first 15 you find.



Collecting a sample

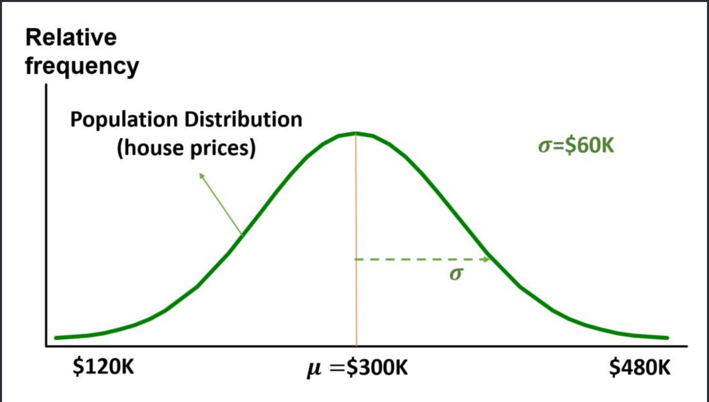
```r
# Create a vector of house prices (you should have 15 prices)
sample_house_prices <- c(327000, 276000, 513000)
# Calculate sample statistics
sample_mean <- mean(sample_house_prices)
sample_variance <- var(sample_house_prices)
sample_standard_deviation <- sd(sample_house_prices)
# Sample mean of first 5 houses
sample_mean_5 <- mean(sample_house_prices[1:5])
```

Sampling Distribution

Distribution of your answers $\longrightarrow$ Sampling distribution

Statistic	**Population**	**Sampling Distribution**
Mean	μ	μ
Standard Deviation	σ	$\sigma/\sqrt{n}$

Sampling Distribution



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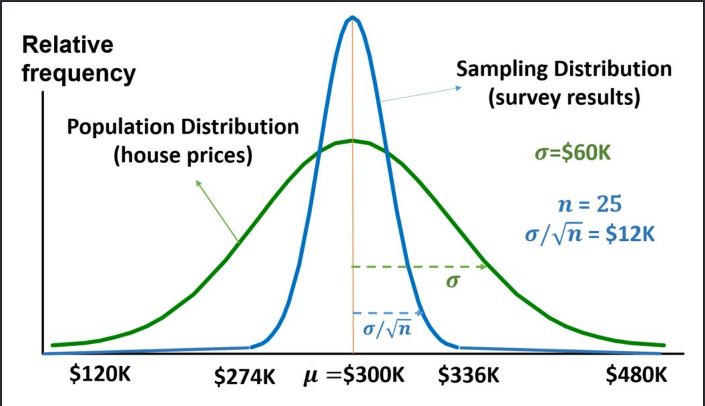

Sampling Distribution

Assume $\mu = \$300K$, $\sigma = \$60K$.

	n	$\sigma/\sqrt{n}$	± 3 SD range (99.7%)
Survey 1	25	\$12K	\$264K \$336K
Survey 2	100	\$6K	\$282K \$318K
Survey 3	3600	\$1K	\$297K ... \$303K

Sampling Distribution

Let’s compare sample mean of 5 houses vs 15 houses.

What do you expect to see?

t Distribution

We often do not know population variance and use sample variance instead.

In that case, the sample mean will have a t distribution.

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- Would you be more comfortable with your conclusion if you had 1000 houses in your survey?

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- Is it possible that, out of bad luck, you picked the cheapest houses?
- Would you be more comfortable with your conclusion if you had 1000 houses in your survey?
- When should you reject the hypothesis? When not?

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α is usually chosen as 0.05 prior to sampling.


"A ROLLICKING COMEDY."
-JOHN ANDERSON, VARIETY
WRITTEN, DIRECTED BY AND STARRING LAKE BELL
IN A WORLD...
SPEAK UP AND LET YOUR VOICE BE HEARD
WINNER
SUNDANCE
www.inaworldmovie.com
SONY
COMING SOON

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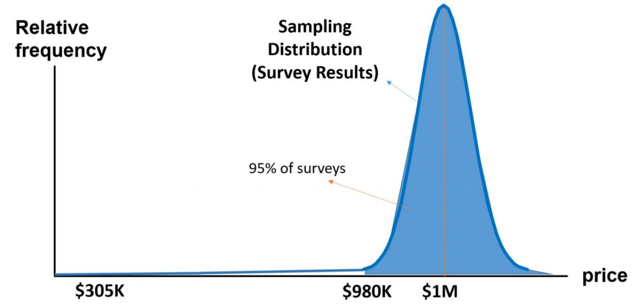

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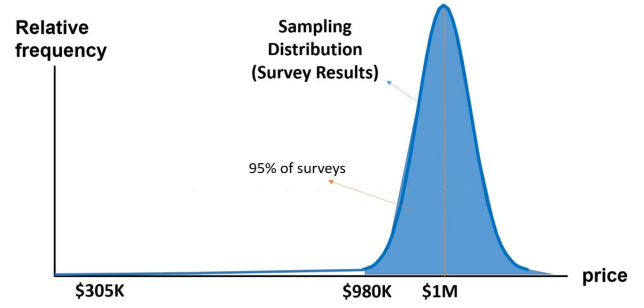

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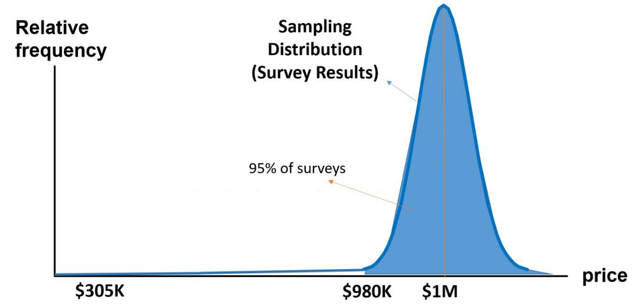

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This is p, the area to the left of $\$305K$. $p < 10^{-100}$!

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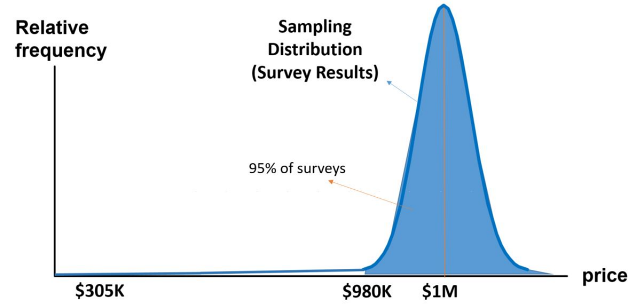

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Since $p < \alpha = 0.05$, reject the null hypothesis!

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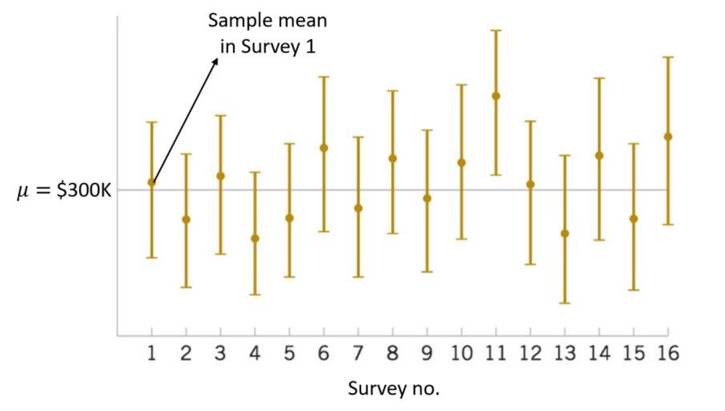
Confidence Intervals

Each sample will have a different 95% confidence interval, and 95% of such intervals will contain the population mean:



Confidence Interval

```
# Calculate 95% confidence interval (default)
avg_price_ci_95 <- t.test(sample_house_prices, conf.level=0.95)
# Calculate 99% confidence interval
avg_price_ci_99 <- t.test(sample_house_prices, conf.level=0.99)
```