THE UNIVERSITY OF TEXAS AT AUSTIN
McCOMBS SCHOOL OF BUSINESS

Probability Review 2

Lecture 3

STA 371G

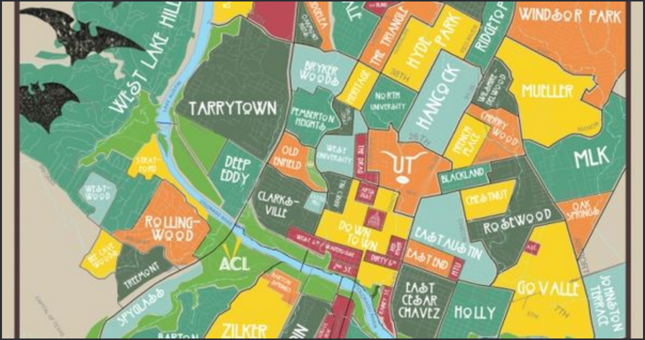
Sample vs Population

Find out the average house price in Austin.

Sample vs Population

Find out the average house price in Austin.

How would you do that?



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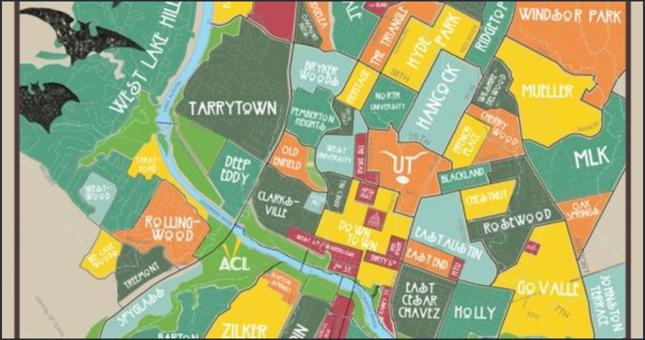

Look at each house price?

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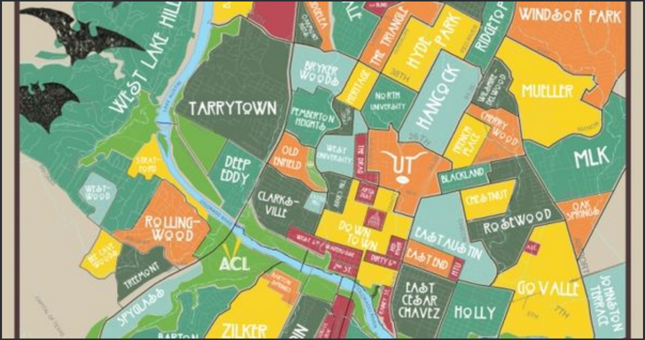

Look at each house price?

360,000 houses in Austin!

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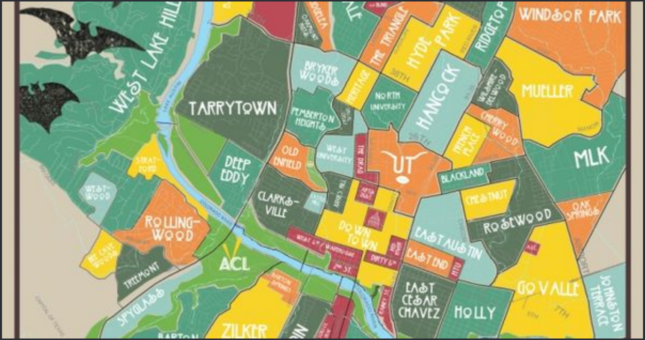

Look at each house price?

360,000 houses in Austin!

Can we do something smarter?

Sample vs Population

A smarter approach:

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- Pick n houses randomly (e.g. $n = 100$)

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Just like making polls to predict election results!

Sample vs Population

	Population	**Sample**
Members	all house prices	prices you picked
Average	population mean	sample mean
Variance	population variance	sample variance

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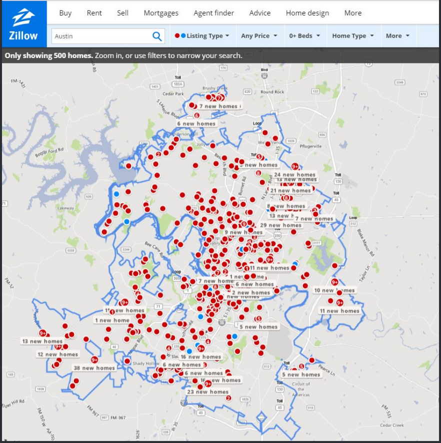
Estimating a population parameter (population mean) based on a sample statistic (sample mean).

Collecting a sample

Zillow.com, “Austin, TX.”

- Click “More Map”
- Select 15 houses, note their prices in an R script.
- Do not discard any price, use the first 15
- Try to represent different regions



Only showing 500 homes. Zoom in, or use filters to narrow your search.

Collecting a sample

Your R script should look like this

```r
# Create a vector of house prices (You should have 15 price data)
sample_house_prices <- c(327000,276000,513000)
# Calculate sample statistics
sample_mean <- mean(sample_house_prices)
sample_variance <- var(sample_house_prices)
sample_standard_deviation <- sd(sample_house_prices)
# Sample mean of first 5 houses
sample_mean_5 <- mean(sample_house_prices[1:5])
# Print them to console
cat("Sample Mean", sample_mean)
cat("Sample Variance", sample_variance)
cat("Sample Standard Deviation", sample_standard_deviation)
cat("Sample Mean of first 5 houses",sample_mean_5)
```

Sampling Distribution

On Learning Catalytics, enter your results.

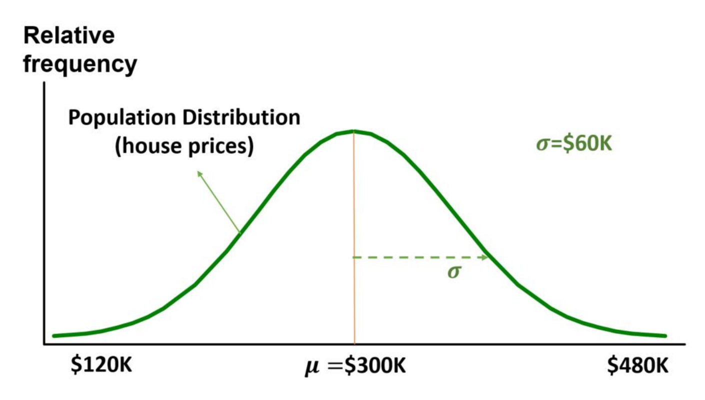
And here is what they look like…

Sampling Distribution

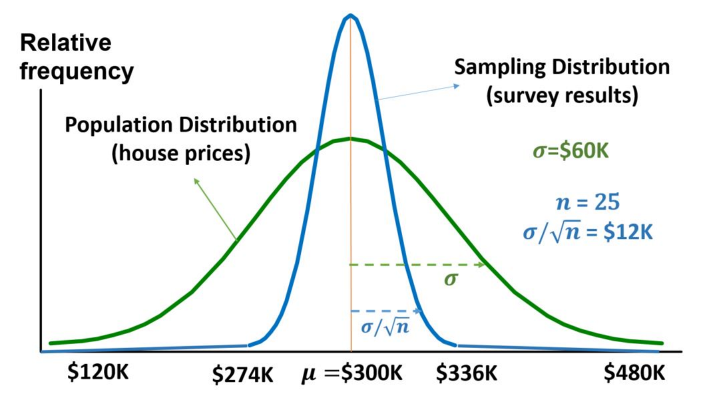
Distribution of your answers ⟶ Sampling distribution

Statistic	**Population**	**Sample Mean**
Mean	μ	μ
Standard Deviation	σ	$\sigma/\sqrt{n}$

Sampling Distribution



Sampling Distribution



Sampling Distribution

Assume $\mu =$\$300K, $\sigma =$\$60K.

	n	$\sigma/\sqrt{n}$	3 std. dev. range (99.7%)
Survey 1	25	\$12K	\$264K \$336K
Survey 2	100	\$6K	\$282K \$318K
Survey 3	3600	\$1K	\$297K ... \$303K

Sampling Distribution

Let’s compare sample mean of 5 houses vs 15 houses.

What do you expect to see?

t Distribution

We often do not know population variance and use sample variance instead.

In that case, the sample mean will have a *t* distribution.

Hypothesis Testing

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- Would you reject the hypothesis? Why?
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- Would you be more comfortable with your conclusion if you had 1000 houses in your survey?

Hypothesis Testing

Hypothesis: Average house price in Austin is $1M.
Your survey on 25 houses: Average house price is $*305K*.

Questions, questions...

- Would you reject the hypothesis? Why?
- Is it possible that, out of bad luck, you picked the cheapest houses?
- Would you be more comfortable with your conclusion if you had 1000 houses in your survey?
- When should you reject the hypothesis? When not?

P-Value

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$H_0 : \mu = \$1M$ (Null hypothesis)
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α is usually chosen as 0.05 prior to sampling.

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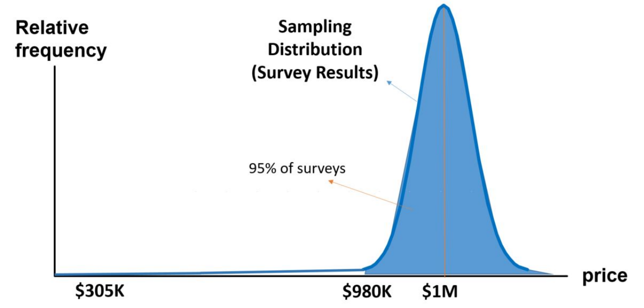

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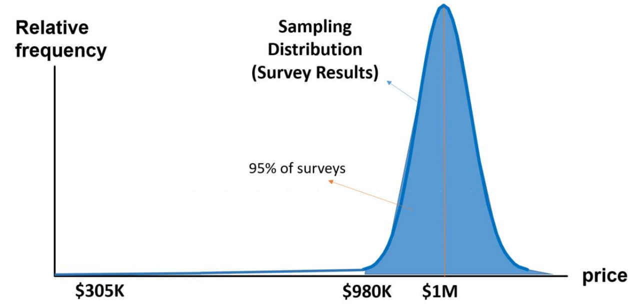

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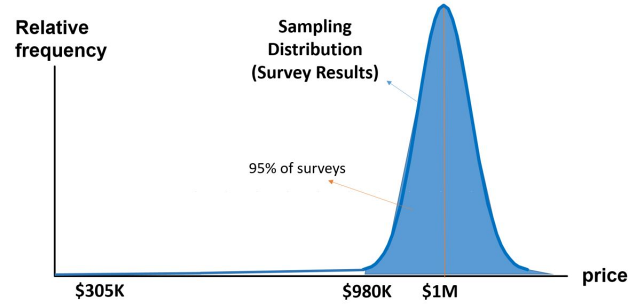

P-value is smaller than 10^{-100}, while $\alpha = 0.05$.
Rather than thinking you are cursed, you simply reject the hypothesis!

P-Value

Learning Catalytics...

P-Value

Learning Catalytics…

Your sample mean = $305K.

P-Value

Learning Catalytics...

Your sample mean = \$305K.

H_0: Average house price is \$390K.
Would you reject the hypothesis?
P-value = 0.01, α=0.05

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H_0: Average house price is \$320K.
Would you reject the hypothesis?
P-value = 0.34, α=0.01

P-Value

Learning Catalytics...

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H_0: Average house price is $320K.
Would you reject the hypothesis?
P-value = 0.34, α=0.01

Confidence Interval

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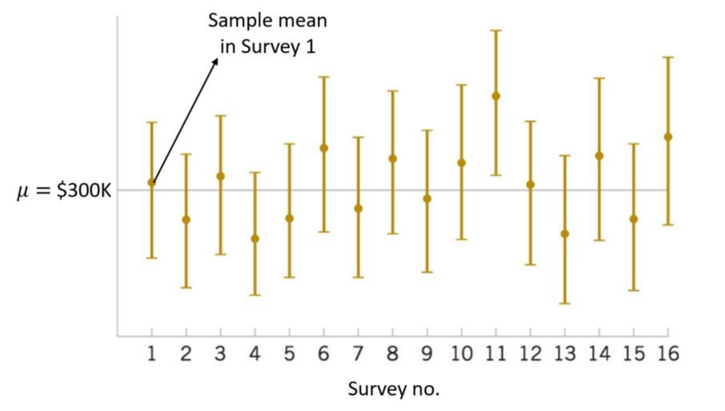

Confidence Interval

Add the following to your R script

```r
# Calculate 95% confidence interval (default)
avg_price_ci_95 <- t.test(sample_house_prices)
# Calculate 99% confidence interval
avg_price_ci_99 <- t.test(sample_house_prices, conf.level = 0.99)
# Display results
cat("95% confidence interval is:", avg_price_ci_95$conf.int)
cat("99% confidence interval is:", avg_price_ci_99$conf.int)
```

Enter your results on Learning Catalytics.