Lab 2: Minecrafting

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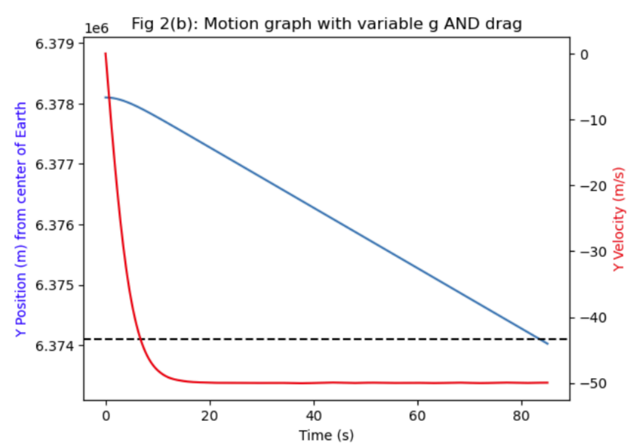
I. Introduction

We have a 4km long mine on our worksite, and I have done the calculations and experiments necessary to calculate the properties of the mineshaft and the times for which a 1kg test mass will fall to the bottom of the mineshaft. For these calculations, I will be using Python and its "solve_ivp" function. These experiments will show the ideal time that the mass reaches the bottom; we will also account for the time with a variable gravity and with drag. Afterwards, we will calculate the Coriolis forces, the forces due to the Earth's rotation, that are applied to the falling mass. We are also going to make calculations with a theoretical mineshaft that goes through all of Earth's diameter. In these calculations, we will have different density distributions of Earth, going from where the earth has the same density throughout to where the core is much denser than the surface.

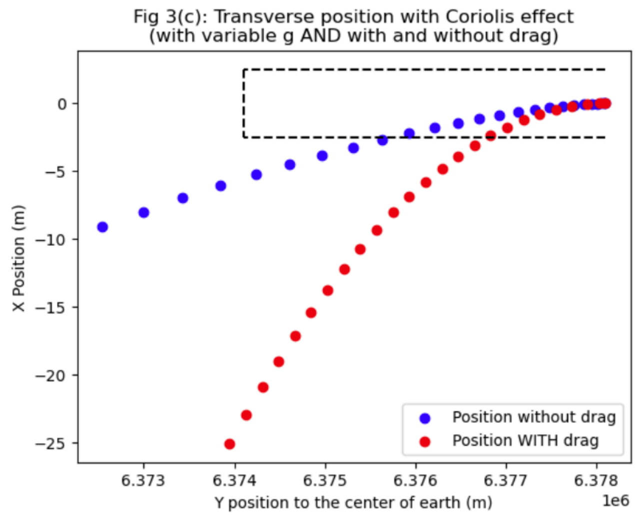
II. Calculating Fall Time

In this part, we will be figuring out how long it takes for the mass to reach the bottom of the mineshaft. In an ideal scenario where the gravity is a constant 9.81 m/s^2 and there is no drag, the fall time is 28.6 seconds. Let's take into account that gravity decreases the further into the Earth you go. We find that the time it takes for the mass to reach the bottom is also 28.6 seconds; this is because we are only dropping the mass to 4km into the earth. The Earth's radius is more than 6,000 km long, which means that the difference in gravity as we go into the Earth isn't going to be large enough to affect the mass's motion. However, if we include drag and claim that the terminal velocity of the mass is 50 m/s, we can see that the mass reaches the bottom at 83.5 seconds, which is a significant difference between the other two times. In Fig 2(b), we can see how the speed levels out at 50 m/s and the position graph reaches the bottom of the shaft after 80 seconds.



III. Feasibility of depth measurement approach (Coriolis Forces)

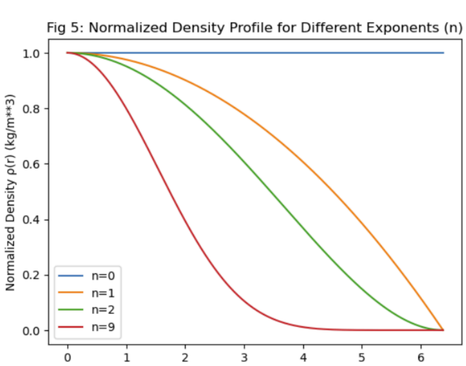
Since our mineshaft is on the equator, we have to take into account the rotational speed and coriolis effect of the Earth at the equator since that will affect the transverse motion of the test mass. As the mass drops, the Earth is rotating, meaning that if the mass is going straight down, but the Earth is rotating, the mass might end up hitting the side of the mineshaft. If we have a 5m wide mineshaft and we drop the test mass from the middle of the entrance, we measure that it hits the side of the mineshaft after 21.9 seconds and 2.4 km away from the surface. If we take into account drag, these values change since we get that it touches the side at 29.6 seconds at 1.3 km from the surface. In Fig 3(c), we can see how the graph with drag (red) touches the side of the mineshaft before the graph without drag (blue). Using this information, we can say that the minshaft is NOT big enough to properly measure the depth by dropping a test mass. We would need a bigger mineshaft if it was an ideal case without drag, and we would need a significantly bigger mineshaft when we account for drag. I would not recommend proceeding with this technique as the resources to expand the mineshaft would be too expensive and laborious. Instead, building a tube and making sure the mass slides freely down the tube would counteract the Coriolis force as the tube guides the mass perfectly down the shaft. To make this calculation more accurate, we could also account for the small friction force.



IV. Calculating crossing times for homogeneus and non-homogeneus Earth

For this section, we will imagine a shaft going from one pole to the other so that the Coriolis effect can be neglected. We will be finding the time it takes for a test mass to reach the middle of the earth for different density distributions. We will be using the equation shown to use different density distributions for n=0,1,2,9. Where n=0 is a constant density, and n=9 is where the center is very dense and the surface is not very dense. The density distribution

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n$$



can be seen in Fig 5, where we can see how the higher we go, the more the density changes between the surface and the center. In the graph where n=0, the time it reaches the center of the Earth is 1267.2 seconds, with a speed of 7906.0 m/s downward. When n=1, the time is 1096.9 seconds, and it passes it with a speed of 10435.3 m/s downwards. When n=2, the time is 1035.1 seconds with a speed of 12200.7 m/s downward. When n=9, the time is 943.9 seconds with a speed of 18392.0 m/s downward. This shows that the higher the n, the faster it reaches the center of the Earth, and it goes at a higher speed.

V. Discussion and Future Work

In these experiments, we have made many idealistic assumptions about the Earth and the mineshaft. We assumed that the air drag coefficient remains constant going down the shaft, but in reality, the temperature and air density changes can affect the drag. We also assumed that the terminal velocity was 50m/s when we could've had more accurate ways to solve for this. Another assumption we made is that the Earth is perfectly spherical, but in reality, it has an oval shape due to its rotation. We could've also used a more accurate density distribution of Earth since it isn't a nice shift between densities, and it's more like layers that each have different densities. After considering these, for future work, we should make the experiment more realistic by having functions that show the transition between different layers/densities in the Earth, and also accounted and also accounted for the fact that the radius changes when you are on the equator versus when you are in one of the poles. Lastly, we could have been more accurate in finding the drag by having a more accurate terminal velocity and a variable drag coefficient. Overall, our assumptions allowed us to have reasonable estimates for measuring the depth and time for a test mass being dropped down a mineshaft; however, by applying the fixes mentioned above, we will have more accurate and realistic measurements.