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Evaluation of the longitudinal parameters of an overhead transmission line with non-homogeneous cross section

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ABSTRACT

This paper presents an accurate method to evaluate the longitudinal parameters of multi-conductor overhead power lines, taking in account skin and proximity effects. This method has the foremost advantage of considering a non-homogeneous ground, composed by separate layers with different conductivities and having surface irregularities, such as hills or valleys. The method splits the system conductors (line and ground) into smaller sub-conductors, with variable rectangular cross-section. The method is validated by comparing the longitudinal impedance results with those obtained using both analytical equations and other numerical methods proposed in the literature. Analytical results presented are concerned to a cylindrical two-wire line and one-phase overhead line. Using the proposed method results are obtained for one-phase overhead line, considering the ground with different conductivity layers and with different topographies.

1. Introduction

Overhead transmission line has fundamental role on power system networks, being necessary the accurate calculation of its characteristic parameters. In this paper an accurate method is proposed, in order to evaluate the transmission line longitudinal parameters, taking into account skin and proximity effects. This paper continues and complements the work presented in reference [1], where a similar methodology is used to obtain the capacitances of multi-conductor power cable.

The skin effect is related to the system conductors (line and ground) with finite conductivity. It depends on the frequency and affects the line longitudinal parameters [2,3]. Due to this phenomenon, for rapid time-varying currents the current density is no longer uniformly distributed inside the conductor and it tends to flow near the conductor's surface, as the frequency increase. The proximity effect affects both longitudinal and transversal transmission line parameters [1,4,5]. This effect occurs whenever the electromagnetic field of one conductor is changed by the presence of other conductors at its neighborhood.

In general, analytical methods are not accurate for cases with irregular conductor configurations or in the presence of inhomogeneous, anisotropic or non-linear media. For these cases, numerical methods must be applied, in particular, for instance, by using the method of moments [1] or the finite element method [6,7].

The method proposed in this paper, based on the method of moments, characterizes each conductor of the system by a set of smaller and rectangular cross-section sub-conductors, with infinite length along z (line longitudinal axis). Splitting up the conductors into sub-conductors demands a special precaution for high frequencies, due to skin effect. As the frequency increases the penetration depth decreases, obliging to use thinner sub-conductors. Making use of uniform cross-section sub-conductors leads to an amount of computer processing time increase and memory consumption. This problem is crucial concerning the splitting up of the line cylindrical conductors. To overcome this problem the developed method uses non-uniform cross-section sub-conductors with thinner sections near the conductor's surface. This methodology can be applied to more complex configuration systems than the traditional one, which usually assumes the line conductors with cylindrical configuration, parallel to a homogeneous and flat ground. The new method allows other geometric configuration conductors facing to a non-homogenous ground, having layers with different conductivity and/or with surface irregularities, due to the presence of hills, valleys, oil pipes, rivers, etc.

In order to validate the method the longitudinal impedance of the conductors system is calculated and the results are

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compared with the ones obtained not only using analytical treatments [2,8–10] but also using other numerical methods proposed in literature. The analytical treatments are adequate to evaluate the external and internal impedance of the cylindrical conductors and to calculate the internal impedance due to the ground (flat and homogeneous).

This paper is structured in five sections. Section 2 deals with the description of the method, being its validation presented in Section 3. In Section 4 the methodology is applied to study systems with non-homogeneous ground and finally, in Section 5 conclusions are presented.

2. Method formulation

The proposed method allows the accurate evaluation of the longitudinal parameters of an overhead line, taking into account skin and proximity effects. This methodology allows the characterization of complex configuration systems, such as an overhead transmission line facing a non-homogeneous ground, having layers with different conductivity and/or with surface irregularities, due to the presence of hills, valleys, oil pipes, rivers, etc. The model is based on the following assumptions:

(1) The cylindrical conductors and the earth are split into partitions consisting of sub conductors, with rectangular cross-section and infinite length along z (line longitudinal axis);
(2) The current density is uniform within each sub-conductor;
(3) The system operates on a steady-state sinusoidal regime.

The division into sub-conductors requires a special care on high frequencies due to skin effect. Increasing the frequency the penetration depth decreases and conductor partitions must be changed to progressively thinner sub-conductors. Definition of the adequate sub-conductors size must constitute the first step of the methodology. Making use of uniform cross-section sub-conductors, computer processing time and memory consumption is then strongly increased. This problem is crucial concerning the splitting up of the line cylindrical conductors. To overcome it, non-uniform cross-section sub-conductors were adopted, using thick sub-conductors in the conductor core and thin sub-conductors near the conductor's surface. This sub-conductor discretization is adequate because the current density is higher near the outer boundaries, when high frequencies are considered, due to the presence of the skin effect.

The following criterion is taken to define the appropriate sub-conductor partition for each cylindrical line conductors, where the Cartesian coordinates centered on the conductor axis are given by:

$$x_i = -r_0 \ \cos\left((i-1)\frac{\pi}{N_d}\right); \quad y_i = -r_0 \ \cos\left((i-1)\frac{\pi}{N_d}\right)$$
$$\text{being } i = 1, \ldots, N_d + 1 \tag{1}$$

where r_0 is the conductor radius and N_d is the number of elements along each coordinate axis. The sub-conductors which characterize the cylindrical conductor will be picked from the resulting grid, with $N_d \times N_d$ rectangles. Only the rectangles which centers are inside the conductor cross-section are selected for use. To better understand the described process, an example with $N_d = 6$ is illustrated in Fig. 1a. The rectangles with black centers are used, whereas the ones with gray centers are not considered.

To evaluate the appropriate number of sub-conductors, related to N_d^2, we must guarantee that the thickness of the sub-conductors adjacent to the conductor's surface is at most equal to tenth of the conductor electromagnetic penetration depth, δ. Taking into account the criterion defined in (1), previous condition means that

$$N_d \geq \frac{\pi}{\arccos(1-((\delta/r_0)/10))} \tag{2}$$

the penetration depth δ being

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{3}$$

where ω is the angular frequency ($\omega = 2\pi f$, being f the frequency), μ and σ are, respectively, the magnetic permeability and the conductivity of the conductor.

Eq. (2) shows the influence of the frequency f on N_d which is obtained through the penetration depth δ (3). Eq. (2) shows also that an adequate criterion for the choice of N_d must depend on the radio δ/r_0. For instance, for a cylindrical conductor with 1.5 cm radius, 5.7×10^7 S/m conductivity and magnetic permeability μ_0, the values of N_d are 9, 19, 29 and 106, for $f = 50$ Hz, 1 kHz, 5 kHz and 1 MHz, respectively.

Besides this, number N_d must be big enough in order that the approximated section of the discretized conductor is comparable to the real in order to guarantee that the obtained section of the discretized conductor can be approximated to the real conductor.

To explain the proposed method a simple case is taken into consideration. Assume an infinite length cylindrical conductor, characterized by N sub-conductors and immersed in a homogenous dielectric environment, with magnetic permeability μ_0. The method formulation begins with the application of the Faraday law to a closed path $\boldsymbol{s}$,

$$\oint_{\boldsymbol{s}} \boldsymbol{E} \cdot d\boldsymbol{s} = -\frac{d\psi_{\boldsymbol{s}}}{dt} \tag{4}$$

where $\boldsymbol{E}$ is the electrical field vector and $\psi_{\boldsymbol{s}}$ is the flux linkage along the path $\boldsymbol{s}$, in other words, the flux which crosses the section $S_{\boldsymbol{s}}$ delimited by the path $\boldsymbol{s}$, Fig. 1b. This path, defined between the longitudinal coordinates z and $z + \Delta z$, passes inside the sub-conductors i and N (reference sub-conductor). Applying the Faraday law and taken into account the steady-state regime, we get:

$$\Delta z E_i + U(z+\Delta z) - \Delta z E_N - U(z) = -j\omega\psi_{\boldsymbol{s}} \tag{5}$$

where $U(z)$ and $U(z+\Delta z)$ are the voltage between i and N sub-conductors, at coordinates z and $z+\Delta z$, respectively. Considering a first order approach,

$$U(z+\Delta z) = U(z) + \frac{dU}{dz}\Delta z \tag{6}$$

and replacing Eq. (6) into Eq. (5), we obtain:

$$(E_i - E_N)\Delta z + \frac{dU}{dz}\Delta z = -j\omega\psi_{\boldsymbol{s}} \tag{7}$$

Knowing that the potential vector, $\boldsymbol{A}$, is related to the magnetic induction field, $\boldsymbol{B}$, through the relation, $\boldsymbol{B} = rot\,\boldsymbol{A}$, then, by application of Stokes' theorem to the path $\boldsymbol{s}$, we obtain:

$$\psi_{\boldsymbol{s}} = \int_{S_{\boldsymbol{s}}} \boldsymbol{B} \cdot \boldsymbol{n}_{\boldsymbol{s}} dS = \oint_{\boldsymbol{s}} \boldsymbol{A} \cdot d\boldsymbol{s} \tag{8}$$

which results,

$$A_i \Delta z - A_N \Delta z = \psi_{\boldsymbol{s}} \tag{9}$$

Replacing Eq. (9) into Eq. (7), and characterizing the voltage drop per unit length along the axial direction by $dU/dz = -\eta$, we obtain,

$$-j\omega(A_i - A_N) = E_i - E_N - \eta \tag{10}$$

As the electric field is related to the current density, $\boldsymbol{J}$, through the equation $\boldsymbol{J} = \sigma\,\boldsymbol{E}$, σ being the conductivity of the conductor, Eq. (10) can be written in terms of the current density related to each

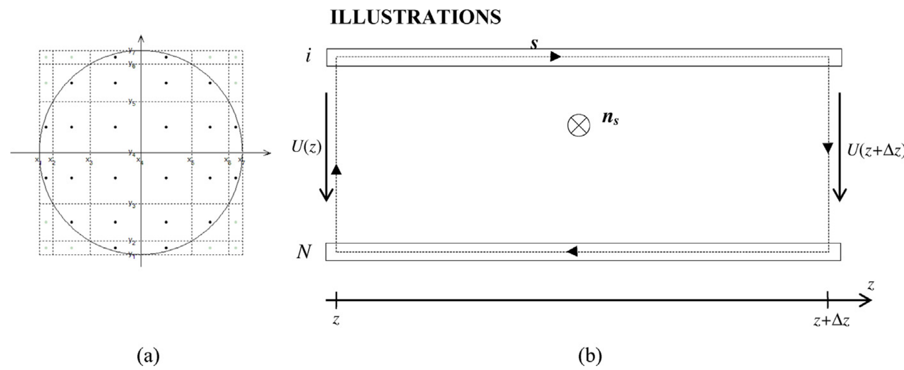

Fig. 1. Rectangular grid used to select the sub-conductors which characterize the cylindrical conductor, (a) and illustration of the path used to apply the induction law to sub-conductors i and N (reference sub-conductor) (b).

sub-conductor $i = 1, \ldots, N$. The system of equations to solve is given by Eq. (10) applied to all system sub-conductors ($i = 1, \ldots, N-1$) and adding one more equation:

$$I = \sum_{i=1}^{N} J_i S_i \tag{11}$$

Eq. (11) considers that the total current I is obtained adding the currents related to each sub-conductor that characterize the cylindrical conductor. Each sub-conductor current is equal to the product of its current density by the area S_i of its transversal section. Solving this system of equations we obtain the current densities related to all sub-conductors of the system.

The vector potential of a filament with infinite length is, according to [11], given by:

$$A = \frac{\mu_0}{2\pi} \ln\left(\frac{1}{r}\right) J_z \Delta S \tag{12}$$

where J_z is the current density and ΔS is the cross section of the filament. The potential vector at any point in space, due to one conductor, is calculated, according to [11], from equation,

$$A = \frac{\mu_0}{2\pi} J_z \iint \ln\left(\frac{1}{r}\right) dS \tag{13}$$

where $r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$ represents the distance between the conductor center, with (x_0, y_0) coordinates, and a generic point (x, y).

For the sub-conductor i (with $i = 1, \ldots, N-1$) Eq. (13) can be written as:

$$A_i = \sum_{k=1}^{N} L_{ik} J_k + A_0 \tag{14}$$

where A_0 is an arbitrary constant that depends on the potential reference, and coefficients L_{ik} are the solution of

$$L_{ik} = \frac{\mu_0}{2\pi} \iint_{S_k} \ln \frac{1}{r_{ik}} dS \tag{15}$$

r_{ik} being the distance between the center of sub-conductor i to the points of the domain S_k of sub-conductor k. Note that coefficients L_{ik} do not mean neither a self-inductance nor mutual inductances but they represent the coefficients of the vector potential development with the form of (14) with the physical dimension of (inductance × displacement). Integrals in (15) are solved by analytical developments used for conductors with rectangular cross sections, and must be calculated for each sub-conductor of the system. Alternate expressions have been adopted, for instance, in [12–14], where analytical solutions for rectangular conductors are presented for finite length conductors, and, also in [15] where alternate expressions of L_{ik} are applied to square infinite sub-conductors.

Consequently, the system equations to solve the current density of each sub-conductor J_k can be formulated using a matrix form,

$$\mathbf{MJ} = \mathbf{H} \tag{16}$$

the elements of the system matrix $\mathbf{M}$ being with dimension $N \times N$, defined by:

$$\begin{aligned} &M_{ii} = j\omega(L_{ii} - L_{Ni}) + \frac{1}{\sigma_i};\ M_{iN} = j\omega(L_{iN} - L_{NN}) - \frac{1}{\sigma_N} \text{ with } i = 1, \ldots, N-1 \\ &M_{ik} = j\omega(L_{ik} - L_{Nk}) \text{ with } i \neq k,\ k = 1, \ldots, N-1;\ M_{Nk} = S_k \text{ with } k = 1, \ldots, N \end{aligned} \tag{17}$$

The voltage drop per unit length along the axial direction, $-\eta$ in Eq. (10), is equal for all sub-conductors of the system and equal to zero, since the electric transversal field does not exist and all sub-conductors are touching each other. We assume that the total current I is equal to one, which is not a limitation of the method due to the system linearity. Thus, the array $\mathbf{H}$ has the first $N-1$ elements equal to zero ($\eta_i = 0$, $i = 1, \ldots, N-1$) and the last element equal to one. For the case of an isolated conductor, the internal longitudinal impedance is evaluated dividing the surface electrical field (related to a sub-conductor located at the conductor surface) by the conductor current.

For a complex system, with N_c conductors, the system equations to solve can be formulated using (16), nevertheless some considerations must be taken. The first one is the meaning of N which, for a multi-conductor system, corresponds to the total number of sub-conductors. The second one is related to the total current of the system which, in this case, must be equal to zero. The last consideration is related to the voltage drop, $dU/dz = -\eta$, that must be a vector, which comes out from the transmission line propagation equation, $\boldsymbol{\eta} = \mathbf{Z}\,\mathbf{I}$, where $\boldsymbol{\eta}$ represents the voltage drop per unit length along the longitudinal direction of each conductor defined in relation with the reference conductor, $\mathbf{I}$ represents the conductor current vector and $\mathbf{Z}$ the longitudinal impedance matrix.

The values of $\boldsymbol{\eta}$, will be equal for all the sub-conductors related to the same conductor and will be zero for all the sub conductors related to the reference. In this way, a reduced impedance matrix as referred in [15], is obtained. The self-impedance values are the diagonal elements of $\mathbf{Z}$ (the real part representing the self-resistances and the imaginary part divided by the angular frequency representing the self-inductances), and the mutual-impedance values are the off-diagonal elements of $\mathbf{Z}$ (the real part representing the mutual-resistances and the imaginary part divided by the angular frequency representing the mutual-inductances).

Considering the system linearity we consider the values of $\boldsymbol{\eta}$ equal to one if the conductor is energized and equal to zero if it is not. To obtain the longitudinal parameters of the multi-conductor system we must solve the new system of Eq. (16), considering that

Table 1
Longitudinal parameters of a two-wire line, with conductors distanced by 4 cm and 30 cm, for three different frequencies (50, 1000 and 5000 Hz). Comparison among analytical and new method results.

D [cm]	Frequency [Hz]	Analytical formula		Proposed method		Error	
		Resistance [mΩ/m]	Inductance [μH/m]	Resistance [mΩ/m]	Inductance [μH/m]	Resistance [%]	Inductance [%]
4	50	0.0556	0.4122	0.0630	0.4694	−13.3	−13.9
	1000	0.1897	0.3461	0.2640	0.3600	−39.2	−4.0
	5000	0.4076	0.3307	0.5922	0.3368	−45.3	−1.8
30	50	0.0556	1.2913	0.0557	1.2918	−0.2	−0.04
	1000	0.1897	1.2253	0.1904	1.2252	−0.4	0.01
	5000	0.4076	1.2098	0.4090	1.2097	−0.3	0.01

only one conductor is energized each time. Then, the current in each conductor, I_m (with $m = 1, \ldots, N_c$), must be calculated as the weighed sum of its sub-conductor current densities, $I_m = \sum_k S_k J_k$, where subscript k characterizes the sub-conductors of conductor m. The last step is to calculate the self and mutual admittance coefficients, related to the energized conductor j:

$$Y_{mj} = \left(\frac{I_m}{\eta_j}\right)_{\eta_n=0, n\neq j} \tag{18}$$

The longitudinal impedance matrix $\mathbf{Z}$ is obtained by inversion of matrix $\mathbf{Y}$ ($\mathbf{Z} = \mathbf{Y}^{-1}$).

3. Method validation

The proposed method is first validated, in Sections 3.1 and 3.2, evaluating the longitudinal impedance obtained by simulation with the results obtained using analytical methods proposed in the literature. These equations are adequate to evaluate both external and internal impedance of cylindrical conductor as well as flat and homogeneous ground conductor. Internal impedance is frequency dependent and is associated to the skin effect phenomenon. The validation is performed using two different conductors systems: two-wire line and one-phase transmission line. In the first one two isolated cylindrical conductors, immersed in air, are considered. In the second application one cylindrical conductor located above a flat and homogeneous ground is used. The internal impedance, Z_{int}, of a cylindrical conductor, with radius r_0 and conductivity σ, is given by [2],

$$Z_{int} = R_{DC} k r_0 \frac{J_0(kr_0)}{2J_1(kr_0)};\ R_{DC} = \frac{1}{\sigma\pi r_0^2};\ k = \sqrt{-j\omega\mu_0\sigma} \tag{19}$$

where $J_0(kr_0)$ and $J_1(kr_0)$ are the Bessel functions of the first kind of order 0 and 1, respectively, with argument (kr_0), and R_{DC} is the DC resistance, per-unit-length. Assuming the two-wire line having conductors with the same radius and distanced each other by D, the longitudinal impedance of the system is given by:

$$Z = 2Z_{int} + j\omega L_e \tag{20}$$

being Z_{int} evaluated using Eq. (19) and the external inductance L_e is calculated using [2],

$$L_e = \frac{\mu_0}{\pi} \ln\left(\frac{D}{2r_0} + \sqrt{\left(\frac{D}{2r_0}\right)^2 - 1}\right) \tag{21}$$

For the one-phase transmission line, assuming the height of the cylindrical conductor above ground equal to h, and the ground conductivity equal to σ_g, the longitudinal impedance is,

$$Z = Z_{int} + Z_D \tag{22}$$

where Z_{int} is the internal impedance of the cylindrical conductor and Z_D contain the external reactance and the internal impedance related to ground skin effect. Impedance Z_D is calculated using Dubanton empirical formula [8–10], which is based on the method of images, applied to a ground fictitious surface located at a complex distance d below the real ground surface, which depends on the field penetration depth, δ, in earth:

$$Z_D = j\omega\frac{\mu_0}{2\pi} \ln\left(\frac{2h+2d}{r_0}\right);\ d = \frac{\delta}{2}(1-j) \tag{23}$$

3.1. Two-wire line system

The two-wire system used here assumes the two cylindrical conductors with equal radius, $r_0 = 1.5$ cm, and conductivity $\sigma = 5.7 \times 10^7$ S/m. In the studies performed the current is assumed to be equal to 1 kA and two different values are used to characterize the distance between conductor centers: $D = 4$ cm and $D = 30$ cm. The longitudinal impedance of the system is obtained for three different frequency values: 50 Hz, 1 kHz and 5 kHz.

Table 1 presents the resistance and inductance values obtained using both analytical expression (20) and proposed method for $D = 4$ cm and $D = 30$ cm. The relative errors among analytical and simulation results, using the analytical ones as reference, are also presented. The results presented show increasing errors with increasing proximity between line conductors. The errors are less than 0.4% for $D = 30$ cm, however for $D = 4$ cm the resistance error is greater than 45%, at 5 kHz, and the inductance error is greater than 13% at 50 Hz. The errors obtained for $D = 4$ cm were expected since the analytical equation used to characterize the internal impedance (19) was developed for an isolated cylindrical conductor. The inductance error reduces with increasing frequency because the internal inductance tends to be negligible compared to the external inductance value. Fig. 2 illustrates the current density distribution on each conductor for $D = 4$ cm and $f = 50$ Hz. The proximity effect is visible since the figures show a higher current density in the region where the two conductors are closer.

3.2. One-phase transmission line

The one-phase line used here has a cylindrical conductor with radius $r_0 = 1.5$ cm, conductivity $\sigma = 5.7 \times 10^7$ S/m and placed at $h = 20$ m above ground. Two different values are used to characterize ground conductivity: $\sigma_g = 0.01$ S/m and $\sigma_g = 0.001$ S/m. The frequency values considered in numerical applications are again 50 Hz, 1 kHz and 5 kHz. It is important to emphasize that the method is not limited on frequency, as far as displacement currents can be neglected. In this way, the frequency upper limit may be extended to about 1 MHz, for the cases with the presence of the soil. However, there are computer restrictions related to the total number of elements used to discretize the conductors of the system (order of matrix $\mathbf{M}$ in (16)). Total number of elements must be limited to about 20,000 for the equipment used in the present work. This limit is almost reached by the cylindrical conductor itself, for the 1 MHz case, which needs about 18,000 sub-conductors. Alternate techniques may be adopted to overcome this problem. For instance, as done in [15], may be convenient to

Table 2
Longitudinal parameters of one-phase transmission line, with ground conductivity $\sigma_g = 0.01$ S/m and $\sigma_g = 0.001$ S/m, for three different frequencies (50, 1000 and 5000 Hz). Comparison among analytical and new method results.

σ [S/m]	Frequency [Hz]	Analytical formula		Proposed method		Error	
		Resistance [mΩ/m]	Inductance [μH/m]	Resistance [mΩ/m]	Inductance [μH/m]	Resistance [%]	Inductance [%]
0.01	50	0.0754	2.2754	0.0757	2.2615	−0.4	0.6
	1000	0.9421	1.9621	0.9285	1.9533	1.4	0.4
	5000	3.7818	1.8228	3.7798	1.8188	0.1	0.2
0.001	50	0.0766	2.5018	0.0762	2.4875	0.5	0.6
	1000	1.0338	2.1754	1.0178	2.1633	1.5	0.6
	5000	4.6270	2.0165	4.5507	2.0063	1.6	0.5

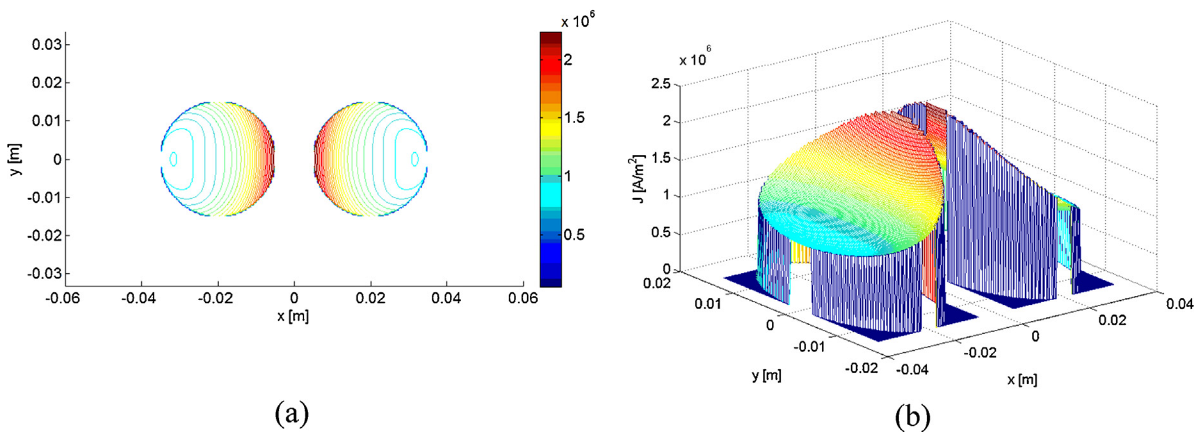

Fig. 2. Two-wire line with $D = 4$ cm, 50 Hz frequency and current equal to 1 kA: (a) current density contours and (b) current density space distribution.

consider high-frequency current distribution corrections at surface sub-conductors.

Fig. 3 shows the discretization used to characterize the cylindrical conductor and the ground for a frequency equal to 50 Hz.

Table 2 presents the resistance and inductance values obtained using both Dubanton empirical formula (23) and proposed method, using $\sigma_g = 0.01$ S/m and $\sigma_g = 0.001$ S/m. The relative errors among analytical and simulation results, using the analytical ones as reference, are also presented. Results show a good agreement between analytical and proposed method results, with errors less than 2%. The errors decrease with the decrease of ground conductivity can be explained by the approach used for ground discretization. As a matter of fact, we considered the area of discretization dependent on penetration depth but we kept constant the density of ground filaments used.

3.3. Eccentric and rectangular conductors

A validation of the presented method is also performed for a case where both skin and proximity effects are taken into account and analytical solution does not correspond to accurate results. The application chosen is described in Fig. 6(a) of [15] for the example of eccentric and rectangular conductors. Fig. 4 presents the results obtained for the resistance (a) and inductance (b), per unit length. Results obtained with the proposed model are shown with solid lines, for the sub-conductor discretization given by using the criterion described in Section 2. Values obtained in [15] are shown with asterisks for comparison purposes.

4. Analysis of results

In this section we analyze the influence of a non-homogeneous ground on line longitudinal parameters, for the one-phase transmission line. The studies performed characterize the non-homogeneous ground with: two layers with different conductivity; surface irregularities, such as a hill and a valley. All studies are carried out using the one-phase line presented in paragraph Section 3.2 and assuming the current equal to 1 kA.

4.1. Two layers with different conductivity

In this paragraph the ground is characterized using two layers with different conductivities. The upper layer has 300 m depth and conductivity σ_{g1}, the lower layer has conductivity σ_{g2}. Two studies are performed, the first one (study 1) assumes $\sigma_{g1} = 0.001$ S/m and $\sigma_{g2} = 0.01$ S/m. The second study (study 2), the conductivities are swapped, $\sigma_{g1} = 0.01$ S/m and $\sigma_{g2} = 0.001$ S/m.

Table 3 presents the simulation values obtained for the resistance and inductance of the line, for the three frequency values

Table 3
Longitudinal parameters of one-phase transmission line with two layers ground. Study 1 with $\sigma_{g1} = 0.001$ S/m and $\sigma_{g2} = 0.01$ S/m and study 2 with $\sigma_{g1} = 0.01$ S/m and $\sigma_{g2} = 0.001$ S/m. Three different frequencies (50, 1000 and 5000 Hz) are used.

Frequency [Hz]	Study 1: $\sigma_{g1} = 0.001$ S/m, $\sigma_{g2} = 0.01$ S/m		Study 2: $\sigma_{g1} = 0.01$ S/m, $\sigma_{g2} = 0.001$ S/m	
	Resistance [mΩ/m]	Inductance [μH/m]	Resistance [mΩ/m]	Inductance [μH/m]
50	0.0617	2.3427	0.0767	2.2788
1000	0.7111	2.1501	0.8640	1.9440
5000	4.4763	2.0156	3.5884	1.8137

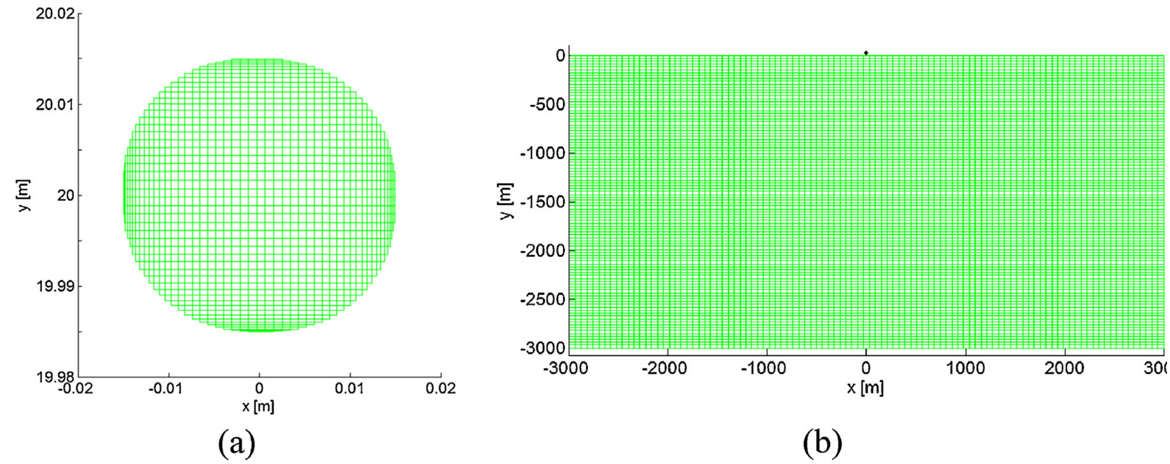

Fig. 3. Discretization used to characterize (a) the cylindrical conductor and (b) the ground, for a frequency equal to 50 Hz.

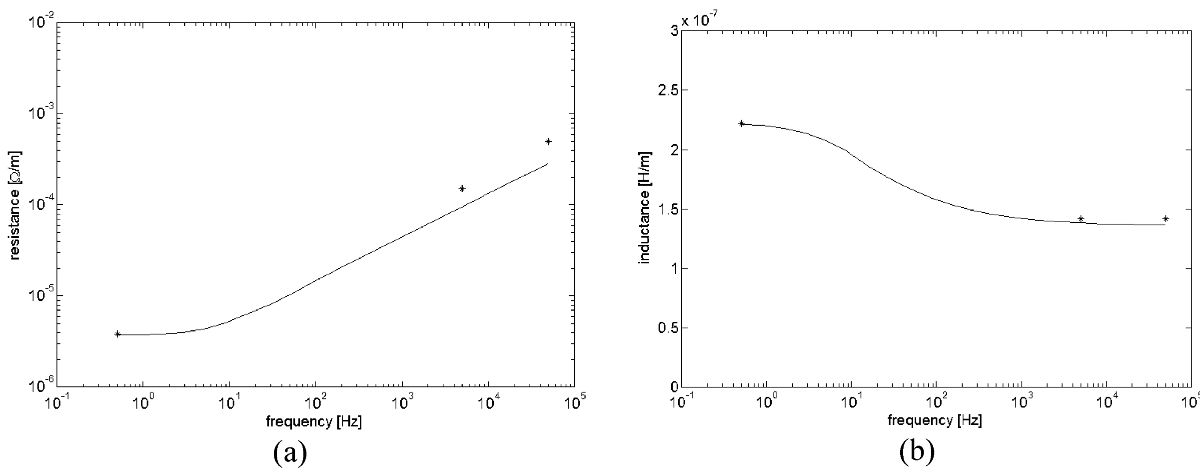

Fig. 4. Results obtained for the example presented in Fig. 6(a) of [15], concerning eccentric and rectangular conductor arrangement. (a) Resistance and (b) inductance, per unit length. The results obtained with the proposed model are presented with solid lines, being the points marked with asterisks obtained from Fig. 7(a) and (b) of [15].

(50 Hz, 1 kHz and 5 kHz), with the ground layer conductivity values defined before.

Results obtained on study 1 show that with low frequency (50 Hz) the current density tends to spread throughout the lower layer since it has higher conductivity. Therefore, the line resistance shows a lower value than the one obtained for homogenous ground, with conductivity equal to 0.01 S/m (Table 2). When high frequency (5 kHz) is considered, the current density spreads mostly in the upper layer and the line parameters tend to those obtained for homogenous ground, with conductivity equals to 0.001 S/m (Table 2).

Results obtained on study 2, with 50 Hz frequency, show that the current density remains enclosed within the upper layer with higher conductivity. Therefore, the obtained resistance shows a higher value than the one obtained for homogenous ground, with conductivity equal to 0.01 S/m (Table 2), since with homogenous ground the current spreads through a larger area. At a frequency of 5 kHz the current distribution in ground remains confined to the upper layer, and the results are analogous to the ones obtained with homogenous ground, with 0.01 S/m conductivity.

4.2. Ground surface irregularities

In the studies presented here the ground surface irregularities are characterized by parables, with 100 m width and 100 m height (hill) or depth (valley). The transmission line is located at the parable vertex, Fig. 5. The ground conductivity is considered to be uniform and equal to 0.01 S/m.

Table 4 presents the simulation values obtained for the resistance and inductance of the line, for the three frequency values (50 Hz, 1 kHz and 5 kHz). The differences among the results obtained with flat ground (Table 2) and these ones are related to the

Table 4
Longitudinal parameters of one-phase transmission line, considering the ground surface with irregularities and uniform conductivity, σ_g = 0.01 S/m. Three different frequencies (50, 1000 and 5000 Hz) are used.

Frequency [Hz]	Hill		Valley	
	Resistance [mΩ/m]	Inductance [μH/m]	Resistance [mΩ/m]	Inductance [μH/m]
50	0.0690	2.2926	0.0810	2.2312
1000	0.7671	2.0401	1.0532	1.8730
5000	3.7700	1.9270	3.9338	1.7220

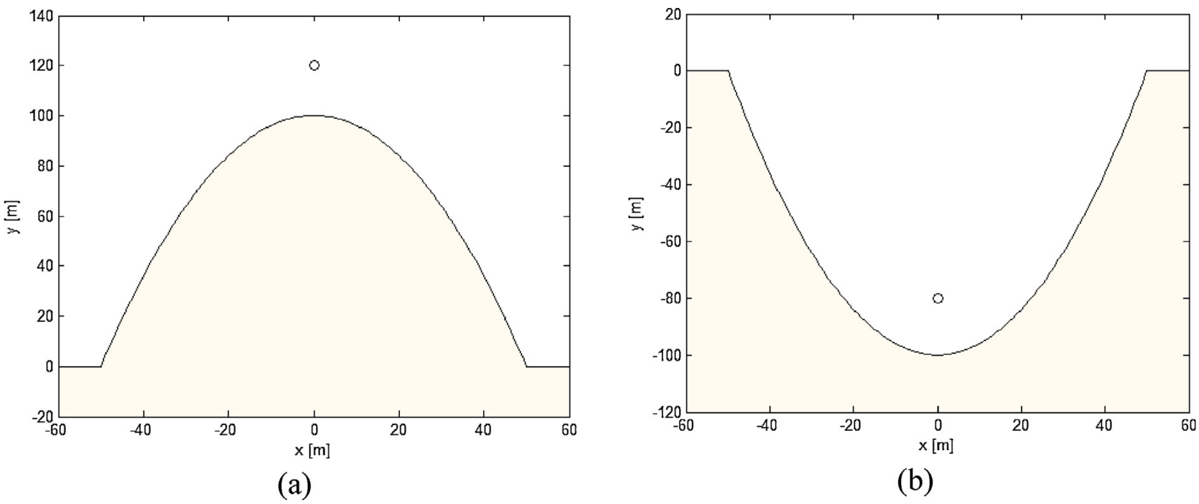

Fig. 5. Ground surface with irregularities: (a) one-phase overhead line at the top of the hill and (b) one-phase overhead line at the bottom of the valley.

average distance between the cylindrical conductor and the ground surface. As the average distance increases the resistance decreases and the inductance increases. When compared to flat ground the average distance is lower for the valley case and higher for the hill case.

5. Conclusions

This paper presents an accurate method to evaluate the longitudinal parameters of multi-conductor overhead power lines, taking in account skin and proximity effects. This methodology allows de characterization of complex configuration systems, such as an overhead transmission line facing a non-homogeneous ground, having layers with different conductivity and/or with surface irregularities, due to the presence of hills, valleys, oil pipes, rivers, etc. The new method characterizes each conductor of the system by a set of smaller and rectangular cross-section sub-conductors, with infinite length along z (line longitudinal axis). Due to skin effect the sub-conductors are characterized by non-uniform cross-sections, presenting thinner sections near the conductor's surface. To validate the proposed method the longitudinal impedance is evaluated and compared with the results obtained not only using analytical formulas but also using numerical methods presented in the literature. The analytical formulas are adequate to evaluate the external impedance of the system and the internal impedance of both isolated cylindrical conductor and flat and homogeneous ground conductor. If the system analyzed is within these conditions the errors are less than 2%, being lower for low frequency values, since the field is more uniform and therefore less sensible to the number of sub-conductors used to split the system real conductor, due to the proximity effect. Only for the particular case, considering two-wire transmission line distanced by 4 cm, the errors obtained reached about 45% for the resistance and about 14% for the inductance. These differences are due to proximity effect which is not considered on the analytical expression used.

Studies are performed in order to evaluate the influence of a non-homogeneous ground on line longitudinal parameters, using one-phase transmission line. The ground non-homogeneity is characterized using: two layers with different conductivity and surface irregularities, such as a hill and a valley. The studies performed with two layers ground, the upper one is considered to have 300 m depth, and the layers conductivity used are 0.01 S/m and 0.001 S/m. Results obtained show that for low frequency the current density tends to spread in the layer with higher conductivity. Consequently, if the lower layer has higher conductivity the resistance value obtained is lower than the one obtained with uniform 0.01 S/m ground, since the current density spreads into a greater area. For high frequency, only the conductivity of the upper layer is important, being the line parameter values similar to the ones obtained with uniform ground with the same conductivity. The study performed with ground surface irregularities show that the line parameter values depend on the average distance between the cylindrical conductor and the ground surface.

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