Single-channel case

In the multi-channel case:

$$f_{\alpha\alpha'} \to e^{i\sigma_\ell} \frac{i}{2} \left[H_\ell^{(-)} \delta_{\ell\ell'} - S_{\ell'} H_{\ell'}^{(+)} \right]$$

with $H^{(\pm)}(\rho) = \exp[\pm i \varphi_\ell]$; $\varphi_\ell = kr - \eta \ln(2kr) + \sigma_\ell$

In the single-channel case

$$f_{\alpha\alpha'}(r) \to f_\alpha(r) = e^{i\sigma_\ell} \frac{i}{2} \left[H_\ell^{(-)} - S_\ell H_\ell^{(+)} \right]$$

$$= e^{i\sigma_\ell} \frac{i}{2} \left[e^{-i\varphi_\ell} - e^{2i\delta_\ell} e^{+i\varphi_\ell} \right] =$$

$$= e^{i\sigma_\ell} \frac{i}{2} e^{i\delta_\ell} \underbrace{\left[e^{-i\varphi_\ell} e^{-i\delta_\ell} - e^{i\delta_\ell} e^{i\varphi_\ell} \right]}_{= -2i \,\mathrm{sen}(\varphi_\ell + \delta_\ell)}$$

Use $\mathrm{sen}(\varphi_\ell + \delta_\ell) = \cos\delta_\ell \,\mathrm{sen}\,\varphi_\ell + \mathrm{sen}\,\delta_\ell \cos\varphi_\ell$

$$\Rightarrow f_\alpha(r) = e^{i(\sigma_\ell + \delta_\ell)} \left[\cos\delta_\ell \,\mathrm{sen}\,\varphi_\ell + \mathrm{sen}\,\delta_\ell \cos\varphi_\ell \right]$$

So, in single-channel problems, one can use the alternative asymptotic condition:

$$\bar{f}_\alpha(r) \to \left[\cos\delta_\ell \,\mathrm{sen}\,\varphi_\ell + \mathrm{sen}\,\delta_\ell \cos\varphi_\ell \right]$$

which has the advantage of being real valued for <u>real potentials</u>