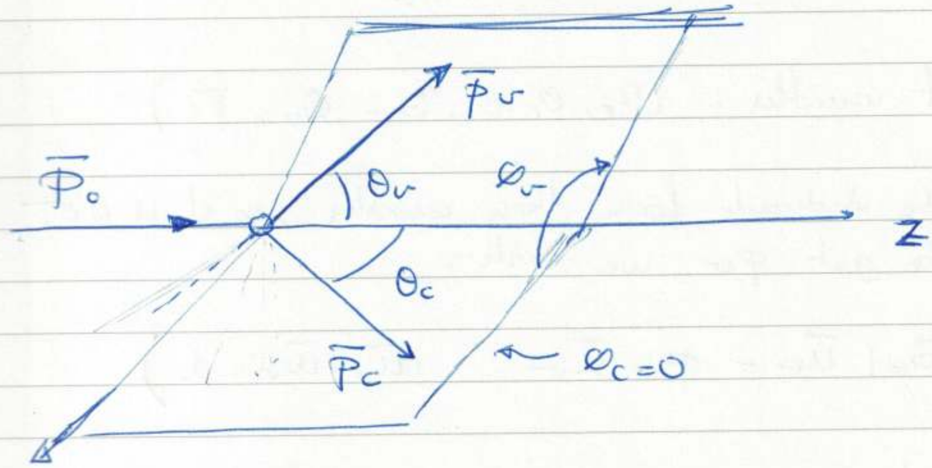
Kinematics for 2-body breakup with core/target excitations

Relevant momenta:

$$a(c+v) + t \longrightarrow c + v + t$$

$$\vec{p}_0 + \vec{p}_t^{(0)} \longrightarrow \vec{p}_c + \vec{p}_v + \vec{p}_t$$

For lab frame: $\vec{p}_t^{(0)} = 0$

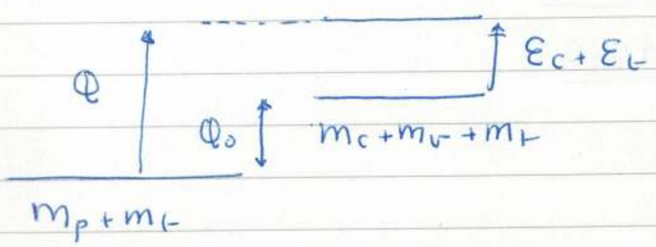


Following Tostevin '01, XZ plane defined by $\vec{p}_0 - \vec{p}_c$ vectors so, in this frame, $\phi_c = 0$ and ϕ_v is then the angle between the $\vec{p}_0 - \vec{p}_v$ plane and the $\vec{p}_0 - \vec{p}_c$ plane.

Energy conservation (LAB)

$$T_p + m_p + m_t = T_c + \varepsilon_c + T_v + T_t + \varepsilon_t + m_v + m_c + m_t$$

$$T_p = T_c + T_v + T_t + \underbrace{\underbrace{(m_v + m_c - m_p)}_{S_{vc} = -Q_0} + \varepsilon_c + \varepsilon_t}_{-Q}$$



$$Q_0 = -S_{vc} < 0$$

Using the momentum and energy conservation equations, we can eliminate the dependence on the target variables:

$$\vec{p}_t = \vec{p}_0 - \vec{p}_c - \vec{p}_\nu \equiv \vec{p}_{ef} - \vec{p}_\nu$$

$$T_t = \frac{p_t^2}{2m_t} = \frac{1}{2m_t}\left[\vec{p}_{ef} - \vec{p}_\nu\right]^2 = \frac{1}{2m_t}\left[p_{ef}^2 + p_\nu^2 - 2\,\vec{p}_{ef}\cdot\vec{p}_\nu\right]$$

$$\frac{1}{2m_t}\left[p_{ef}^2 + p_\nu^2 - 2\,\vec{p}_{ef}\cdot\vec{p}_\nu\right] + T_c + \frac{p_\nu^2}{2m_\nu} - Q = T_p \qquad (I)$$

Independent variables: $(\theta_c, \phi_c = 0, \theta_\nu, \phi_\nu, p_c)$

p_ν can be deduced from these variables, so it is NOT independent. To get p_ν, we write:

$$\vec{p}_\nu \equiv |\vec{p}_\nu|\,\vec{u}_\nu = p_\nu\,\vec{u}_\nu \qquad (\vec{u}_\nu\cdot\vec{u}_\nu = 1)$$

From (I):

$$\frac{1}{2}\left(\frac{1}{m_\nu} + \frac{1}{m_t}\right)p_\nu^2 - \frac{\vec{p}_{ef}\cdot\vec{u}_\nu}{m_t}\,p_\nu + \frac{p_{ef}^2}{2m_t} - T_p + T_c - Q = 0$$

Define:

$$\begin{cases} a = \frac{1}{2}\left(\frac{1}{m_\nu} + \frac{1}{m_t}\right) \\ b = -\frac{1}{m_t}\,\vec{p}_{ef}\cdot\vec{u}_\nu \\ c = \frac{p_{ef}^2}{2m_t} - T_p + T_c - Q = \frac{p_{ef}^2}{2m_t} + \frac{p_c^2}{2m_c} - (T_p + Q) \end{cases}$$

$$p_\nu = \frac{-b \pm \sqrt{b^2 - 4a.c}}{2a}$$

N.b.: In chosen frame:

$$\vec{p}_0 = \begin{pmatrix} 0 \\ 0 \\ p_0 \end{pmatrix} \qquad \vec{p}_c = p_c \begin{pmatrix} \sin\theta_c \\ 0 \\ \cos\theta_c \end{pmatrix} \qquad \vec{p}_\nu = p_\nu \begin{pmatrix} \sin\theta_\nu \cos\phi_\nu \\ \sin\theta_\nu \sin\phi_\nu \\ \cos\theta_\nu \end{pmatrix}$$