# Ejercicio 15

## Métodos Numéricos

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Ejercicio 15: Se desea interpolar la función f(x)=ln(x) en los puntos de abscisas 1, 2, 3 mediante un polinomio de grado adecuado

a) Calcule el polinomio de interpolación utilizando las fórmulas de Lagrange y de Newton

## Lagrange

$$P_n = \sum_{k=0}^{n} y_k \, l_k(x)$$

$$l_k = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}$$

$$f(x) = \ln(x)$$

| | $x$ | $y$ |
|---|---|---|
| $x_1$ | 1 | $\ln(1) = 0$ |
| $x_2$ | 2 | $\ln(2)$ |
| $x_3$ | 3 | $\ln(3)$ |

$$l_2(x) = \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} = \frac{(x-1)(x-3)}{(2-1)(2-3)} = -(x-1)(x-3)$$

$$l_3(x) = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{(x-1)(x-2)}{2}$$

$$\boxed{P_2(x) = -(x-1)(x-3)\ln(2) + \frac{(x-1)(x-2)}{2}\ln(3)}$$

# Newton

| $x$ | $f(x)$ |
|---|---|
| $1$ | $\ln(1)=0$ |
| $2$ | $\ln(2)$ |
| $3$ | $\ln(3)$ |

$$\frac{\ln(2)-0}{2-1} = \ln(2)$$

$$\frac{\ln(3)-\ln(2)}{3-2} = \ln\left(\frac{3}{2}\right)$$

$$\frac{\ln(3/2)-\ln(2)}{3-1} =$$

$$= \frac{\ln\left(\frac{3/2}{2}\right)}{2} = \frac{1}{2}\ln\left(3/4\right) = \ln\left(\sqrt{3/4}\right) = \ln\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{P_2(x) = 0 + \ln(2)(x-1) + \ln\left(\frac{\sqrt{3}}{2}\right)(x-1)(x-2)}$$

b) Obtenga una cota lo más ajustada posible del error de interpolación en el intervalo [1, 3]

$$|e(x)| = \left| \frac{f^{n+1}(\varepsilon)}{(n+1)!}(x-1)(x-2)(x-3) \right| =$$

$$= \left| \frac{f^{(3)}(\varepsilon)}{3!}(x-1)(x-2)(x-3) \right| \qquad \varepsilon \in [1, 3]$$

$$f(x) = \ln(x)$$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = \frac{-1}{x^2}$$

$$f'''(x) = \frac{2}{x^3}$$

$$\left|\frac{f^{(3)}(\varepsilon)}{3!}(x-1)(x-2)(x-3)\right| = \left|\frac{\frac{2}{\varepsilon^3}}{6}(x-1)(x-2)(x-3)\right| =$$

$$= \left|\frac{2}{6\varepsilon^3}(x^2-3x+2)(x-3)\right| =$$

$$= \left|\frac{1}{\varepsilon^3}\cdot\frac{2}{6}(x^3-6x^2+11x-6)\right| =$$

$$= \left|\frac{1}{\varepsilon^3}\right|\cdot\left|\frac{2}{6}\right|\cdot\left|x^3-6x^2+11x-6\right|$$

$$\varepsilon \in [1,3] \Rightarrow 1 \leq \varepsilon \leq 3 \Longrightarrow 1^3 \leq \varepsilon^3 \leq 3^3 \Longrightarrow$$

$$\Longrightarrow \underset{1}{\underset{\shortparallel}{\frac{1}{1^3}}} \geq \frac{1}{\varepsilon^3} \geq \frac{1}{3^3}$$

$$\left|\frac{1}{\varepsilon^3}\right| \cdot \left|\frac{2}{6}\right| \cdot \left|x^3 - 6x^2 + 11x - 6\right| \leq 1 \cdot \left|\frac{2}{6}\right| \cdot \left|x^3 - 6x^2 + 11x - 6\right|$$

$$g(x) = x^3 - 6x^2 + 11x - 6$$

$$g'(x) = 3x^2 - 12x + 11$$

$$3x^2 - 12x + 11 = 0$$

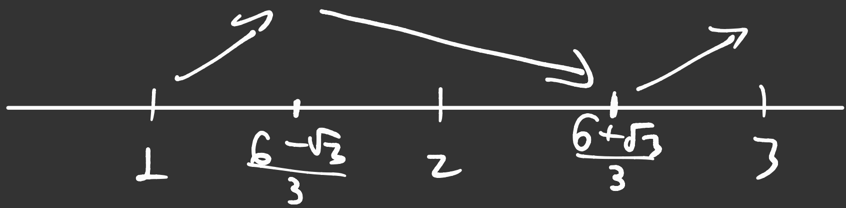
$$x = \frac{+12 \pm \sqrt{12^2 - 4 \cdot 3 \cdot 11}}{2 \cdot 3} = \frac{12 \pm \sqrt{12}}{6} = \frac{12 \pm 2\sqrt{3}}{6} =$$

$$\frac{6 \pm \sqrt{3}}{3} = \begin{cases} \dfrac{6+\sqrt{3}}{3} \approx 2'577 \\[2ex] \dfrac{6-\sqrt{3}}{3} \approx 1'4226 \end{cases}$$

$$f'(1) = 3\cdot 1^2 - 12\cdot 1 + 11 = 2 > 0 \quad \nearrow$$

$$f'(2) = 3\cdot 2^2 - 12\cdot 2 + 11 = -1 < 0 \quad \searrow$$

$$f'(3) = 3\cdot 3^2 - 12\cdot 3 + 11 = 2 > 0 \quad \nearrow$$




$$x = \frac{6-\sqrt{3}}{3} \Longrightarrow \text{Máximo}$$

$$x = \frac{6+\sqrt{3}}{3} \Longrightarrow \text{Mínimo}$$

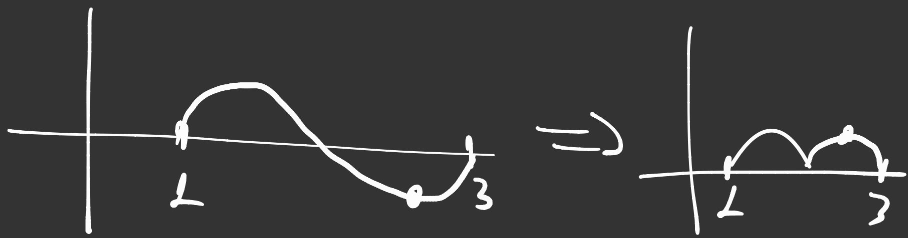
$$g(1) = 1^3 - 6 \cdot 1^2 + 11 \cdot 1 - 6 = 0$$

$$g(3) = 3^3 - 6 \cdot 3^2 + 11 \cdot 3 - 6 = 0$$

$$g\left(\frac{6+\sqrt{3}}{3}\right) = \frac{-2\sqrt{3}}{9} = -0'3849$$

$$g\left(\frac{6-\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{9} = 0'3849$$

$$|g(x)| = \begin{cases} +g(x) & g(x) \geq 0 \\ -g(x) & g(x) < 0 \end{cases}$$



$$|g(x)| \leq \left|\frac{2\sqrt{3}}{9}\right| = \left|\frac{-2\sqrt{3}}{9}\right| = 0'3849 \leq 0'385$$

$$1 \cdot \left|\frac{2}{6}\right| \cdot \left|x^3 - 6x^2 + 11x - 6\right| = \frac{2}{6} \cdot \|g(x)\| \leq$$

$$\leq \frac{2}{6} \cdot 0'385 = 0'1283$$

$$\boxed{|e(x)| \leq 0'1283}$$