(5) $S''(-1) = S''(1) = S(-1) = S(1) = 0$ $\qquad S(0) = 1$

x	-1	0	1
y	0	1	0

$h_i = 1$

$$\Delta = \begin{cases} \Delta_0 = \frac{1-0}{1} = 1 \\ \Delta_1 = \frac{0-1}{1} = -1 \end{cases}$$

$$\begin{bmatrix} \frac{2}{h_0} & \frac{1}{h_0} & 0 \\ \frac{1}{h_0} & 2\left(\frac{1}{h_0}+\frac{1}{h_1}\right) & \frac{1}{h_1} \\ 0 & \frac{1}{h_1} & \frac{2}{h_1} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$3\begin{bmatrix} \frac{\Delta_0}{h_0} \\ \frac{\Delta_0}{h_0}+\frac{\Delta_1}{h_1} \\ \frac{\Delta_1}{h_1} \end{bmatrix} = 3\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} d_0 \\ d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & 7 & 2 & -3 \\ 0 & 1 & 2 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & 7 & 2 & -3 \\ 0 & 0 & 12 & -18 \end{array}\right) \Rightarrow$$

$$\Rightarrow d_2 = \frac{-18}{12} = -\frac{3}{2} \qquad d_1 = \frac{-3-2(d_2)}{7} = \frac{-3-2\left(\frac{-3}{2}\right)}{7} = \frac{-3+3}{7} = 0$$

$$d_0 = \frac{3}{2}$$

$x \in [-1, 0]$

-1	0	$3/2$		
-1	0		$-1/2$	
0	1	1		$\frac{-1}{2}$
0	1	0	-1	

$$P_0(x) = 0 + \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}(x+1)(x) = \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}x(x+1)$$

$x \in [0, 1]$

0	1	0		
0	1		-1	
1	0	-1		$\frac{1}{2}$
1	0	$-3/2$	$\frac{-1}{2}$	

$$P_1(x) = 1 + 0(x) - (x)^2 + \frac{1}{2}(x)^2(x-1) = 1 - x^2 + \frac{1}{2}x^2(x-1)$$

$$S(x) = \begin{cases} \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}x(x+1)^2 & x \in [-1, 0] \\ 1 - x^2 + \frac{1}{2}x^2(x-1) & x \in [0, 1] \end{cases}$$