

$$⑤ \quad S''(-1) = S''(1) = S(-1) = S(1) = 0$$

$$S(0) = 1$$

$$\begin{array}{c|ccc} x & -1 & 0 & 1 \\ \hline y & 0 & 1 & 0 \end{array}$$

$$h_0 = 1$$

$$\Delta = \begin{cases} \Delta_0 = \frac{1-0}{1} = 1 \\ \Delta_1 = \frac{0-1}{1} = -1 \end{cases}$$

$$\begin{bmatrix} \frac{2}{h_0} & \frac{1}{h_0} & 0 \\ \frac{1}{h_0} & 2\left(\frac{1}{h_0} + \frac{1}{h_1}\right) & \frac{1}{h_1} \\ 0 & \frac{1}{h_1} & \frac{2}{h_1} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$3 \begin{bmatrix} \frac{\Delta_0}{h_0} \\ \frac{\Delta_0}{h_0} + \frac{\Delta_1}{h_1} \\ \frac{\Delta_1}{h_1} \end{bmatrix} = 3 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -3 \end{pmatrix}$$

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$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & 7 & 2 & -3 \\ 0 & 1 & 2 & -3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 0 & 7 & 2 & -3 \\ 0 & 0 & 12 & -18 \end{array} \right) \Rightarrow$$

$$\Rightarrow d_2 = \frac{-18}{12} = -\frac{3}{2}$$

$$d_1 = \frac{-3 - 2(d_2)}{7} = \frac{-3 - 2\left(-\frac{3}{2}\right)}{7} = \frac{-3+3}{7} = 0$$

$$d_0 = \frac{3}{2}$$

$$x \in [-1, 0]$$

$$\begin{array}{ccc|c} -1 & 0 & & \\ -1 & 0 & & \\ 0 & 1 & & \\ 0 & 1 & & \end{array} \begin{array}{l} \searrow \\ \searrow \\ \searrow \\ \searrow \end{array} \begin{array}{l} 3/2 \\ 1 \\ 0 \end{array} \begin{array}{l} \searrow \\ \searrow \\ \searrow \end{array} \begin{array}{l} -1/2 \\ -1 \end{array} \begin{array}{l} \searrow \\ \searrow \end{array} \begin{array}{l} -1/2 \\ -1/2 \end{array}$$

$$P_0(x) = 0 + \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}(x+1)(x) = \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}x(x+1)$$

$$x \in [0, 1]$$

$$\begin{array}{ccc|c} 0 & 1 & & \\ 0 & 1 & & \\ 1 & 0 & & \\ 1 & 0 & & \end{array} \begin{array}{l} \searrow \\ \searrow \\ \searrow \\ \searrow \end{array} \begin{array}{l} 0 \\ -1 \\ -1 \\ -3/2 \end{array} \begin{array}{l} \searrow \\ \searrow \\ \searrow \end{array} \begin{array}{l} -1 \\ -1/2 \end{array} \begin{array}{l} \searrow \\ \searrow \end{array} \begin{array}{l} 1/2 \\ 1/2 \end{array}$$

$$P_1(x) = 1 + 0(x) - (x)^2 + \frac{1}{2}(x)^2(x-1) = 1 - x^2 + \frac{1}{2}x^2(x-1)$$

$$S(x) = \begin{cases} \frac{3}{2}(x+1) - \frac{1}{2}(x+1)^2 - \frac{1}{2}x(x+1) & x \in [-1, 0] \\ 1 - x^2 + \frac{1}{2}x^2(x-1) & x \in [0, 1] \end{cases}$$