Circuit Theory and Electronics Fundamentals

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1º Laboratory Report

March 25, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to compare the mesh method and nodal method to the values of current and voltage of the simulated circuit and see how accurate they are. The circuit can be seen in Figure 1.

In section 2, a theoretical analysis of the circuit is presented. In section 5, the circuit is analyzed by simulation, and the results are compared to the theoretical results obtained in section 2. Finally, in section 6, we conclude our study.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analyzed theoretically, by two different methods, which use the characteristics of the components present in the circuit and one of the two Kirchhoff Laws.

For the Mesh method we used Kirchhoff's voltage law and for the nodal method we used Kirchhoff's current law. Besides the independent voltage sources and current sources, the other components of the circuit are, the resistors, which obey Ohm's law, and dependent voltage sources and current sources.

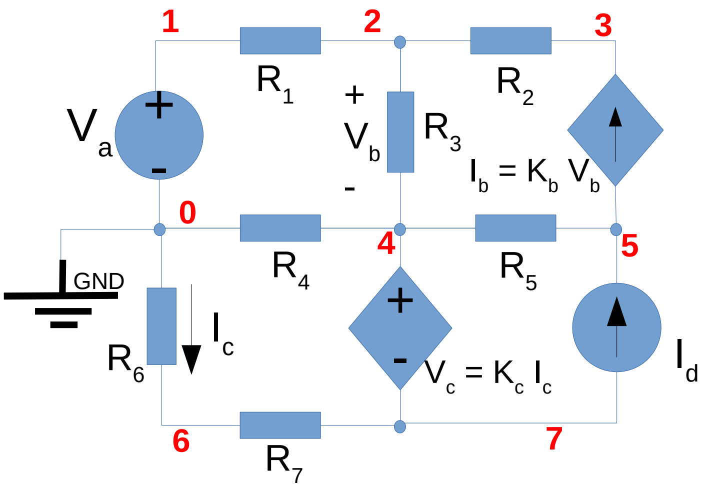

Figure 1: Nodal representation of the circuit.

Name	Value [mA]
§R1	1.04309563061e+03
§R2	2.01744623407e+03
§R3	3.13691375104e+03
§R4	4.15429988186e+03
§R5	3.07915362723e+03
§R6	2.02592738504e+03
§R7	1.04226655522e+03
§H1	8.25247516035e+03
£G1	7.31630468385e-03
Va	5.23936486299e+00
@Id	1.03899051042e-03

Table 1: Constants provided by Python. A variable preceded by @ is of type *current* and expressed in Ampere;a variable preceded by § is of type *resistence* and expressed in Ohms;a variable preceded by £ is of type *conductance* and expressed in Siemens; other variables are of type *voltage* and expressed in Volt.

The simulator uses a dummy voltage source placed in Node 6 to measure the current that passes through I_c, thus creating the Node 8, that has the same voltage as Node 6.

3 Mesh Method

To determine the values of the current that goes through the branches of circuit in the Figure 1 by the Mesh method, there's only the need to use 4 equations by applying Kirchhoff's voltage law or other relations to obtain the values of 4 unknown values of the currents I_1, I_2, I_3 and I_4, as the current of each branch can be obtained by a linear combination of these currents. The first equation can be writen as:

$$I_d = I_4. \tag{1}$$

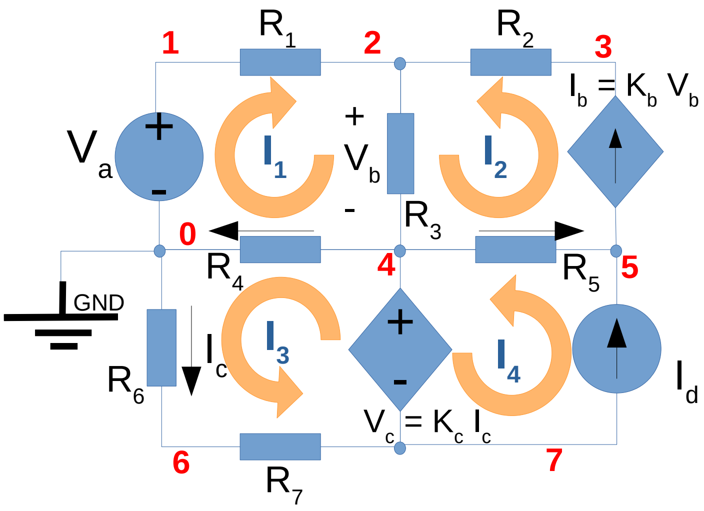

Figure 2: Circuit analyzed in the lab.

Applying Kirxhhoff's Voltage Law (KVL) to the Mesh with I_3 and to the Mesh with I_1, we get the following second and third equations:

$$0 = R_4 I_1 + (R_4 + R_6 + R_7 - K_c)I_3, \tag{2}$$

and

$$V_a(t) = (R_1 + R_3 + R_4)I_1 + R_4 I_3 R_3 I_2. \tag{3}$$

Knowing that:

$$I_2 = I_b, \tag{4}$$

And

$$v_b = \frac{I_b}{V_b} = \frac{I_2}{V_b} \tag{5}$$

We get the fourth equation:

$$I_1(R_3 K_b) + I_2(R_3 K_b - 1) = 0 \tag{6}$$

4 Nodal Method

In the nodal method, the circuit requires 6 equations to determine the values of V2, V3, V4, V5, V6, V7, which are obtained by applying Kirchhoff's current Law and a relation between the potential difference of two nodes and two other nodes, whose potential difference is imposed by a dependent voltage source.

Since we considered V_0 to be the ground, which means that V_o = 0 and therefore:

$$V_1 = V_a. \tag{7}$$

We get the following equations by applying Kirchhoff's current law at different nodes:
For node 2 we have:

$$V_a \frac{1}{R_1} = V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + V_3 \frac{-1}{R_2} + V_4 \frac{-1}{R_3}. \tag{8}$$

Name	Value [mA]
@I1[i]	0.240224
@I2[i]	-0.251168
@I3[i]	0.968913
@I4[i]	1.038991
@G1[i]	-0.251168
@Id[current]	1.038991
@R1[i]	0.240224
@R2[i]	-0.251168
@R3[i]	-0.010944
@R4[i]	1.209137
@R5[i]	-1.290158
@R6[i]	0.968913
@R7[i]	0.968913

Table 2: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere

Node 3:

$$0 = V_2(\frac{-1}{R_2} - K_b) + V_3\frac{1}{R_2} + V_4(K_b). \tag{9}$$

Node 5:

$$I_d = V_2(K_b) + V_4(\frac{-1}{R_5} - K_b) + V_5\frac{1}{R_5}. \tag{10}$$

Node 6:

$$0 = V_6(\frac{1}{R_6} + \frac{1}{R_7}) + V_7\frac{-1}{R_7}. \tag{11}$$

Node 7:

$$0 = V_4 + V_6(\frac{K_c}{R_6}) - V_7. \tag{12}$$

And finally for node 4:

$$-I_d = V_2\frac{-1}{R_3} + V_4(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}) + V_5\frac{-1}{R_5} + V_6\frac{-1}{R_7} + V_7\frac{1}{R_7}. \tag{13}$$

Name	Value [V]
V(1)	5.239365
V(2)	4.988788
V(3)	4.482071
V(4)	5.023118
V(5)	8.995714
V(6)	-1.962948
V(7)	-2.972813
V(8)	-1.962948

Table 3: Operating point. The variables are of type *voltage* and expressed in Volt.

5 Simulation Analysis

The simulated values we obtained are represented in table 4 and in the following subsection we will compare this result to the theoretical values obtained earlier.

Name	Value [A or V]
@g1[i]	-2.51168e-04
@id[current]	1.038991e-03
@r1[i]	2.402240e-04
@r2[i]	-2.51168e-04
@r3[i]	-1.09438e-05
@r4[i]	1.209137e-03
@r5[i]	-1.29016e-03
@r6[i]	9.689131e-04
@r7[i]	9.689131e-04
v(1)	5.239365e+00
v(2)	4.988788e+00
v(3)	4.482071e+00
v(4)	5.023118e+00
v(5)	8.995714e+00
v(6)	-1.96295e+00
v(7)	-2.97281e+00
v(8)	-1.96295e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

5.1 Operating point Analysis

Name	Value [A or V]
@G1[i]	0 %
@Id[current]	0 %
@R1[i]	0 %
@R2[i]	0 %
@R3[i]	1.8275e-03 %
@R4[i]	0 %
@R5[i]	1.5502e-04 %
@R6[i]	1.03208e-05 %
@R7[i]	1.03208e-05 %
V(1)	0%
V(2)	0%
V(3)	0%
V(4)	0%
V(5)	0%
V(6)	1.01887e-04%
V(7)	1.0091e-04%
V(8)	1.01887e-04%

Table 5: Relative errors between the simulation values and the values obtained by the theoretical methods. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

As seen in Table 5, comparing with the other tables, the errors we obtained were very close to zero and therefore are negligible as expected because we analyzed a very simple circuit without any temporal variation. Some of the errors obtained were equal to zero, but in reality, they are actually greater than zero. This happens due to rounding done by the simulator and by Octave, that also caused the rise of some non-zero errors.

6 Conclusion

In conclusion, the values we obtained in the simulation agree with the theoretical values obtained by the mesh and nodal methods, showing negligible errors. These are excellent results which show that the methods we used are legitimate, as expected, and can be used by the simulation program to simulate the circuit.

Nevertheless, the simulator and methods used might produce solutions, which may not occur in a real circuit due to various factors including the Joule effect in the cables that connect the components in the circuit, and other random and systematic errors, which compromise the precision and accuracy of the results. However, the results might still be a good approximation of the real values, which can be verified by analyzing a real representation of the circuit.