Circuit Theory and Electronics Fundamentals

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2º Laboratory Report

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Contents

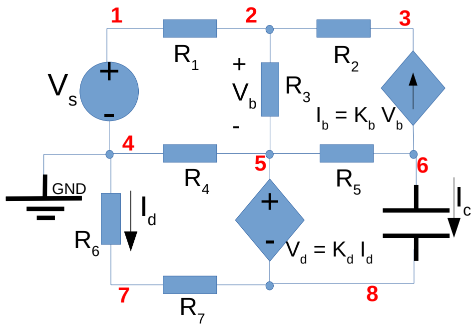

Figure 1: Nodal representation of the circuit.

1 Introduction

Name	Value
§R(1)	1.043096 K
§R(2)	2.017446 K
§R(3)	3.136914 K
§R(4)	4.154300 K
§R(5)	3.079154 K
§R(6)	2.025927 K
§R(7)	1.042267 K
!!C	1.038991 u
Vs	5.239365
£Kb	7.316305 m
§Kd	8.252475 K

Table 1: Constants provided by Python. A variable preceded by !! is of type *capacitance* and expressed in Farad ;a variable preceded by § is of type *resistence* and expressed in Ohm;a variable preceded by £ is of type *conductance* and expressed in Siemens; other variables are of type *voltage* and expressed in Volt.

The objective of this laboratory assignment is to find the natural, forced and total solution, with the use of a nodal analysis an the equivalent resistance, which will be used to plot the frequency response of the functions $v_c(f)$ and $v_6(f)$. The circuit can be seen in Figure 1.

In section 2, a theoretical analysis of the circuit is presented. In section 5, the circuit is analyzed by simulation, and the results are compared to the theoretical results obtained in section 2. Finally, in section 6, we conclude our study.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically.

2.1 Nodal Analysis

The first step we took was analyzing the circuit for t¡0 and by applying Kirchhoff's Current and voltage Laws we determined the currents in branches and voltage in nodes respectively. This are the equations used by us.

In node 1 we also considered:

$$V_1 = V_s | V_4 = 0 \tag{1}$$

Using the equations above, we get the following matrix equation:

$$\begin{bmatrix} G_1 & -(G_1+G_2+G_3) & G_2 & G_3 & 0 & 0 & 0 \\ 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & -(K_b+G_5) & G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(G_6+G_7) & G_7 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_bG_6 & -1 \\ 0 & G_3 & 0 & -(G_3+G_4+G_5) & G_5 & G_7 & -G_7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \end{bmatrix}$$

Using octave we obtain the following table:

Name	Value [mA]
V(1)	5.239365
V(2)	4.988788
V(3)	4.482071
V(4)	0.000000
V(5)	5.023118
V(6)	5.796502
V(7)	-1.962948
V(8)	-2.972813

Table 2: Voltage of the 8 nodes present in the circuit

Using Ohms law, we can calculate the current flowing through each resistor, which are represented in the following table:

Name	Value [mA]
@G1[i]	-0.251168
@C[current]	0.000000
@R1[i]	-0.240224
@R2[i]	-0.251168
@R3[i]	-0.010944
@R4[i]	1.209137
@R5[i]	-0.251168
@R6[i]	0.968913
@R7[i]	0.968913

Table 3: Current values of the 8 nodes present in the circuit

2.2 Equivalent Resistance

A set of equations were used, bearing in mind that the Thévenin Theorem was applied. By applying this theorem, all the independent tension sources were considered null, to study the influence of the condenser in the circuit, in order to obtain the circuit's equivalent resistance in the perspective of the circuit extremities. We assumed that the tension $V_s = 0V$ and there was a passage from a constant value to null value, which was the result of the passage of a non-null current passing through the condenser. Also, since the voltage source interior resistance was null, it's the equivalent to cause a short-circuit to that source.

The Kirchhoff's Laws can also be applied here, leading to the same equations used in the section before. The rest of the equations can be obtained using the following equation:

$$V_x = V_6 - V_8 \tag{2}$$

With former equations, we can form the matrix equation used bellow:

$$\begin{bmatrix} G_1 & -(G_1+G_2+G_3) & G_2 & G_3 & 0 & 0 & 0 & \\ 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & \\ 0 & 0 & 0 & 0 & -(G_6+G_7) & G_7 & & \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & K_dG_6 & -1 & \\ -G_1 & G_1 & 0 & -(G_4+G_6) & G_4 & 0 & G_7 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_x \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using octave we get the following data:

Name	Value [mA]
V(1)	0.000000
V(2)	-0.000000
V(3)	-0.000000
V(4)	0.000000
V(5)	-0.000000
V(6)	8.769316
V(7)	0.000000
V(8)	0.000000

Table 4: Theoretical voltage values for t = 0

Name	Value [mA]
@G1[i]	-0.000000
@C[current]	-0.002848
@R1[i]	-0.000000
@R2[i]	-0.000000
@R3[i]	-0.000000
@R4[i]	-0.000000
@R5[i]	2.847963
@R6[i]	-0.000000
@R7[i]	-0.000000

Table 5: Theoretical current values for t = 0. A variable preceded by @ is of type *current* and expressed in Ampere.

2.3 Natural Solution

In this section we will start by analyzing the natural solution , where t varies from 0 to 20 ms, by using the equivalent resistance we calculated before.

Assim that $V_0 = V_x$, the natural solution of the capacitor is

$$V_6n(t) = V_x e^{-(t)/CR_eq} \tag{3}$$

We can now plot the graph of the solution for t raging from 0 to 20 ms:

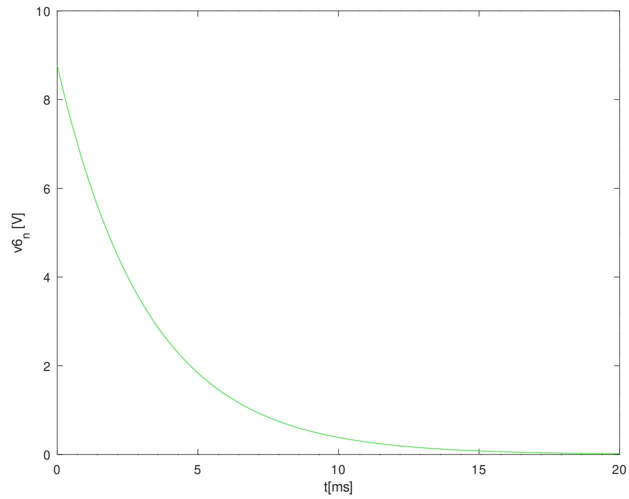

Figure 2: Natural solution

2.4 Forced Solution

The nodal analysis of this circuit, is done with the tension phasor being equal to 1 and the condenser has an impedance of $\frac{1}{iwc}$. After this, we make the same analysis, which can be represented by the following matrix:

$$\begin{bmatrix} G_1 & -(G_1+G_2+G_3) & G_2 & G_3 & 0 & 0 & 0 \\ 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & -(K_b+G_5) & Y_c+G_7 & 0 & -Y_c \\ 0 & 0 & 0 & 0 & 0 & -(G_6+G_7) & G_7 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_dG_6 & -1 \\ 0 & G_3 & 0 & -(G_3+G_4+G_5) & G_5+Y_c & G_7 & -(G_7+Y_c) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Name	Value [V]
V(1)	1.000000
V(2)	0.952174
V(3)	0.855461
V(4)	0.000000
V(5)	0.958727
V(6)	0.569359
V(7)	0.374654
V(8)	0.567400

Table 6: Eletric potential of the 8 nodes in the circuit

2.5 Total Solution

We can obtain $v_6t(t)$ (the total solution), by superimposing the natural(section 2.2.) and forced solutions (2.3) obtained before. Since we determined the function of the natural solution in section 2.2 we will now determine the forced solution.

The complex equation of the phasor can be written as:

$$v_6f(t) = V_6sen(\omega t + \varphi_s) \tag{4}$$

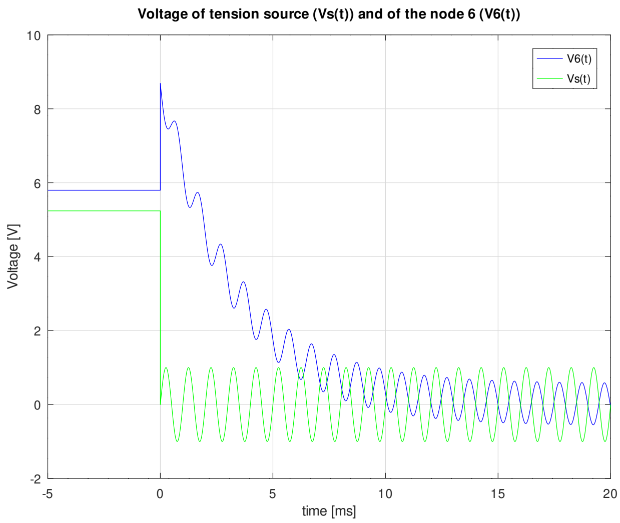

Figure 3: Total solution $v_6(t)$

In the equation above, $\omega = 2\pi f = 2000\pi$ and Φ is the argument of

By superimposing the equations of both the natural and forced solutions, we obtain this equation:

$$v_6(t) = v_6 n(t) + V_6 f(t) = V_6 sen(\omega t + \varphi_s) + V_x e^{-(1)/R_s C} \quad (5)$$

By plotting this function, for t in range of -5 to 20 ms, we get the graph below:

2.6 Frequency Response

Lastly, we will analyze the frequency response of each phasor, by applying the same method used in section 2.4, with the exception that Y_c wil remain as a variable ($Y_c(f) = j2\pi f C$).

Considering:

$$V_c(t) = V_6(t) - V_8(t) \quad (6)$$

Next we can plot the graphs of $V_c(t)$ and $V_6(t)$, as functions of f in range of $10^(-1)$ to $10^(6)$ Hz, which will be defined as logarithmic scales.

The magnitude plot has values in dB, which respect the following equation:

$$V_d B = 20 \log_1 0(V) \quad (7)$$

The graphics of the plotted functions $v_c(f)$ and $v_6(f)$:

3 Simulation Analysis

It's important to mention that the node V_4, V_5, V_6 and V_7, from the simulation, correspond respectively to the nodes V_5, V_6, V_7 and V_8 from the theoretical part and, the node 8 has the same voltage and current as the node 6 of the simulation, due to the existence of a null voltage source, used to measure that current that passes through those two nodes. Besides this, there's an extra node in the simulation, which is the node 8 mentioned earlier.

3.1 Operating point Analysis (for t¡0 and Vs(t) = 0)

Ngspice was used to perform an operating point analysis for t¡0 (obtaining the table below) and by setting $v_s = 0$ and replacing the capacitor with a voltage sorce $V_x = V_6 - V_8$, the voltage and current values, present in tables below, were obtained.

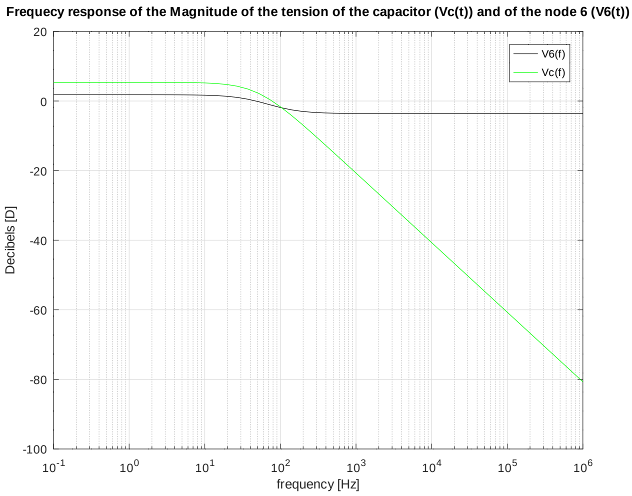

Figure 4: Magnitude plot

Name	Value [mA]
@c[i]	0.000000e+00
@g1[i]	-2.51168e-04
@r1[i]	2.402239e-04
@r2[i]	-2.51168e-04
@r3[i]	-1.09438e-05
@r4[i]	1.209137e-03
@r5[i]	-2.51168e-04
@r6[i]	9.689132e-04
@r7[i]	9.689132e-04
v(1)	5.239365e+00
v(2)	4.988788e+00
v(3)	4.482071e+00
v(4)	5.023118e+00
v(5)	5.796502e+00
v(6)	-1.96295e+00
v(7)	-2.97281e+00
v(8)	-1.96295e+00

Table 7: Current and voltage results obtained with Ngspice for t¡0

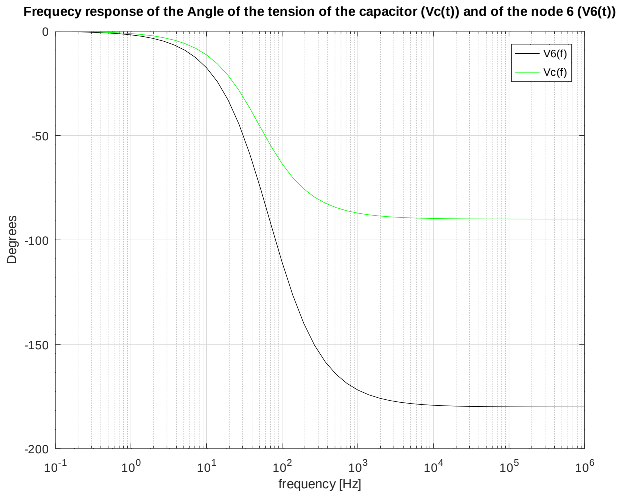

Figure 5: Phase plot

Name	Value [mA]
@g1[i]	6.189645e-18
@r1[i]	-5.91995e-18
@r2[i]	6.189645e-18
@r3[i]	2.696940e-19
@r4[i]	1.282784e-18
@r5[i]	-2.84796e-03
@r6[i]	8.673617e-19
@r7[i]	1.836888e-20
v(1)	0.000000e+00
v(2)	6.175078e-15
v(3)	1.866235e-14
v(4)	5.329071e-15
v(5)	8.769316e+00
v(6)	-1.75721e-15
v(7)	-1.77636e-15
v(8)	-1.75721e-15

Table 8: Current and voltage results obtained with Ngspice for Vs = 0

3.2 Transient Analysis - Natural Response

The first step in order to compute the natural response of v_6 is to set $v_s(t) = 0$ and set initial conditions so that v_6 and v_s have the same values obtained in table 10, which ensures that in the start of the transient analysis the capacitor is fully charged. The graphic in figure 11 was obtained by executing a transient analysis, with t ranging from o to 20 ms.

By analyzing the graphic we can see a quick drop in voltage from just a little above 8 ms to 0, which is due to the fact that the capacitor is just discharging and since there are no independent voltage sources, there is no voltage being provided to v_6.

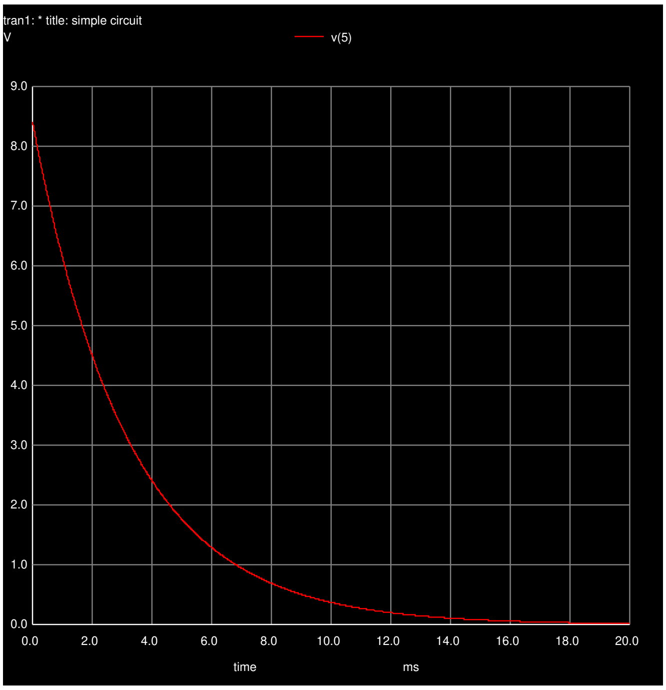

Figure 6: Natural response

3.3 Transient Analysis - total Response

Taking in consideration that $v_s(t) = \sin(2\pi f t)$ and f = 1000Hz, the total response can be obtained by performing a transient analysis and the figure 12 can be obtained by plotting $v_s(t)$ and $v_6(t)$.

3.4 Frequency Response

By excuting a frequency sweep, throughout the length of the interval [0.1, 1000000], we are able to plot the magnitude and phase, which appear below respectively.

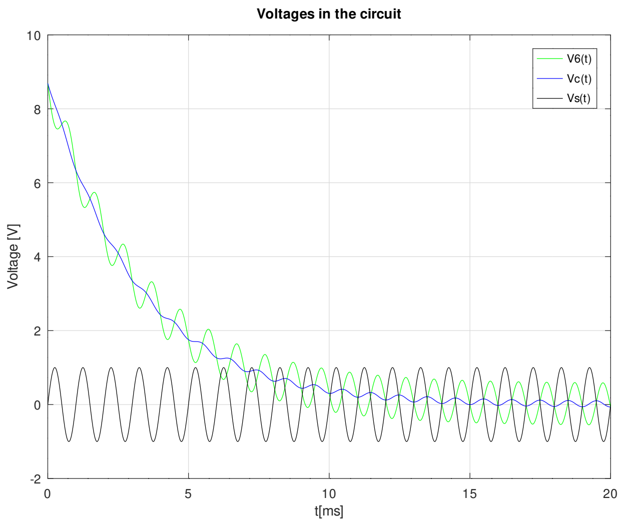

Figure 7: Stinulus voltage ,$v_s(t)$, shown in black and total response, $v_6(t)$, shown in green

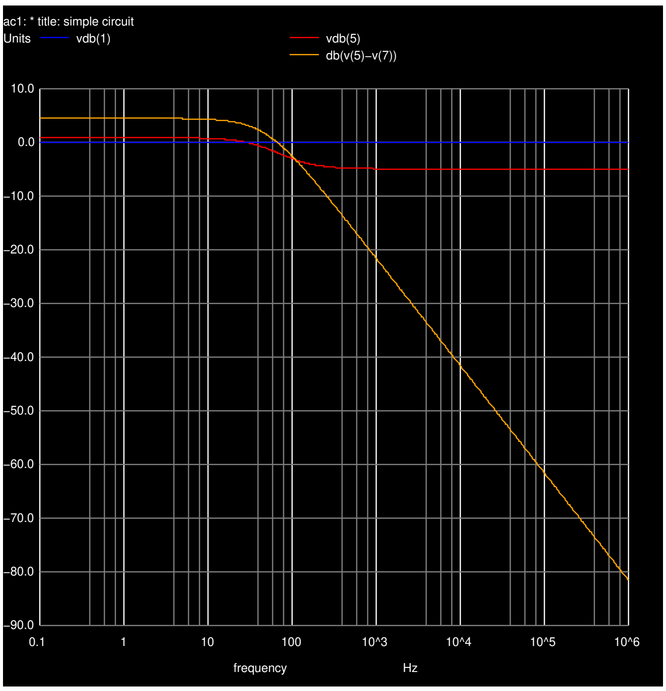

Figure 8: Magnitude plot

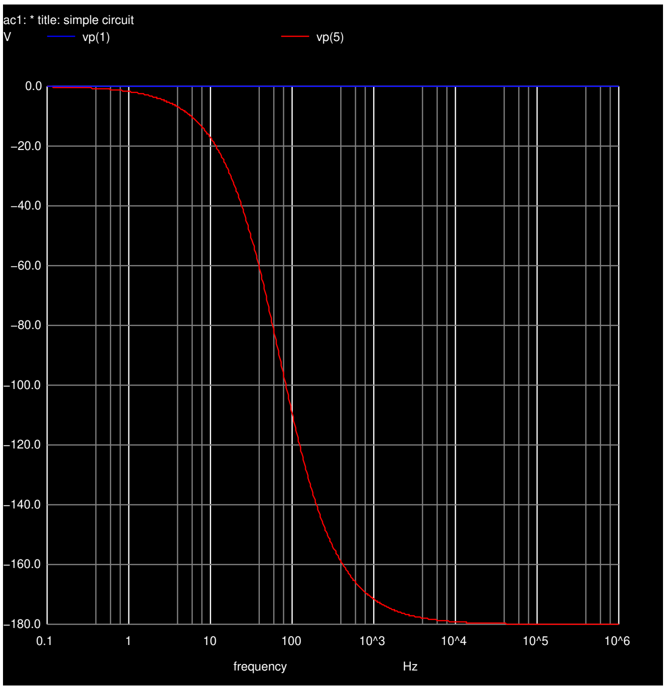

Figure 9: Phase plot

4 Conclusion

4.1 V_s is constant an non-null

In conclusion, the values we obtained in the simulation agree with the theoretical values obtained for the dc voltage source V_s, showing negligible errors. These are excellent results which show that the methods we used are legitimate, as expected, and can be used by the simulation program to simulate the circuit.

Nevertheless, the simulator and methods used might produce solutions, which may not occur in a real circuit due to various factors including the Joule effect in the cables that connect the components in the circuit, and other random and systematic errors, which compromise the precision and accuracy of the results. However, the results might still be a good approximation of the real values, which can be verified by analyzing a real representation of the circuit. It-s also safe to say that while some relative errors were very close to zero, or in some cases even zero, this is due to approximations made by the simulator (ngspice) and octave.

Name	Value [mA]
V(1)	5.239365
V(2)	4.988788
V(3)	4.482071
V(4)	0.000000
V(5)	5.023118
V(6)	5.796502
V(7)	-1.962948
V(8)	-2.972813

Table 9: Theoretical voltage results obtained with octave

Name	Value [mA]
@c[i]	0.000000e+00
@g1[i]	-2.51168e-04
@r1[i]	2.402239e-04
@r2[i]	-2.51168e-04
@r3[i]	-1.09438e-05
@r4[i]	1.209137e-03
@r5[i]	-2.51168e-04
@r6[i]	9.689132e-04
@r7[i]	9.689132e-04
v(1)	5.239365e+00
v(2)	4.988788e+00
v(3)	4.482071e+00
v(4)	5.023118e+00
v(5)	5.796502e+00
v(6)	-1.96295e+00
v(7)	-2.97281e+00
v(8)	-1.96295e+00

Table 10: Simulation results obtained with Ngspice

4.2 $V_s = 0$ and capacitor are replaced by the voltage source

By analysing the tables below, we can see a pattern of repeated voltages, whose order of magnitude is around $10^(-5)$:

Name	Value [mA]
V(1)	0.000000
V(2)	-0.000000
V(3)	-0.000000
V(4)	0.000000
V(5)	-0.000000
V(6)	8.769316
V(7)	0.000000
V(8)	0.000000

Table 11: Theoretical voltage results obtained with octave

Name	Value [mA]
@g1[i]	6.189645e-18
@r1[i]	-5.91995e-18
@r2[i]	6.189645e-18
@r3[i]	2.696940e-19
@r4[i]	1.282784e-18
@r5[i]	-2.84796e-03
@r6[i]	8.673617e-19
@r7[i]	1.836888e-20
v(1)	0.000000e+00
v(2)	6.175078e-15
v(3)	1.866235e-14
v(4)	5.329071e-15
v(5)	8.769316e+00
v(6)	-1.75721e-15
v(7)	-1.77636e-15
v(8)	-1.75721e-15

Table 12: Simulation results obtained with Ngspice