Seminar 3 - 2025

1. A company receives a shipment of 25 hard drives. Before accepting the shipment, 6 of them will be randomly selected and tested. Only if all 6 meet specifications, will the shipment be accepted. 4 of the 25 drives (of the shipment) are defective, what is the probability that the shipment will not be accepted?

A: There are 21 hard drives that meet the specifications. The required probability is $1 - \dfrac{C_{21}^6}{C_{25}^6}$.

2. In a bag there are marbles numbered from 0 to 5:

Number on the marble	0	1	2	3	4	5
Frequency (in the bag)	9	11	12	13	14	11

4 marbles are drawn randomly without replacement. Let X_k denote the k-th drawn number, $k = \overline{1,4}$. Compute the following probabilities:

(a) $P(X_1 = 1, X_2 = 2, X_3 = 3, X_4 = 4)$;
(b) $P(\{X_1 = X_2 = 0\} \cap \{X_3 = X_4 = 5\}$;
(c) $P(\{X_1 = X_2 = X_3\} \cap \{X_2 < 2\})$;
(d) $P(\{X_1 \neq 1\} \cup \{X_2 \neq 2\} \cup \{X_3 \neq 3\})$;
(e) $P(X_3 = X_4 = 1 | X_1 = X_2 = 1)$.

A: (a) $P(X_1 = 1, X_2 = 2, X_3 = 3, X_4 = 4) = \frac{11}{70} \cdot \frac{12}{69} \cdot \frac{13}{68} \cdot \frac{14}{67}$
(b) $P(\{X_1 = X_2 = 0\} \cap \{X_3 = X_4 = 5\}) = \frac{9}{70} \cdot \frac{8}{69} \cdot \frac{11}{68} \cdot \frac{10}{67}$
(c) $P(\{X_1 = X_2 = X_3\} \cap \{X_2 < 2\}) = \frac{9}{70} \cdot \frac{8}{69} \cdot \frac{7}{68} + \frac{11}{70} \cdot \frac{10}{69} \cdot \frac{9}{68}$
(d) $P(\{X_1 \neq 1\} \cup \{X_2 \neq 2\} \cup \{X_3 \neq 3\}) = 1 - \frac{11}{70} \cdot \frac{12}{69} \cdot \frac{13}{68}$
(e) $P(X_3 = X_4 = 1 | X_1 = X_2 = 1) = \frac{9}{68} \cdot \frac{8}{67}$.

3. Let M be a subset with 3 randomly chosen (without replacement) elements of the set $\{2, 3, 4, 5, 6\}$ and N be a subset with 2 randomly chosen (without replacement) elements of the set $\{0, 1, 7, 8\}$. Let $U = M \cup N$. Compute the probabilities of the following events:
A:"U contains only odd numbers."
B:"U contains only consecutive numbers."
C:"$\{0, 4\} \subset U$."
D:"U contains at least two even numbers."

A: $P(A) = 0$; $P(B) = \frac{2}{C_5^3 C_4^2} = \frac{2}{60} = \frac{1}{30}$; $P(C) = \frac{C_4^2 C_3^1}{C_5^3 C_4^2} = \frac{6 \cdot 3}{60} = \frac{3}{10}$;

$P(\text{"no even number"}) = P(A) = 0;\ P(\text{"exactly one even number"}) = \frac{C_3^1}{C_5^3 C_4^2} = \frac{3}{60} \Rightarrow P(D) = 1 - \frac{3}{60} = \frac{19}{20}$.

4. A person rolls a die and tosses a coin.
a) Write a sample space for this experiment.
b) What is the probability of the event E:"the person obtains the number 6 **or** heads"?

A: a) Let $S = \{(d, c) : d \in \{1, 2, 3, 4, 5, 6\}, c \in \{0, 1\}\}$, where we represent "heads" by 1 and "tails" by 0.
b) Let E_1:"the person obtains the number 6" and E_2:"the person obtains heads". The event E_1 is identified with the subset $\{(6, c) : c \in \{0, 1\}\}$ of S and the event E_2 is identified with the subset $\{(d, 1) : d \in \{1, 2, 3, 4, 5, 6\}\}$ of S.
S is a finite sample space, the σ-field is $\mathcal{K} = \mathcal{P}(S)$ (the collection of all subsets of S) and we use the classical definition of probability

$$P(A) = \frac{\text{the number of favorable outcomes for the occurrence of } A}{\text{number of all possible outcomes within the experiment}}, \text{ where } A \subseteq S.$$

Note that $(S, \mathcal{K}, P)$ is a probability space.
We have

$$P(E) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{2}{12} + \frac{6}{12} - \frac{1}{12} = \frac{7}{12}.$$

Here we used that $E_1 \cap E_2 = \{(6, 1)\}$.

5. 9 persons get randomly on a train with 3 cars. Compute the probabilities of the following events:
A: "There are exactly 3 persons in the first car."
B: "There are 3 persons in each car."
C: "There is 1 person in a car and there are 4 persons in each of the other two cars."
D: "There is at least a person in each car."

A: $P(A) = \frac{C_9^3 \cdot 2^6}{3^9}$; $P(B) = \frac{C_9^3 \cdot C_6^3}{3^9}$; $P(C) = \frac{3 \cdot 9 \cdot C_8^4}{3^9}$;
$P(D) = 1 - P(\text{"exact one car is empty"}) - P(\text{"exact two cars are empty"}) = 1 - \frac{3 \cdot (2^9 - 2)}{3^9} - \frac{3}{3^9} = \frac{3^9 - 3 \cdot 2^9 + 3}{3^9}$.