Quadrature formulas

A

1. Find the degree of exactness of the following quadrature formula: $\int_0^1 f(x)\,dx = \frac{1}{4}f(0) + \frac{3}{4}f\left(\frac{2}{3}\right) + R(f)$.

2. Let $\int_{-1}^1 f(x)\,dx = f(-a) + f(a) + R(f),\ a \in (0,1]$. Prove that its degree of exactness d is $d \geq 1$ for any $a \in (0,1]$. Find the value of a for which the degree of exactness d is maximum and specify the value of d in this case.

3. Find $n \in \mathbb{N}$ such that $\int_1^2 x \ln x\,dx$ is approximated by the repeated trapezium formula with precision $\epsilon = 10^{-5}$.

4. Approximate
$$\int_1^3 \frac{x}{x^2+4}\,dx$$
using the repeated Simpson formula with $n = 2$.

B

1. Find the degree of exactness of the following quadrature formula: $\int_{-1}^1 f(x)\,dx = \frac{2}{3}\left[f(-1) + f(0) + f(1)\right] + R(f)$.

2. Check if $\int_0^b f(x)\,dx = \frac{b}{3}\left[2f\left(\frac{b}{4}\right) - f\left(\frac{b}{2}\right) + 2f\left(\frac{3b}{4}\right)\right] + R(f)$ has the degree of exactness 3.

3. Find $n \in \mathbb{N}$ such that $\int_1^2 x \ln x\,dx$ is approximated by the repeated Simpson formula with precision $\epsilon = 10^{-5}$.

4. Approximate
$$\int_1^3 \frac{x}{x^2+4}\,dx$$
using the repeated trapezium formula with $n = 2$.