Laboratory 2: Solving Differential Equations with MAPLE

1. Find the general solution of the following differential equations and plot some solutions graphs

   (a) $y' = 2x\,(1 + y^2)$

   (b) $(x^2 - 1)\,y' + 2xy^2 = 0$

   (c) $2x^2y' = x^2 + y^2$

   (d) $y'' + y = \sin x + \cos x$

   (e) $y'' - y = e^{2x}$

   (f) $y'' - y' = \frac{1}{1+e^x}$

2. Solve the following IVPs and draw the solution graph:

   (a) $y' = 1 + y^2$, $y\,(0) = 1$

   (b) $y' = \frac{1}{1-x^2}y + 1 + x$, $y\,(0) = 0$

   (c) $y' - 2y = -x^2$, $y\,(0) = \frac{1}{4}$

   (d) $y'' - 5y' + 4y = 0\,,\ y(0) = 5,\ y'(0) = 8;$

   (e) $y'' - 4y' + 5y = 2x^2e^x\ ,\ y(0) = 2,\ y'(0) = 3;$

   (f) $y'' + 4y = 4\,(\sin 2x + \cos 2x),\ y(\pi) = y'(\pi) = 2\pi;$

3. Consider the differential equation

$$y'\,(x) + \frac{k}{x}y\,(x) = x^3,$$

   where $k \in \mathbb{R}$. Find the general solution

   (a) Find the general solution.

   (b) For $k = 1$ draw some solutions graphs.

   (c) Solve the IVP $\begin{cases} y'\,(x) + \frac{1}{x}y\,(x) = x^3 \\ y(1) = 0 \end{cases}$ and draw the solution graph.

   (d) Use `animate` command to see the solution dependence of the IVP $\begin{cases} y'\,(x) + \frac{k}{x}y\,(x) = x^3 \\ y(1) = 0 \end{cases}$ with respect to the parameter k.

4. Find parameters a and b such that the solution of the IVP $\begin{cases} y'\,(x) = ay\,(x) + b \\ y(0) = 1 \end{cases}$ passes through the points $(2, 2e^2 - 1)$ and $(3, 2e^3 - 1)$. Plot the solution graph.

5. Find the solution of the IVP

$$\begin{cases} y'' - y' - 2y = 0 \\ y(0) = a \\ y'(0) = 2 \end{cases}$$

   and the parameter a such that $y(x) \to 0$ as $x \to +\infty$.