Seminar 3 - 2025

Exercise 1
The time, in minutes, it takes to reboot a certain system is a continuous variable with the density function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} c(4-x)^2, & \text{if } 0 < x < 4 \\ 0, & \text{otherwise} \end{cases}$$

a. Compute the constant c.

b. Compute the probability that the system takes between 1 and 2 minutes to reboot.

c. Compute the probability that the system takes at least 1 minute to reboot.

Exercise 2
The time to failure T, in hours of operating time, of a television set subject to random voltage surges has exponential $\operatorname{Exp}\left(\frac{1}{500}\right)$ distribution.

a. Compute the cumulative distribution function of T.

b. Compute the probability that the unit operates successfully more than 400 hours.

c. Suppose the unit has operated successfully for 400 hours. What is the (conditional) probability it will operate for another 500 hours?

$$T \sim \operatorname{Exp}\left(\frac{1}{500}\right) \Longleftrightarrow f_T(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ \frac{1}{500} e^{-\frac{t}{500}}, & \text{if } t > 0. \end{cases}$$

Exercise 3
The length of time for one individual to be served at a cafeteria is a random variable T having an exponential distribution with a mean of 5 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?

$$T \sim \operatorname{Exp}(\lambda) \Longleftrightarrow f_T(t) = \begin{cases} 0, & \text{if } t \leq 0 \\ \lambda e^{-t\lambda}, & \text{if } t > 0. \end{cases}$$

Exercise 4
It is known that the density function of the random time T (in minutes) in which a device performs a certain task is $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \begin{cases} a(b-x), & \text{if } x \in (0,b) \\ 0, & \text{otherwise,} \end{cases}$ where $a, b \in \mathbb{R}$ are parameters. Compute a and b if the expected time to complete the task is 1 minute.

Exercise 5
A factory produces graphite-core wood pencils with the standard length of 190 mm and an error (in mm) which follows the normal distribution $N(1, 0.5)$. Compute the expectation and the variance of the length of a pencil. $X \sim N\left(\mu, \sigma^2\right)$, then the density function of X is $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, x \in \mathbb{R}$.