

Lab 11

Direct methods for solving linear systems

Gauss method with partial pivoting

Consider the linear system $Ax = b$, with $A = (a(i, j))_{i,j=1,n}$ and $b = (b(1), \dots, b(n))'$.

Algorithm:

Input: n -order of the system; A -matrix of coefficients; b -vector of free terms;

Output: x -vector of the solutions or a message in case of incompatibility of the system

1. For $p = 1, \dots, n - 1$
 - Let $abs(a(q, p)) = \max(abs(a(p : n, p)))$.
 - If $a(q, p) = 0$ then "Message"; Exit
 - If $q \neq p$ interchange the lines p and q from A and b .
 - Perform the necessary operations for obtaining zeros on the column p , below $a(p, p)$.
 - Apply the transformations also to the vector b .
2. If $a(n, n) = 0$ then "Message"; Exit
3. For $i = n : -1 : 1$ do
 - Compute $x(i)$.
4. Display x .

Problems:

1. Implement Gauss method for solving linear systems, using partial elimination. Solve the following system of equations:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 1 & 5 \\ -1 & 1 & -5 & 3 \\ 3 & 1 & 7 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 31 \\ -2 \\ 18 \end{bmatrix}.$$

2. Find LU decomposition of the following matrix using Doolittle method.

$$A = \begin{bmatrix} 6 & 2 & 1 & -1 \\ 2 & 4 & 1 & 0 \\ 1 & 1 & 4 & -1 \\ -1 & 0 & -1 & 3 \end{bmatrix}.$$

For $b = \begin{bmatrix} 8 \\ 7 \\ 5 \\ 1 \end{bmatrix}$, solve the system $Ax = b$.