

Seminar 2 - 2025

1. Consider the English alphabet with 21 consonants and 5 vowels (lowercase letters). In how many ways can we choose 6 letters such that we get 4 distinct consonants and 2 distinct vowels, if: a) the letters are not ordered; b) the letters are ordered? Examples: a) {s,e,a,r,c,h}; b) (a,r,c,h,e,s), (c,a,s,h,e,r), (c,h,a,s,e,r).

A: Applying the basic principle of counting, we get: a) $C_{21}^4 \cdot C_5^2$; b) $A_{21}^4 \cdot A_5^2 \cdot C_6^4$.

2. Two numbers are obtained by rolling two dice. Compute the probabilities of the following events:

a) A: "the numbers are equal".

b) B: "the sum of the numbers is even".

c) C: "the sum of the numbers is at most equal to 10."

A: a) $P(A) = \frac{6}{36} = \frac{1}{6}$; b) $P(B) = \frac{3^2+3^2}{36} = \frac{1}{2}$; c) $P(C) = 1 - \frac{3}{36} = \frac{11}{12}$.

3. For this problem, we assume that the birthday of every person falls equally likely on any month of the year (i.e., the probability that a person was born in a certain month is $\frac{1}{12}$). Compute the probability that

a) in a group of 5 persons there are at least 2 persons that celebrate their birthdays in the same month?

b) in a group of 5 persons all the birthdays fall in at most two months?

A: a) $1 - \frac{A_{12}^5}{12^5} \approx 62\%$. b) $\frac{C_{12}^1 + C_{12}^2(2^5 - 2)}{12^5}$, where C_{12}^1 is number of cases when all the persons are born in the same month, C_{12}^2 is the number of ways to choose the two months for the cases when all the birthdays fall in exactly 2 months, while $2^5 - 2$ is the number of surjective functions from the persons to chosen 2 months.

4. For this problem, we assume that the birthday of every person falls equally likely on any day of the year 2003 (i.e., the probability that a person was born in a certain day is $\frac{1}{365}$). Which is the minimum number n such that the probability of the event "at least two persons share a birthday in an arbitrary group of n persons born in 2003" is at least 50%?

A: The probability that at least two out of n share a birthday is $1 - \frac{A_{365}^n}{365^n} = 1 - \frac{365}{365} \cdot \frac{365-1}{365} \cdot \dots \cdot \frac{365-(n-1)}{365}$, if $2 \leq n \leq 365$. The minimum number n such that $1 - \frac{364}{365} \cdot \dots \cdot \frac{365-(n-1)}{365} \geq 0.5$ can be found by running the following script in Python:

```
n = 2
Q = 364 / 365

while Q >= 0.5:
    n += 1
    Q *= (365 - (n - 1)) / 365

print(n)
```

The answer is 23.

5. A person sends 10 memes by choosing for each meme a recipient from a list of 20 friends. Compute the probability that the first friend in the list receives exactly 5 memes?

A: $\frac{C_{10}^5 19^5}{20^{10}}$, where 20^{10} is the number of possible cases, which is the number of functions from the memes to friends, C_{10}^5 is the number of ways to choose the 5 emails for the first friend and 19^5 is the number of functions from the remaining memes to the remaining friends.

6. 5 balls numbered from 1 to 5 are randomly placed on a line. Compute the probability that:

a) the first and the last balls have even numbers;

b) the first two balls have odd numbers;

c) the balls with even numbers are next to each other;

d) at least two balls that are placed next to each other have the same parity.

A: a) $\frac{2! \cdot 3!}{5!} = \frac{1}{10}$. b) $\frac{A_3^2 \cdot 3!}{5!} = \frac{3}{10}$. c) $\frac{2 \cdot 4 \cdot 3!}{5!} = \frac{2}{5}$. d) $1 - \frac{2! \cdot 3!}{5!} = \frac{9}{10}$.

7. In how many ways can we split the following marbles among 3 persons:

a) 1 red marble, 1 blue marble, 1 green marble, 1 yellow marble and 1 orange marble?

b) 5 red marbles;

c) 5 red marbles and 3 blue marbles;

d) 5 red marbles, 3 blue marbles and 4 green marbles.

A: a) 3^5 (number of functions from the set of 5 marbles to the set of 3 persons); we apply the result of Problem 5 b) and the basic principle of counting to get: b) $C_{5+3-1}^{3-1} = C_7^2$; c) $C_{3+5-1}^{3-1} \cdot C_{3+3-1}^{3-1} = C_7^2 \cdot C_5^2$; d) $C_{3+5-1}^{3-1} \cdot C_{3+3-1}^{3-1} \cdot C_{3+4-1}^{3-1} = C_7^2 \cdot C_5^2 \cdot C_6^2$.