Laboratory 6: Equilibrium points. Stability

Exerciţiul 1 *Find the equilibrium solutions for each of the given autonomous equations. Study their stability by the graphical method and by the stability method in the first approximation:*

(a) $x'(t) = x^2(t) - 2x(t)$

(b) $x'(t) = x(t) \cdot (x(t) - 1) \cdot (x(t) - 2)$

(c) $x'(t) = \sin(x(t))$ *(Hint: to find all the solution of the equation* $\sin(x) = 0$ *use the command _EnvAllSolutions := true:)*

Exerciţiul 2 *Draw the phase portrait of the following linear systems and specify the stability of the origin* $(0; 0)$

(a) $\begin{cases} x'(t) = 2x(t) + y(t) \\ y'(t) = x(t) + 2y(t) \end{cases}$

(b) $\begin{cases} x'(t) = -3x(t) + 4y(t) \\ y'(t) = -2x(t) + 3y(t) \end{cases}$

(c) $\begin{cases} x'(t) = -x(t) - y(t) \\ y'(t) = x(t) - 3y(t) \end{cases}$

(d) $\begin{cases} x'(t) = -2x(t) \\ y'(t) = -4x(t) - 2y(t) \end{cases}$

(e) $\begin{cases} x'(t) = x(t) + 4y(t) \\ y'(t) = x(t) + y(t) \end{cases}$

(f) $\begin{cases} x'(t) = 2x(t) - y(t) \\ y'(t) = x(t) + 2y(t) \end{cases}$

(g) $\begin{cases} x'(t) = -y(t) \\ y'(t) = x(t) \end{cases}$

(h) $\begin{cases} x'(t) = x(t) - 4y(t) \\ y'(t) = 5x(t) - 3y(t) \end{cases}$

Exerciţiul 3 *Find the equilibrium points of the following nonlinear systems and study their stability. Draw in each case the corresponding phase portrait:*

(a) $\begin{cases} x'(t) = y(t) \\ y'(t) = x(t) \cdot (1 - x^2(t)) + y(t) \end{cases}$

(b) $\begin{cases} x'(t) = -2x(t) + y(t) + 2 \\ y'(t) = x(t) \cdot y(t) \end{cases}$

(c) $\begin{cases} x'(t) = y^2(t) \\ y'(t) = x(t) \end{cases}$

(d) $\begin{cases} x'(t) = x^2(t) - y^2(t) \\ y'(t) = x(t) \cdot y(t) - 1 \end{cases}$

Exerciţiul 4 *Let' consider the following prey-predator system:*

$$\begin{cases} x'(t) = 2 \cdot x(t) - 1.2 \cdot x(t)y(t) \\ y'(t) = -y(t) + 0.9 \cdot x(t)y(t) \\ x(0) = 0.5 \\ y(0) = 2 \end{cases}$$

(a) Draw the IVP solution;

(b) Find the equilibrium points and study their stability.

(c) Draw the system phase portrait.