## Lab 13

## Numerical methods for solving nonlinear equations

1. Consider $f(x) = xe^x - 1 = 0$ with solution $\alpha = 0.5671432$. Let the first fixed point iteration be $x_{k+1} = e^{-x_k}$, the second $x_{k+1} = \frac{1+x_k}{e^{x_k}+1}$ and the third $x_{k+1} = x_k + 1 - x_k e^{x_k}$. Consider $x_0 = 0.5$ and $\varepsilon = 10^{-10}$. For each case, find the aproximation and the number of iterations needed to reach the required precision (Use *format long*).

2. Solve the equation
$$x = cosx.$$
using Newton's method for: $x_0 = \frac{\pi}{4}$, $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$.

3. For finding the position of a satellite for $t = 9$ minutes, we have to solve Kepler's equation
$$f(E) = E - 0.8 \sin E - \frac{2\pi}{10} = 0.$$
Type the results obtained applying Newton's method 6 times, starting with $E = 1$. (Notice the quadratic precision.)

4. Use the secant method with $x_0 = 1$ and $x_1 = 2$ to solve $x^3 - x^2 - 1 = 0$, with $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$.

5. Let $f : [1, 2] \to \mathbb{R}$, $f(x) = (x - 2)^2 - \ln x$. Solve the equation $f(x) = 0$, using bisection and false position methods, for $\varepsilon = 10^{-4}$ and maximum number of iterations $N = 100$. (Use $abs(f(c) < \varepsilon$ as a stopping criterion.)

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(*Facultative*) **1.** The function $f(x) = x - 0.2sinx - 0.5$ has exactly one zero between $x_0 = 0.5$ and $x_1 = 1$, since $f(0.5)f(1) < 0$, while $f'(x)$ does not vanish on $[0.5, 1]$. Locate the zero correct to six significant digits using Newton's, secant and bisection methods. Type how many steps each method would require to produce six significant digits and compute the errors at each step. (Exact solution is 0.61546850).

**2.** Check the performances of Newton's method in two versions:

$$\text{standard: } x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$\text{root of multiplicity } m\text{: } x_{k+1} = x_k - m\frac{f(x_k)}{f'(x_k)}$$

to approximate the multiple zero $\alpha = 1$ of the function $f(x) = (x^2-1)^p \log x$ (for $p \geq 1$ and $x > 0$). The desired root has multiplicity $m = p+1$. Consider the value $p = 2$ and $x_0 = 0.8$, $\varepsilon = 10^{-10}$. Type the number of iterations required to converge for each case.