Seminar 8 - 2025

Exercise 1

1. Explain the difference between point estimation and interval estimation.
2. Let $Z \sim N(0,1)$. Define z_α in probabilistic terms.
3. Explain the meaning of:
$$\mathbb{P}(-z_{1-\alpha/2} \leq Z \leq z_{1-\alpha/2}) = 1 - \alpha.$$
4. Interpret "95% confidence" in plain words.

Exercise 2

A machine produces rods with normally distributed length. Population standard deviation is $\sigma = 5$ mm. A sample of $n = 100$ rods produces mean $\bar{x} = 52$ mm.

1. Construct a 95% confidence interval for μ.
2. Construct a 99% confidence interval.
3. Explain the effect of confidence level on interval width.
4. Explain why μ is not random but the interval is.

Exercise 3

Observed lifetimes (in years) of a device:

$$32,\ 30,\ 34,\ 31,\ 29,\ 35,\ 33$$

1. Compute $\bar{x}$ and s.
2. Construct a 95% confidence interval for μ.
3. Explain why $T(n-1)$ is used instead of $N(0,1)$.

Exercise 4

A manufacturer wants a CI for the mean with maximum length $\Delta = 1$, known standard deviation $\sigma = 4$, confidence 95%.

1. Derive and compute required n.
2. Explain how sample size depends on σ and confidence.

Exercise 5

Let $X_1, \ldots, X_n \sim \text{Exponential}(\lambda)$.

1. Derive a method of moments estimator.
2. Compute the maximum likelihood estimator.
3. Compare the two.
4. Show whether the MLE is unbiased.

Exercise 6

Let $X \sim N(\mu, \sigma^2)$, with σ known.

1. Compute the Fisher information for μ.
2. Use the Cramér–Rao theorem to derive the lower bound.
3. Show that $\bar{X}$ is efficient.
4. Explain the meaning of "efficiency".