Laboratory 1 - Exercises

1. Evaluate:

   (a) $12 + 4 - 5$

   (b) 2^{10}

   (c) $\sin 0.1$

   (d) $(a + b)(a - b)$ (use the command **expand()**)

2. Find the derivatives of the functions:

   (a) $y(x) = 3x^3 + 2x^2 - 5$

   (b) $y(x) = \sqrt{1 + x^4}$

   (c) $y(x) = e^x \sin(x) \cos(x)$

3. Calculate the integrals:

   (a) $\int_0^1 \left(3x^3 + 2x^2 - 5\right) dx$

   (b) $\int_0^\infty \frac{1}{x^2} dx$

   (c) $\int_{-\infty}^{\infty} e^{-x^2} dx$

4. Calculate the limits:

   (a) $\lim_{x \to 0} \frac{\sin x}{x}$

   (b) $\lim_{x \to \infty} \frac{x^3 + 3x^2 - 5}{2x^3 - 7x}$

   (c) $\lim_{x \to \pi} \frac{\cos x + 1}{x - \pi}$

5. Draw the graphs of the functions:

   (a) $f(x) = e^{-x} - 1, x \in [-2; 2]$

   (b) $f(x) = \frac{200 \cdot e^{r \cdot x}}{2\left(e^{r \cdot x} - 1\right) + 100}$, $x \in [0; 50]$, pentru $r = 0.5$ şi $r = -0.5$

   (c) $f(x) = x \cdot \sin\left(\frac{1}{x}\right)$, $x \in [-3; 3]$

6. Draw the graphs of the following curves given in parametric form:

   (a) $\begin{cases} x(t) &= (1 - \cos t)\cos t \\ y(t) &= (1 - \cos t)\sin t \end{cases}$, $t \in [0; 2\pi]$ (Cardioid)

   (b) $\begin{cases} x(t) &= \sin(3t)\cos t \\ y(t) &= \sin(3t)\sin t \end{cases}$, $t \in [0; 2\pi]$

   (c) $\begin{cases} x(t) &= t - \sin t \\ y(t) &= 1 - \cos t \end{cases}$, $t \in [0; 6\pi]$ (Cycloid)

7. Let's consider the function:

$$f(t,s) = 1 - \frac{s \cdot \cos(4t) \cdot \cos(t)}{\sqrt{1 - s^2 \cdot \cos^2(4t) \cdot \sin^2(t)}}$$

and the curve given in parametric form

$$\begin{cases} x(t) &= f\left(t - \frac{\pi}{2}, s\right) \\ y(t) &= f(t,s) \end{cases}, t \in [0; 2\pi]$$

(a) Draw the curve for $s = 0.5$;

(b) Draw in the same window the graphs of the curves corresponding to the values $s = 0.1$, 0.2, ..., 1.

8. Draw the graphs of the following curves given in implicit form:

(a) $x^2 + y^2 - 2x - 4y + 4 = 0$

(b) $x^2 - 2xy - y^2 = 1$

9. Draw the surfaces:

(a) $z(x,y) = 4x^2 e^y - 2x^4 - e^{4y},\ -3 \le x \le 3,\ -1 \le y \le 1$

(b) $z(x,y) = 4x^2 - y^2,\ \ -100 \le x \le 100,\ -100 \le y \le 100$

10. Solve the following equations:

(a) $x^4 - \frac{3}{2}x^3 - x + \frac{3}{2} = 0$

(b) $\sqrt{x^2 + 2x} = 3 - x$

(c) $\begin{cases} 2x + y = & 3 \\ x + 3y = & -1 \end{cases}$

(d) $\begin{cases} 2x + xy + 2y = & 59 \\ 3x - 2xy + 3y = & -34 \end{cases}$

11. Load the linear algebra package (with(linalg):). Let

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix},\ \ B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix},\ \ C = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 4 & 2 & 2 \end{pmatrix}$$

Calculate:

(a) $2A - BC$

(b) B^{-1}

(c) eigenvalues and eigenvectors of the matrix C.