Lab 10

Conditioning number of a linear system

1. a) Solve the following linear system:
$$\begin{bmatrix} 400 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}.$$

b) Solve the linear system obtained by changing an element of the coefficient matrix:
$$\begin{bmatrix} 401 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}.$$

c) Find the conditioning number. (Use Matlab function *cond.*)

2. Consider the system:
$$Ax = b,$$

with
$$A = \begin{bmatrix} 10 & 7 & 8 & 7 \\ 7 & 5 & 6 & 5 \\ 8 & 6 & 10 & 9 \\ 7 & 5 & 9 & 10 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 32 \\ 23 \\ 33 \\ 31 \end{bmatrix}.$$

a) Solve this system and find its conditioning number. (Use Matlab function *cond.*)

b) Solve the system
$$A\tilde{x} = \tilde{b},$$

where
$$\tilde{b} = \begin{bmatrix} 32.1 \\ 22.9 \\ 33.1 \\ 30.9 \end{bmatrix}.$$

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Compute the input relative error $\frac{\|b - \tilde{b}\|}{\|b\|}$ and the output relative error $\frac{\|x - \tilde{x}\|}{\|x\|}$.

Compute $\frac{\|x - \tilde{x}\|}{\|x\|} \Big/ \frac{\|b - \tilde{b}\|}{\|b\|}$

c) Solve the system

$$\bar{A}\bar{x} = b,$$

with

$$\bar{A} = \begin{bmatrix} 10 & 7 & 8.1 & 7.2 \\ 7.08 & 5.04 & 6 & 5 \\ 8 & 5.98 & 9.89 & 9 \\ 6.99 & 4.99 & 9 & 9.98 \end{bmatrix}.$$

Compute the input relative error $\frac{\|A - \bar{A}\|}{\|A\|}$ and the output relative error $\frac{\|x - \bar{x}\|}{\|x\|}$. Compute $\frac{\|x - \bar{x}\|}{\|x\|} \Big/ \frac{\|A - \bar{A}\|}{\|A\|}$.

3. Find the conditioning numbers of the Hilbert matrices $H_n = (h_{ij})_{\substack{1 \leq i \leq n \\ 1 \leq j \leq n}}$, with $h_{ij} = \frac{1}{i+j-1}$, for $n = 10 : 15$.

4. a) Find the conditioning numbers of the Vandermonde matrices $V(t_k)$ for the points $t_k = \frac{1}{k}$, $k = \overline{1, n}$, for $n = \overline{10, 15}$.

b) Find the conditioning numbers of the Vandermonde matrices $V(t_k)$ for the points $t_k = -1 + \frac{2}{n}k \in [-1, 1]$, with $k = \overline{1, n}$, for $n = \overline{10, 15}$.