Seminar 3 - 2025

Exercise 1
A sample $(X_1, \ldots, X_{10})$ is drawn from a distribution with a probability density function

$$f(x) = \frac{1}{2}\left(\frac{1}{\theta}e^{-x/\theta} + \frac{1}{10}e^{-x/10}\right), \quad 0 < x < \infty$$

The sum of all 10 observations equals 150. Estimate θ by the method of moments.

Exercise 2
Given the following independent observations (X_i, Y_i), $i = 1, \ldots, 5$,

$$X_1 = 0.5, X_2 = 1.2, X_3 = 0.9, X_4 = 0.7, X_5 = 1.5$$
$$Y_1 = 2, Y_2 = 1.8, Y_3 = 2.5, Y_4 = 1.2, Y_5 = 2.1$$

with exponential distribution and parameter vector (θ_1, θ_2), $\theta_1 > 0$, $\theta_2 > 0$, find the 2-dimensional estimator using the MLE. Note that the pdf is given by

$$f(x) = \theta e^{-\theta x}. \tag{1}$$

Exercise 3 Consider the example on slides 111-113 in Lecture 3. Explain why

$$\theta_{\text{post}} = \frac{\sigma_{\text{data}}^2 \cdot \mathbb{E}[\theta] + n\sigma_{\text{prior}}^2 \cdot \bar{x}}{n\sigma_{\text{prior}}^2 + \sigma_{\text{data}}^2}.$$