Direct and iterative methods for solving linear systems

A

Consider the system $A\mathbf{x} = b$, where

$$A = \begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 2 & 1 & -1 \\ 2 & -2 & -2 & 9 \\ 3 & -4 & 4 & 15 \end{pmatrix}, \quad b = \begin{pmatrix} -1 \\ 3 \\ -11 \\ -4 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

Solve the system using:

1. Gauss elimination with **partial pivoting**;
2. LU factorization;
3. 2 iterations of Jacobi method with $\mathbf{x}^{(0)} = (1\ 1\ 1\ 1)^T$;
4. 2 iterations of Gauss-Seidel method with $\mathbf{x}^{(0)} = (1\ 1\ 1\ 1)^T$.

B

Consider the system $A\mathbf{x} = b$, where

$$A = \begin{pmatrix} -3 & 1 & 0 & 2 \\ -3 & 2 & 5 & 3 \\ -6 & 2 & -1 & 7 \\ -12 & 6 & 9 & 17 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 11 \\ 2 \\ 24 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

Solve the system using:

1. Gauss elimination with **partial pivoting**;
2. LU factorization;
3. 2 iterations of Jacobi method with $\mathbf{x}^{(0)} = (0\ 0\ 0\ 0)^T$;
4. 2 iterations of Gauss-Seidel method with $\mathbf{x}^{(0)} = (0\ 0\ 0\ 0)^T$.