Cubic splines. Least squares approximation

A

1. Check if the following function is a natural cubic spline on the interval $[-1,1]$:

$$S(x) = \begin{cases} x^3+3x^2+7x-5, & x \in [-1,0] \\ -x^3+3x^2+7x-5, & x \in [0,1] \end{cases}.$$

2. A clamped cubic spline S for a function f is defined by

$$S(x) = \begin{cases} 1+ax+2x^2-2x^3, & x \in [0,1] \\ 1+b(x-1)-4(x-1)^2+7(x-1)^3, & x \in [1,2] \end{cases}.$$

   Determine $f'(0)$ and $f'(2)$.

3. Determine a constant function, a line and a quadratic polynomial that best fit the data:

x	1	2	3	4
y	0	1	1	2

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B

1. Check if the following function is a natural cubic spline on the interval $[0,2]$:

$$S(x) = \begin{cases} x^3+x-1, & x \in [0,1] \\ -(x-1)^3+3(x-1)^2+4(x-1)+1, & x \in [1,2] \end{cases}.$$

2. A natural cubic spline S is defined by

$$S(x) = \begin{cases} 1+a(x-1)-b(x-1)^3, & x \in [1,2] \\ 1+c(x-2)-\frac{3}{4}(x-2)^2+d(x-2)^3, & x \in [2,3] \end{cases}.$$

   If S interpolates the data $(1,1),\ (2,1),\ (3,0)$, find $a,\ b,\ c,\ d$.

3. Determine a constant function, a line and a quadratic polynomial that best fit the data:

x	1	2	3	4
y	7	4	3	12

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