Turing Machines

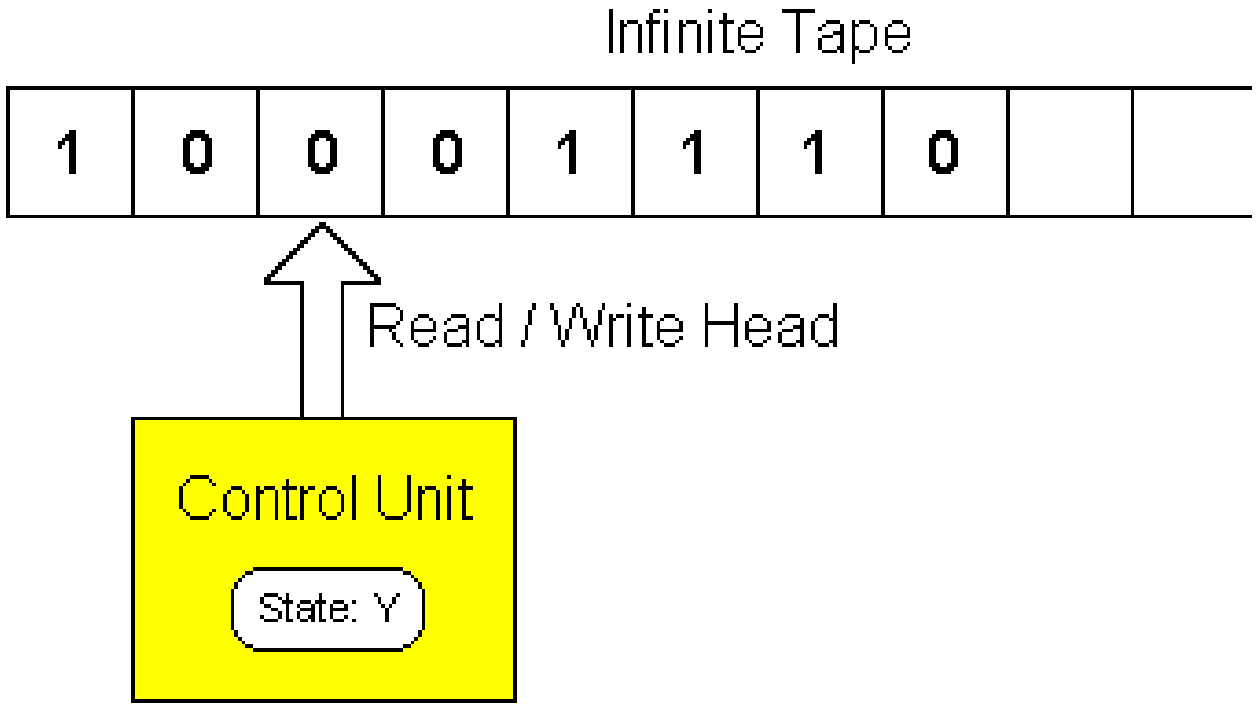
Alan Turing

- Enigma (criptography)
- Turing test
- Turing machine (1937)

Turing Machine

- Mathematical model for computation
- Abstract machine
- Can simulate any algorithm

Turing Machine



- Input band (infinite) divided into cells
- Reading head
- Control Unit: states
- Transitions / moves

Turing machine – definition

7-tuple $M = (Q, \Gamma, b, \Sigma, \delta, q_0, F)$ where:

- Q – finite set of states
- Γ - alphabet (finite set of band symbols)
- $b \in \Gamma$ - blank (symbol)
- $\Sigma \subseteq \Gamma \setminus \{b\}$ – input alphabet
- $\delta : (Q \setminus F) \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\}$ –transition function
- $q_0 \in Q$ – initial state
- $F \subseteq Q$ – set of final states

L = left
R = right

Example – palindrome over {0,1}

- 001100, 00100, 101101 a.s.o. accepted
- 00110, 1011 a.s.o. not accepted

0011<u>0</u><u>0</u>

Example – palindrome over {0,1}

	0	1	b
q_0	(p_1,b,R)	(p_2,b,R)	(q_f,b,R)
p_1	$(p_1,0,R)$	$(p_1,1,R)$	(q_1,b,L)
p_2	$(p_2,0,R)$	$(p_2,1,R)$	(q_2,b,L)
q_1	(q_r,b,L)		(q_f,b,R)
q_2		(q_r,b,L)	(q_f,b,R)
q_r	$(q_r,0,L)$	$(q_r,1,L)$	(q_0,b,R)
q_f			

Delete 0 in left side; search 0 in right side

Delete 1 in left side; search 1 in right side

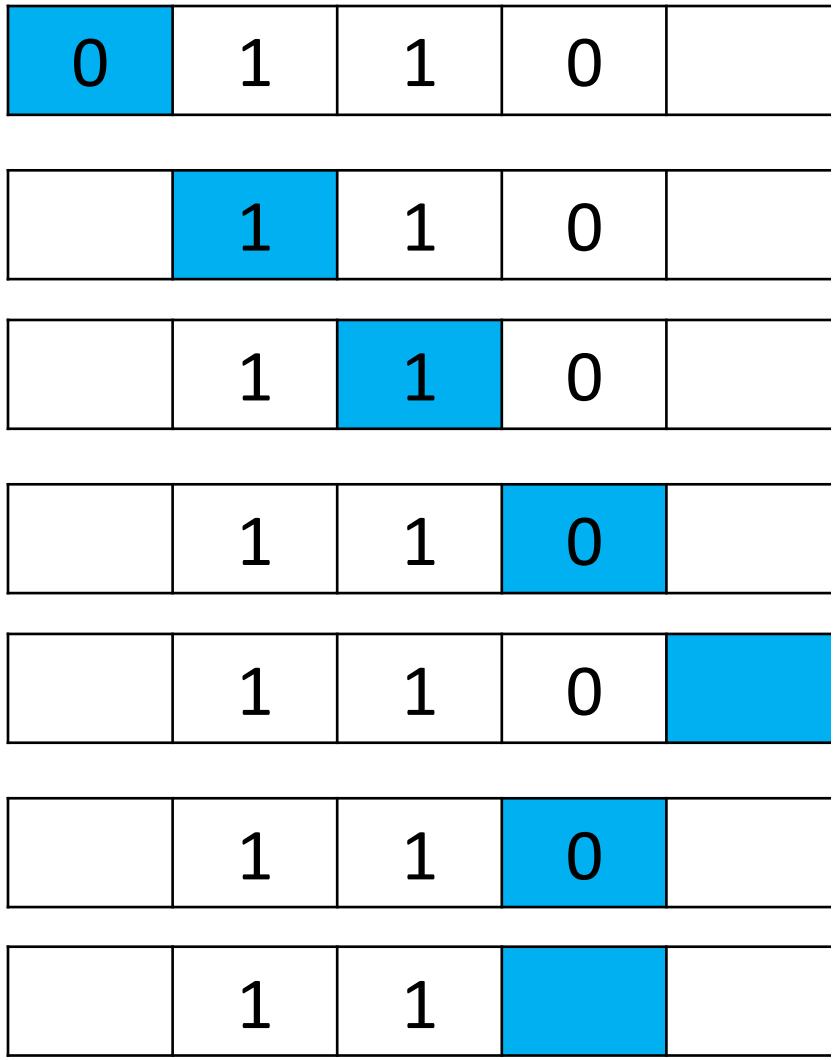
On right is 0 or 1?

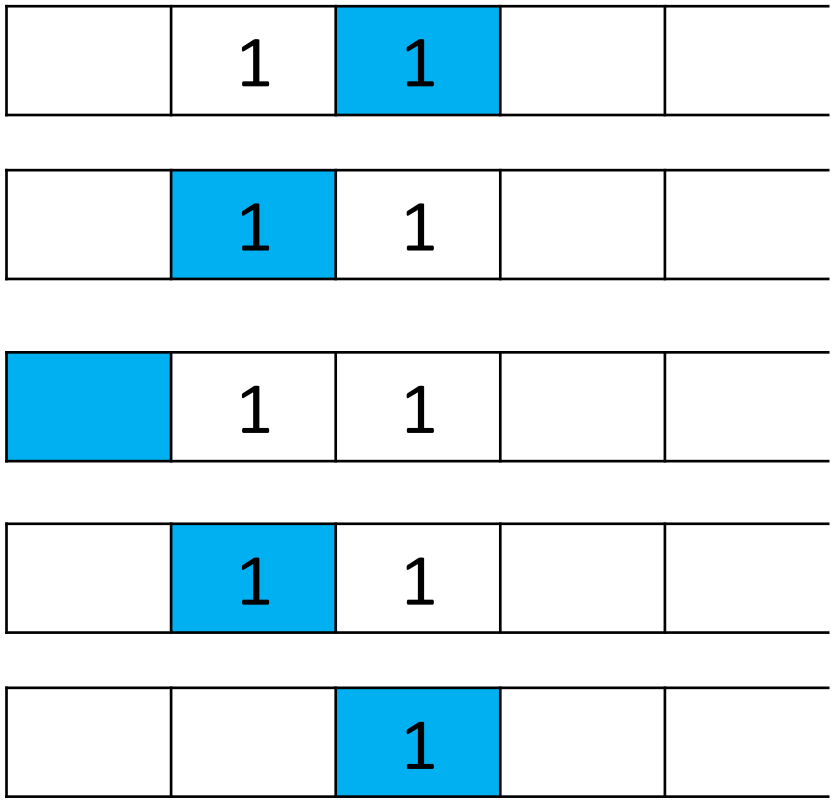
Shift right

q_1 and q_2 – process 0 and 1 on the right

qf –final state

0110





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$(q_0,\underline{0}110) \vdash (p_1, \underline{1}10) \vdash (p_1, 1\underline{1}0)$

$\vdash (p_1, 11\underline{0}) \vdash (p_1, 110\underline{b}) \vdash (q_1, 11\underline{0})$

$\vdash (q_r, 1\underline{1}) \vdash (q_r, \underline{1}1) \vdash (q_r, \underline{b}11)$

$\vdash (q_0, \underline{1}1) \vdash \ldots$

	0	1	b
q_0	(p_1,b,R)	(p_2,b,R)	(q_f,b,R)
p_1	$(p_1,0,R)$	$(p_1,1,R)$	(q_1,b,L)
p_2	$(p_2,0,R)$	$(p_2,1,R)$	(q_2,b,L)
q_1	(q_r,b,L)		(q_f,b,R)
q_2		(q_r,b,L)	(q_f,b,R)
q_r	$(q_r,0,L)$	$(q_r,1,L)$	(q_0,b,R)
q_f			

https://turingmachinesimulator.com

Finite Automata & Turing Machine

- Simple models for computation
- Input band & input alphabet
- Q – finite number of states
- Transition function – determined by state & symbol

Finite Automata vs Turing Machine

- Read from input band
- Reading head - move to the right
- Finite tape – sequence
- Accept: yes/no

- Read and write on input band
- Reading head - move to the right or left
- Infinite tape
- Also compute