Seminar 7

1. Let $n \in \mathbb{N}$, $n \geq 2$. Prove the group isomorphism

$$(GL_n(\mathbb{R})/SL_n(\mathbb{R}), \cdot) \simeq (\mathbb{R}^*, \cdot)$$

by using the first isomorphism theorem.

2. Prove the group isomorphism

$$(\mathbb{C}/\mathbb{R}, +) \simeq (\mathbb{R}, +)$$

by using the first isomorphism theorem.

3. Let $m, n \in \mathbb{N}$ be such that $(m, n) = 1$. Prove the group isomorphism

$$(\mathbb{Z}_{mn}, +) \simeq (\mathbb{Z}_m \times \mathbb{Z}_n, +).$$

4. Determine the subgroups and the factor groups of the group $(\mathbb{Z}_{12}, +)$ by using the third isomorphism theorem.

5. Compute the composition (product) of the following permutations of 4 elements, and then determine the signature and the inverse of the result:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 3 & 1 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

6. Determine the orbits of each element of the set $\{1, 2, 3, 4, 5\}$ relative to the permutation:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}.$$

7. Decompose into products of disjoint cycles and into products of transpositions the following permutations:

$$\sigma_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 6 & 1 & 5 & 7 & 3 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 4 & 7 & 2 & 5 & 1 & 8 & 9 & 3 \end{pmatrix}.$$

8. Determine the order of each element and the cyclic subgroups of the group $(S_3, \circ)$.