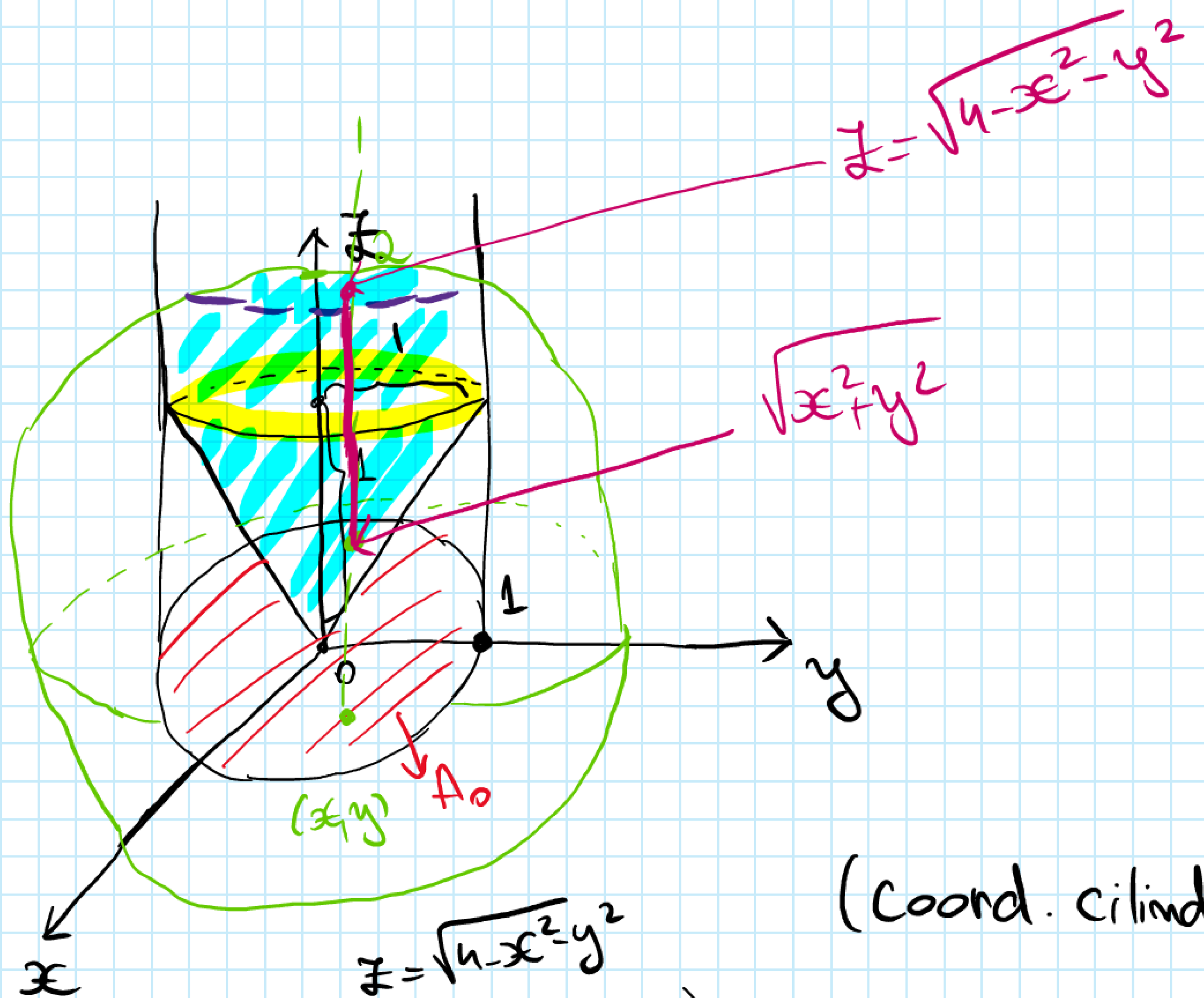
Seminar 14- Calculus 2 05. 06. 2025

Monday, April 14, 2025 4:47 PM

[2] Find the volume and the center of mass of the homogeneous Solid lying inside the cone $z=\sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2=1$ and inside the sphere $x^2+y^2+z^2=4$



(Coord. cilindrice / Fubini)

$$V=\iiint_A dx\,dy\,dz=\iint_{A_0}\left(\int_{z=\sqrt{x^2+y^2}}^{z=\sqrt{4-x^2-y^2}} dz\right)dx\,dy$$

$$=\iint_{A_0}\left(\sqrt{4-x^2-y^2}-\sqrt{x^2+y^2}\right)dx\,dy=$$

$\sqrt{4-\rho^2}=t$

$4-\rho^2=t^2$

$-\cancel{2}\rho\,d\rho=\cancel{2}t\,dt$

$x=\rho\cos\theta \qquad \rho\in[0;1]$

$y=\rho\sin\theta \qquad \theta\in[0;2\pi]$

$$V=\int_0^1\int_0^{2\pi}\left(\sqrt{4-\rho^2}-\rho\right)\cdot\rho\,d\rho\,d\theta=\left(\int_0^1\rho\cdot\sqrt{4-\rho^2}\,d\rho-\underbrace{\int_0^1\rho^2\,d\rho}_{\left.\frac{\rho^3}{3}\right|_0^1}\right)\cdot\int_0^{2\pi}d\theta$$

$$=2\pi\cdot\left(-\int_2^{\sqrt3}t^2\,dt-\frac{1}{3}\right)$$

$$=2\pi\left(\frac{8-3\sqrt3-1}{3}\right)$$

$G\in Oz \Rightarrow x_G=y_G=0$

$$z_G=\frac{\iiint_A z\,dx\,dy\,dz}{V}$$

$$\iiint_A z\,dx\,dy\,dz=\iint_{A_0}\left(\int_{z=\sqrt{x^2+y^2}}^{z=\sqrt{4-x^2-y^2}} z\,dz\right)dx\,dy=\iint_{A_0}\left(\frac{4-x^2-y^2-x^2-y^2}{2}\right)dx\,dy$$

$$=\iint_{A_0}\frac{4-2(x^2+y^2)}{2}=\int_0^1\int_0^{2\pi}(2-\rho^2)\rho\,d\rho\,d\theta=2\pi\cdot\int_0^1 2\rho-\rho^3\,d\rho$$

$$=2\pi\cdot\left(1-\frac{1}{4}\right)=2\pi\cdot\frac{3}{4}=\frac{3\pi}{2}$$

$$z_G=\frac{3\cancel{\pi}}{2}\cdot\frac{3}{2\cancel{\pi}(7-3\sqrt3)}=\frac{9}{28-12\sqrt3}$$

[3]