Laboratory 5: Higher order linear differential equations

Exercise 1 *Check if the specified functions are solutions for the given differential equation:*

(a) $xy'' - (x+1)\,y' - 2\,(x-1)\,y = 0$, $\varphi_1(x) = e^{2x}$, $\varphi_2(x) = x^2 + 1$;

(b) $y'' - tg\,(x)\,y' + 2y = 0$, $\varphi_1(x) = \cos(x)$, $\varphi_2(x) = \sin(x)$;

(c) $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$, $\varphi_1(x) = x$, $\varphi_2(x) = x^2$;

(d) $xy'' - (2x+1)\,y' + (x+1)\,y = 2x^2e^x$, $\varphi_1(x) = \frac{2}{3}x^3e^x$, $\varphi_2(x) = x^2e^x$;

Exercise 2 *Find a solution of the specified form for the given differential equation:*

(a) $xy'' - (2x+1)\,y' + 2y = 0$, $\varphi(x) = e^{ax}$;

(b) $y'' + y' - \dfrac{y}{x} = 0$, $\varphi(x) = ax + b$;

(c) $xy'' + 2y' - xy = 0$, $\varphi(x) = \dfrac{e^{ax}}{bx+c}$;

(d) $xy''' - y'' - xy' + y = -x^2$, $\varphi(x) = ax^2 + bx + c$;

Exercise 3 *Show that the specified functions system S is a fundamental system of solutions for the given linear homogeneous differential equation:*

(a) $S = \{x,\ e^x\}$, $(x-1)\,y'' - xy' + y = 0$;

(b) $S = \left\{\dfrac{e^x}{x},\ \dfrac{e^{-x}}{x}\right\}$, $xy'' + 2y' - xy = 0$;

(c) $S = \{x,\ e^x,\ e^{-x}\}$, $xy''' - y'' - xy' + y = 0$;

(d) $S = \left\{x,\ \dfrac{1}{x},\ 2x \cdot \ln(x) + 2\right\}$, $x^2\,(2x-1)\,y''' + (4x-3)\,xy'' - 2xy' + 2y = 0$;

Exercise 4 *Construct the linear homogeneous differential equation for the given fundamental system of solutions S:*

(a) $S = \{\cos(x),\ \sin(x)\}$;

(b) $S = \{e^{2x},\ x+1\}$;

(c) $S = \left\{x,\ x^3,\ \dfrac{1}{x}\right\}$;

(d) $S = \left\{e^x,\ x,\ \dfrac{e^x}{x}\right\}$;

Exercise 5 *Using variation of the constants method, find a particular solution of the following linear nonhomogeneous differential equations knowing that the given S is a fundamental system of solutions*

(a) $(x-1)\,y'' - xy' + y = 3$, $S = \{x,\ e^x\}$;

(b) $(2x+1)\,y'' + 4xy' - 4y = (2x+1)^2$, $S = \{x,\ e^{-2x}\}$;

(c) $xy'' + 2y' - xy = e^x$, $S = \left\{\dfrac{e^x}{x},\ \dfrac{e^{-x}}{x}\right\}$;

(d) $xy''' - y'' - xy' + y = -x^2$, $S = \{x,\ e^x,\ e^{-x}\}$;