Lab 8

Quadrature formulas (1)

Problems

1. a) Approximate the integral

$$I = \int_0^1 f(x)dx, \qquad \text{for } f(x) = \frac{2}{1+x^2},$$

using trapezium formula.

b) Plot the graph of the function f and the graph of the trapezium with vertices $(0,0), (0,f(0)), (1,f(1))$ and $(1,0)$.

c) Approximate the integral I using Simpson's formula.

2. Approximate the following double integral

$$\int_{1.4}^{2}\int_{1}^{1.5} \ln(x+2y)dydx$$

using trapezium formula for double integrals, given in (1). (*Result*: 0.4295545)

3. Evaluate the integral that arises in electrical field theory:

$$H(p,r) = \frac{60r}{r^2-p^2}\int_0^{2\pi}\left[1-\left(\frac{p}{r}\right)^2\sin x\right]^{1/2}dx,$$

for $r = 110$, $p = 75$, using the repeated trapezium formula for two given values of n. (*Result:* 6.3131)

4. Find the smallest value of n that gives an approximation of the integral $\int_1^2 x\ln(x)dx$ which is correct to three decimals, using the repeated trapezium formula. Apply the repeated trapezium formula for the obtained value of n to approximate the integral. (*Result: 0.636294368858383*)

5. Evaluate the integral

$$\int_0^{\pi}\frac{dx}{4+\sin 20x}$$

using the repeated Simpson's formula for $n = 10$ and 30. (*Result: 0.8111579*)

6. The error function $erf(x)$ is defined by

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt.$$

Use the repeated Simpson's formula to evaluate $erf(0.5)$ with $n = 4$ and $n = 10$. Estimate the accuracy of your result and compare with the correct value $erf(0.5) = 0.520499876$.

Trapezium formula for double integral

Applying succesively trapezium formula with respect to y, and with respect to x, we get

$$\int_a^b \int_c^d f(x,y)dydx \approx \frac{(b-a)(d-c)}{16} \left[f(a,c) + f(a,d) + f(b,c) + f(b,d) \right. \quad (1)$$
$$+ 2f\left(\frac{a+b}{2}, c\right) + 2f\left(\frac{a+b}{2}, d\right) + 2f\left(a, \frac{c+d}{2}\right)$$
$$\left. + 2f\left(b, \frac{c+d}{2}\right) + 4f\left(\frac{a+b}{2}, \frac{c+d}{2}\right)\right]$$