Seminar 6

1. Let $n \in \mathbb{N}$, $n \geq 2$ and

$$SL_n(\mathbb{R}) = \{A \in M_n(\mathbb{R}) \mid \det(A) = 1\}\,,$$

$$GL_n(\mathbb{R}) = \{A \in M_n(\mathbb{R}) \mid \det(A) \neq 0\}\,.$$

Show that $SL_n(\mathbb{R})$ is a normal subgroup of the group $(GL_n(\mathbb{R}), \cdot)$.

2. Show that the center

$$Z(G) = \{x \in G \mid x \cdot g = g \cdot x, \forall g \in G\}$$

of a group $(G, \cdot)$ is a normal subgroup.

3. Determine the (normal) subgroups and the factor groups of the group $(\mathbb{Z}, +)$.

4. Determine the (normal) subgroups of the group $(\mathbb{Z}_6, +)$, and draw the Hasse diagram of their lattice. Then determine the factor groups and fill in the operation table for one of them.

5. Determine the (normal) subgroups of Klein's group $(K, \cdot)$, and draw the Hasse diagram of their lattice. Thwn determine the factor groups, and fill in the operation table for one of them.

6. Determine the subgroups and the normal subgroups of the group $(S_3, \circ)$ (compute S_3/r_H and S_3/r'_H for $H \leq S_3$), and draw the Hasse diagrams of their lattices. Then determine the factor groups, and fill in the operation table for one of them.

7. Determine the subgroups and the normal subgroups of the quaternion group $(Q, \cdot)$, and draw the Hasse diagrams of their lattices. Then determine the factor groups, and fill in the operation table for one of them.

8. Show that the rotations from the dihedral group $(D_4, \cdot)$ form a normal subgroup and determine the corresponding factor group.