

Lecture 7

# Relational Algebra

# Query Language for the Relational Model

- for handling and retrieving the data from a database
- the Relational Model offers support for simple and powerful query languages
  - formal and based on logic
  - allow the optimization
- Query Language NOT THE SAME WITH Programming Language
  - are not used for complex calculus
  - offers a simple and efficient way to access big sets of data
  - not expected to be Turing complete
- 2 query languages with a significant influence on SQL
  - **Relational Algebra** – operational and useful for the execution plan (of the queries)
  - **Relational Calculus** – allows the users to describe *what* and *not how* to get what they want (non-operational, declarative)

# Relational Algebra

- used in the DBMSs to represent the query execution plans
- the query is applied to the instance of the relation and the result of the query represent such an instance of the relation
  - the structure of a relation from a query is fixed
  - the structure of the result that is returned by a query is fixed and determined by the definitions of the structured used in the query language
- each operation returns a relation
- the operators used can be composed
- the result returned by an algebra expression is a relation / table and this relation has a set or records / tuples
- in the relational algebra for the multisets, the duplicates are not eliminated
- the relational algebra query:
  - is organized by using a collection of operators
  - describes each step that is performed in computing the result
  - is evaluated on the input instances of the relations considered
  - return an output instance relation

# Relational Algebra

## Conditions that can be used with the Relational Algebra

- all the conditions that are used for the algebraic operators
- are similar with the conditions used in the SELECT
  - attribute\_name relational\_operator value\_expression
  - attribute\_name IS [NOT] IN column\_relation
    - this condition tests if a value belongs to a set of values
  - relation IS [NOT] IN | = | <> relation
    - the relations must be compatible
- condition
- NOT condition
- condition1 AND | OR condition2

# Relational Algebra

## Operators from Relational Algebra

- The SELECT statements can be also specified by using the expressions from relational algebra

### *Basic operations*

- **projection** ( $\pi$ ) – eliminates all the un-wished attributes from the relation
- **selection** ( $\sigma$ ) – selects a subset of tuples from the relation
- **cross product** ( $\times$ ) – allows the combination of two relations
- **set-difference** ( $-$ ) – selects the tuples that are in a relation and not in the other relation
- **union** ( $\cup$ ) – selects the tuples that are in both of the relations

### *Additional operations – very useful*

- intersection
- join
- division
- rename

Each operation returns a relation / table, and so, the operations can be composed (algebra is *closed*)

# Relational Algebra

**Projection** ( $\pi$ ) – eliminates all the un-wished attributes from the relation

- Notation:  $\pi_{\alpha}(R)$
- $\alpha$  can have a set of expressions that specify the name of the columns of the relation R considered (also, can be computed)
- R – the relation considered
- Result set: a relation with all the attributes that are mentioned in  $\alpha$ 
  - schema: attributes in  $\alpha$
  - tuples: each record from R that is projected on  $\alpha$
- equivalent with

```
SELECT DISTINCT  $\alpha$ 
FROM R
```

```
not (SELECT  $\alpha$  FROM R)
```

- The Relational Algebra operates on SETS, so there will NOT BE DUPLICATES

# Relational Algebra

**Projection** ( $\pi$ ) – example:

$\pi_{StudentId, Grade}(Exam) \Leftrightarrow SELECT \textbf{DISTINCT} StudentId, Grade FROM Exam$

StudentId	Courseld	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

$\pi_{StudentId, Grade} ($

$) =$

StudentId	Grade
4	9
5	10
4	10
6	7
7	9
7	6
7	10

# Relational Algebra

**Selection** ( $\sigma$ ) – selects a subset of tuples from the relation, tuples that satisfy a specified condition

- Notation:  $\sigma_C(R)$
- $C$  is the condition, called *selective predicate*, that has to be satisfied by the returned tuples from the relation  $R$
- $R$  – the relation considered
- Result set: a relation with all the attributes from the relation  $R$  that satisfy the condition  $C$ 
  - schema:  $R$  schema
  - tuples: the records from  $R$  that satisfy condition  $C$
- equivalent with

```
SELECT DISTINCT *  
FROM R  
WHERE C
```

- condition  $C$  has the form: *Term Operator Term*, where *Term* is an attribute or a constant, and *Operator* is a logical operator ( $<$ ,  $<=$ ,  $>$ ,  $>=$ ,  $=$ ,  $\neq$ , ...)
- $C1 \wedge C2$ ,  $C1 \vee C2$ ,  $\neg C1$  are conditions that contain the operators  $\wedge$  (AND),  $\vee$  (OR),  $\neg$  (NOT), and  $C1$ ,  $C2$  are also conditions



# Relational Algebra

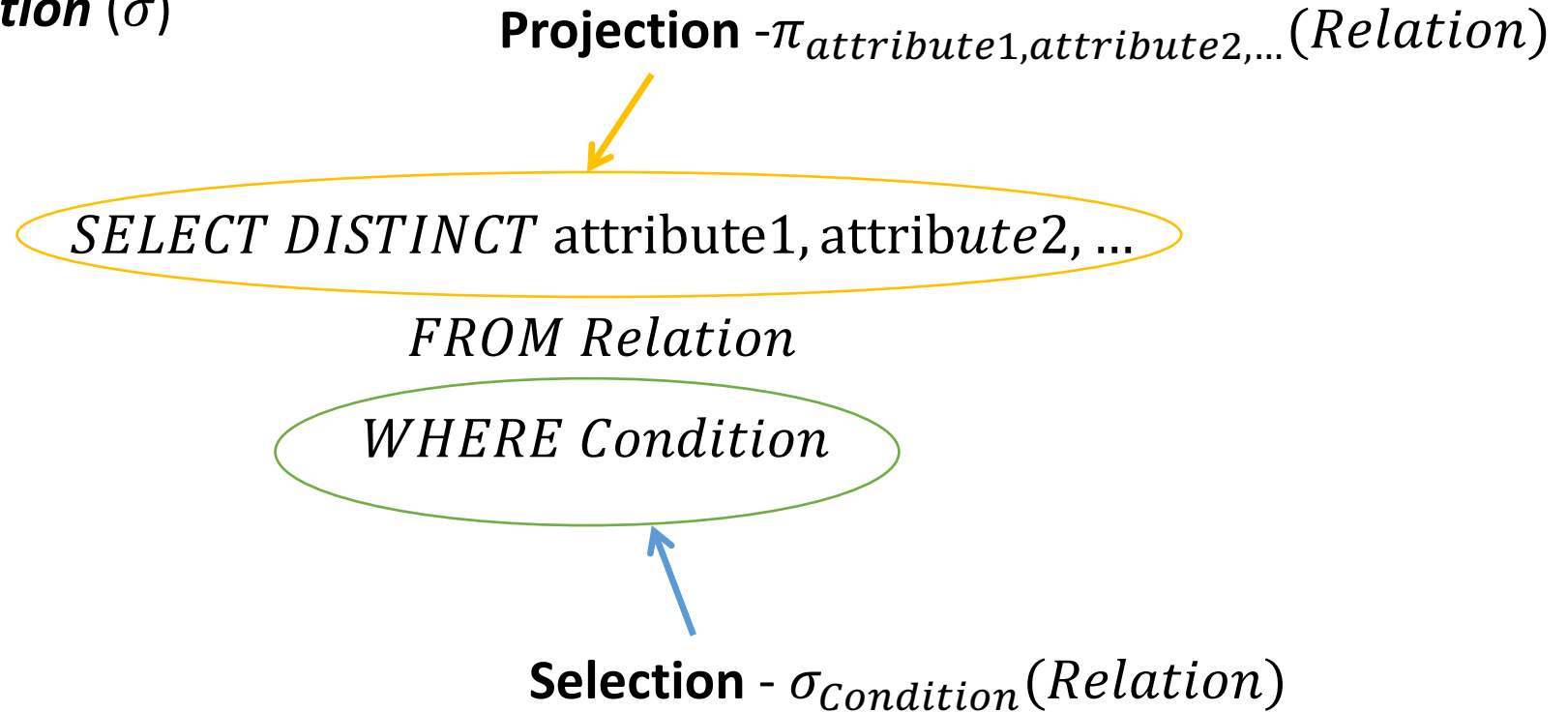
**Selection** ( $\sigma$ )– example:

$$\sigma_{Grade > 7}(Exam) \Leftrightarrow SELECT DISTINCT * FROM Exam WHERE Grade > 7$$

$\sigma_{Grade>7}$ (	<table><tr><th>StudentId</th><th>CourseId</th><th>Grade</th></tr><tr><td>4</td><td>11</td><td>9</td></tr><tr><td>5</td><td>11</td><td>10</td></tr><tr><td>4</td><td>21</td><td>10</td></tr><tr><td>4</td><td>22</td><td>9</td></tr><tr><td>6</td><td>21</td><td>7</td></tr><tr><td>7</td><td>22</td><td>9</td></tr><tr><td>7</td><td>21</td><td>6</td></tr><tr><td>7</td><td>11</td><td>10</td></tr></table>	StudentId	CourseId	Grade	4	11	9	5	11	10	4	21	10	4	22	9	6	21	7	7	22	9	7	21	6	7	11	10	)	=	<table><tr><th>StudentId</th><th>CourseId</th><th>Grade</th></tr><tr><td>4</td><td>11</td><td>9</td></tr><tr><td>5</td><td>11</td><td>10</td></tr><tr><td>4</td><td>21</td><td>10</td></tr><tr><td>4</td><td>22</td><td>9</td></tr><tr><td>7</td><td>22</td><td>9</td></tr><tr><td>7</td><td>11</td><td>10</td></tr></table>	StudentId	CourseId	Grade	4	11	9	5	11	10	4	21	10	4	22	9	7	22	9	7	11	10
	StudentId	CourseId	Grade																																																	
	4	11	9																																																	
	5	11	10																																																	
	4	21	10																																																	
	4	22	9																																																	
	6	21	7																																																	
	7	22	9																																																	
7	21	6																																																		
7	11	10																																																		
StudentId	CourseId	Grade																																																		
4	11	9																																																		
5	11	10																																																		
4	21	10																																																		
4	22	9																																																		
7	22	9																																																		
7	11	10																																																		

# Relational Algebra

**Projection** ( $\pi$ ) and **Selection** ( $\sigma$ )



# Relational Algebra

**Projection** ( $\pi$ ) and **Selection** ( $\sigma$ ) - composition

$\pi_{StudentId, Grade}(\sigma_{Grade > 7}(Exam))$

$\Leftrightarrow SELECT DISTINCT StudentId, Grade FROM Exam WHERE Grade > 7$

$\pi_{StudentId, Grade}(\sigma_{Grade > 7}(Exam))$	StudentId	CourseId	Grade	)	=	StudentId	Grade
	4	11	9			4	9
	5	11	10			5	10
	4	21	10			4	10
	4	22	9			7	9
	6	21	7			7	10
	7	22	9				
	7	21	6				
	7	11	10				

What about  $\sigma_{Grade > 7}(\pi_{StudentId, Grade}(Exam))$  ? Are equivalent? Is important the order of  $\pi$  and  $\sigma$ ?

- $\sigma$  before  $\pi$  only when it is a subquery

# Relational Algebra

**Cross product** (X) – allows the combination of two relations

- Notation:  $R_1 \times R_2$
- 2 relations are combined
- $R_1, R_2$  – the relations considered
- Result set: a relation with all the attributes that are in  $R_1$  and  $R_2$ , combined
  - schema: the attributes from  $R_1$  followed by the attributes from  $R_2$
  - tuples: every tuple  $r_1 \in R_1$  is concatenated with every tuple  $r_2 \in R_2$
- equivalent with

```
SELECT *  
FROM  $R_1$  CROSS JOIN  $R_2$ 
```

or

```
SELECT *  
FROM  $R_1, R_2$ 
```

# Relational Algebra

**Cross product** ( $\times$ ) example: Student[StudentId, Age] and Course[CourseId, NoOfCredits]  
 $Student \times Course \Leftrightarrow SELECT DISTINCT * FROM Student CROSS JOIN Course$

Student

StudentId	Age
4	19
5	19
6	21
7	20

Course

CourseId	NoOfCredits
11	6
12	5

$Student \times Course$

StudentId	Age	CourseId	NoOfCredits
4	19	11	6
4	19	12	5
5	19	11	6
5	19	12	5
6	21	11	6
6	21	12	5
7	20	11	6
7	20	12	5

# Relational Algebra

## ***Union, Intersection, Set-Difference***

- Notation:  $R_1 \cup R_2$ ,  $R_1 \cap R_2$ ,  $R_1 - R_2$
- $R_1$  and  $R_2$  must be union-compatible
  - same number of columns
  - compatible = the columns has to correspond from left to right and have compatible data types / domain
- equivalent with

$R_1 \cup R_2$	$R_1 \cap R_2$	$R_1 - R_2$
SELECT DISTINCT * FROM $R_1$ UNION SELECT DISTINCT * FROM $R_2$	SELECT DISTINCT * FROM $R_1$ INTERSECT SELECT DISTINCT * FROM $R_2$	SELECT DISTINCT * FROM $R_1$ EXCEPT SELECT DISTINCT * FROM $R_2$

- UNION ALL – does not eliminate the duplicates

# Relational Algebra

**Union, Intersection, Set-Difference** example: Student[StudentId, Age] and Course[CourseId, NoOfCredits]

$Student \cup Course \Leftrightarrow SELECT DISTINCT * FROM Student UNION SELECT DISTINCT * FROM Course$

$Student \cap Course \Leftrightarrow SELECT DISTINCT * FROM Student INTERSECT SELECT DISTINCT * FROM Course$

$Student - Course \Leftrightarrow SELECT DISTINCT * FROM Student EXCEPT SELECT DISTINCT * FROM Course$

Student

StudentId	Age
4	19
5	19
6	21
7	20

Course

CourseId	NoOfCredits
11	6
12	5

$Student \cup Course$

StudentId	Age
4	19
5	19
6	21
7	20
11	6
12	5

$Student \cap Course$

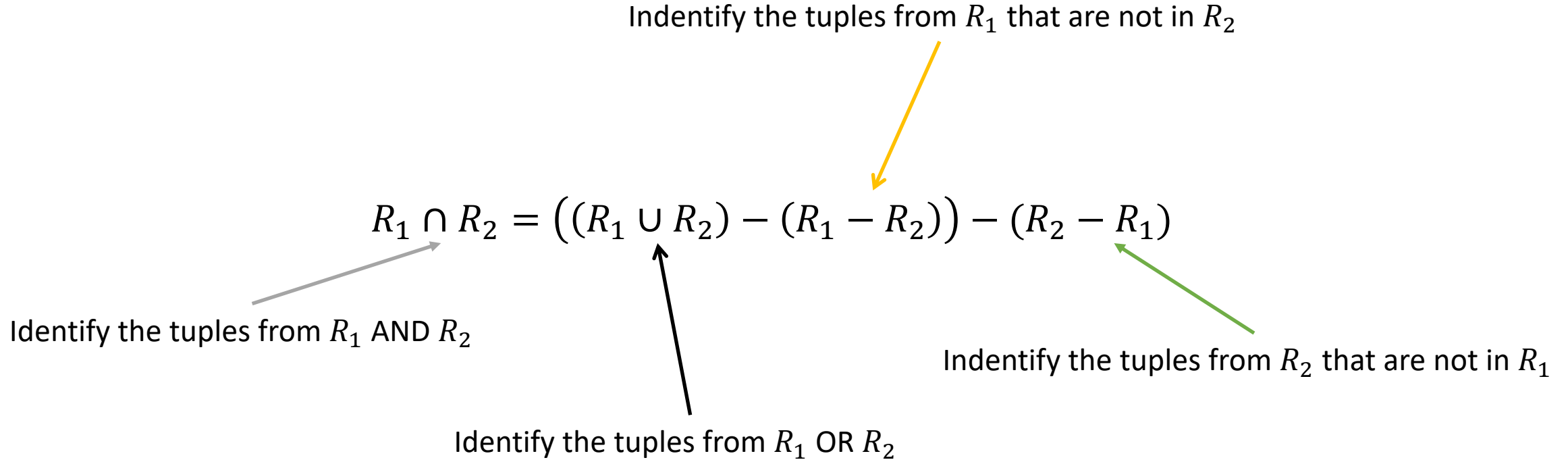
StudentId	Age
-----------	-----

$Student - Course$

StudentId	Age
4	19
5	19
6	21
7	20

# Relational Algebra

## ***Union, Intersection, Set-Difference***





# Relational Algebra

## **Join Operator ( $\otimes_\theta$ ) – or $\theta$ (theta) join**

- Notation:  $R_1 \otimes_\theta R_2$
- 2 relations are combined with respect to condition  $\theta$
- $R_1, R_2$  – the relations considered
- Result set: the records from the cross-product of  $R_1$  and  $R_2$  that satisfy the condition  $\theta$
- $R_1 \otimes_\theta R_2 = \sigma_\theta(R_1 \times R_2)$
- equivalent with

```
SELECT DISTINCT *  
FROM  $R_1$  INNER JOIN  $R_2$  ON  $\theta$ 
```

Example:  $\text{Student} \otimes_{\text{Student.StudentId}=\text{Exam.StudentId}} \text{Exam} \Leftrightarrow$

```
SELECT DISTINCT *  
FROM Student, Exam  
WHERE Student.StudentId = Exam.StudentId
```

```
SELECT DISTINCT *  
FROM Student INNER JOIN Exam  
ON Student.StudentId = Exam.StudentId
```

# Relational Algebra

## ***Equi Join ( $\otimes_{E(\theta)}$ )***

- Notation:  $R_1 \otimes_{E(\theta)} R_2$
- 2 relations are combined with respect to the composed condition  $E(\theta)$
- $R_1, R_2$  – the relations considered
- Result set: the records from  $R_1$  and  $R_2$  that satisfy the composed condition  $E(\theta)$ : only equalities between the attributes that are in  $R_1$  and  $R_2$  and project only one of the redundant attributes (because are equal)

# Relational Algebra

**Equi Join ( $\otimes_{E(\theta)}$ )** – example

$\text{Student} \otimes_{E(\text{Student.StudentId}=\text{Exam.StudentId})} \text{Exam}$

Student

StudentId	Age
4	19
5	19
6	21
7	20

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

$\text{Student} \otimes_{E(\text{Student.StudentId}=\text{Exam.StudentId})} \text{Exam}$

StudentId	Age	CourseId	Grade
4	19	11	9
5	19	11	10
4	19	21	10
4	19	22	9
6	21	21	7
7	20	22	9
7	20	21	6
7	20	11	10

# Relational Algebra

## **Natural Join (\*)**

- Notation:  $R_1 * R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just one of the redundant attributes
- Result set: a relation with all the attributes that are in  $R_1$  and  $R_2$ , combined
  - schema: the union of the attributes of the two relations: attributes with the same name in  $R_1$  and  $R_2$  that appear just once in the result set
  - tuples: obtained from the tuples  $(r_1, r_2)$ , where  $r_1 \in R_1$  and  $r_2 \in R_2$  and  $r_1, r_2$  agree on the common attributes of  $R_1$  and  $R_2$
- Let  $R_1[\alpha], R_2[\beta], \alpha \cap \beta = \{A_1, A_2, \dots, A_n\}$ . Then

$$R_1 * R_2 = \pi_{\alpha \cup \beta} (R_1 \otimes_{R_1.A_1=R_2.A_1 \text{ AND } \dots \text{ AND } R_1.A_n=R_2.A_n} R_2)$$

- equivalent with

```
SELECT DISTINCT *  
FROM R1 NATURAL JOIN R2
```

# Relational Algebra

## ***Natural Join*** (\*) – example

Student

StudentId	Age
4	19
5	19
6	21
7	20

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Student \* Exam

StudentId	Age	CourseId	Grade
4	19	11	9
5	19	11	10
4	19	21	10
4	19	22	9
6	21	21	7
7	20	22	9
7	20	21	6
7	20	11	10

# Relational Algebra

## **Left Outer Join ( $\bowtie_C$ )**

- Notation:  $R_1 \bowtie_C R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just one of the redundant attributes + the attributes from  $R_1$  that have no correspondent in  $R_2$  (*null* value)
- Result set:
  - schema: a relation with the attributes from  $R_1$  followed by the attributes from  $R_2$
  - tuples: tuples from the condition join  $R_1 \Join_C R_2$  + tuples from  $R_1$  that are not used in  $R_1 \Join_C R_2$  combined with the *null* value for the attributes of  $R_2$
- equivalent with

```
SELECT DISTINCT *  
FROM  $R_1$  LEFT OUTER JOIN  $R_2$  ON C
```

# Relational Algebra

## *Left Outer Join* – example

Student

StudentId	Age
4	19
5	19
6	21
7	20
8	21

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Student ⋈<sub>Student.StudentId=Exam.StudentId</sub> Exam

StudentId	Age	CourseId	Grade
4	19	11	9
5	19	11	10
4	19	21	10
4	19	22	9
6	21	21	7
7	20	22	9
7	20	21	6
7	20	11	10
8	21	NULL	NULL

# Relational Algebra

## **Right Outer Join ( $\bowtie_C$ )**

- Notation:  $R_1 \bowtie_C R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just one of the redundant attributes + the attributes from  $R_2$  that have no correspondent in  $R_1$  (*null* value)
- Result set:
  - schema: a relation with the attributes from  $R_1$  followed by the attributes from  $R_2$
  - tuples: tuples from the condition join  $R_1 \Join_C R_2$  + tuples from  $R_2$  that are not used in  $R_1 \Join_C R_2$  combined with the *null* value for the attributes of  $R_1$
- equivalent with

```
SELECT DISTINCT *  
FROM  $R_1$  RIGHT OUTER JOIN  $R_2$  ON C
```



# Relational Algebra

## *Right Outer Join* – example

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Course

CourseId	NoOfCredits
11	5
21	6
22	5
23	4

Exam  $\bowtie_{\text{Exam.CourseId=Course.CourseId}}$  Course

StudentId	CourseId	Grade	NoOfCredits
4	11	9	5
5	11	10	5
4	21	10	6
4	22	9	5
6	21	7	6
7	22	9	5
7	21	6	6
7	11	10	5
NULL	23	NULL	4

# Relational Algebra

## **Full Outer Join ( $\bowtie_C$ )**

- Notation:  $R_1 \bowtie_C R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just one of the redundant attributes + the attributes from  $R_1$  that have no correspondent in  $R_2$  (*null* value) + the attributes from  $R_2$  that have no correspondent in  $R_1$  (*null* value)
- Result set:
  - schema: a relation with the attributes from  $R_1$  followed by the attributes from  $R_2$
  - tuples: tuples from the condition join  $R_1 \Join_C R_2$  + tuples from  $R_1$  that are not used in  $R_1 \Join_C R_2$  combined with the *null* value for the attributes of  $R_2$  + tuples from  $R_2$  that are not used in  $R_1 \Join_C R_2$  combined with the *null* value for the attributes of  $R_1$
- equivalent with

```
SELECT DISTINCT *  
FROM  $R_1$  FULL OUTER JOIN  $R_2$  ON C
```

# Relational Algebra

## Full Outer Join – example

Student

StudentId	Age
4	19
5	19
6	21
7	20
8	21

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Course

CourseId	NoOfCredits
11	5
21	6
22	5
23	4

Student ⋈<sub>Student.StudentId=Exam.StudentId</sub> Exam ⋈<sub>Exam.CourseId=Course.CourseId</sub> Course

StudentId	Age	CourseId	Grade
4	19	11	9
5	19	11	10
4	19	21	10
4	19	22	9
6	21	21	7
7	20	22	9
7	20	21	6
7	20	11	10
8	21	NULL	NULL
NULL	NULL	23	NULL

# Relational Algebra

## **Left Semi Join ( $\triangleright$ )**

- Notation:  $R_1 \triangleright R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just the attributes from  $R_1$  that have no correspondent in  $R_2$  (*null* value)
- Result set:
  - schema: a relation with the attributes from  $R_1$
  - tuples: tuples from  $R_1$  that are not used in the condition join  $R_1 \otimes_C R_2$  ( $R_1$  combined with the *null* value for the attributes of  $R_2$  )

Student	StudentId	Age
	4	19
	5	19
	6	21
	7	20
	8	21

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Student  $\triangleright$  Exam

StudentId	Age	CourseId	Grade
8	21	NULL	NULL

Exam

# Relational Algebra

## **Right Semi Join ( $\lhd$ )**

- Notation:  $R_1 \lhd R_2$
- 2 relations are combined with respect to the attributes that have the *same name* and display just the attributes from  $R_2$  that have no correspondent in  $R_1$  (*null* value)
- Result set:
  - schema: a relation with the attributes from  $R_2$
  - tuples: tuples from  $R_2$  that are not used in the condition join  $R_1 \otimes_C R_2$  ( $R_2$  combined with the *null* value for the attributes of  $R_1$  )

Exam

StudentId	CourseId	Grade
4	11	9
5	11	10
4	21	10
4	22	9
6	21	7
7	22	9
7	21	6
7	11	10

Course

CourseId	NoOfCredits
11	5
21	6
22	5
23	4

Exam  $\lhd$  Course

StudentId	CourseId	Grade	NoOfCredits
NULL	23	NULL	4

# Relational Algebra

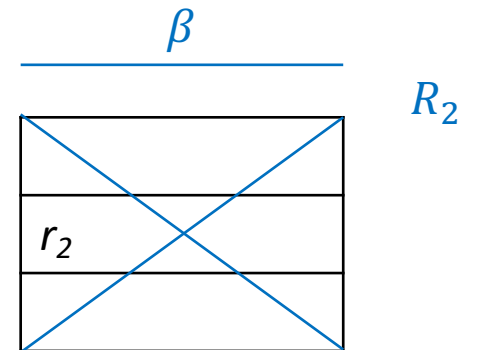
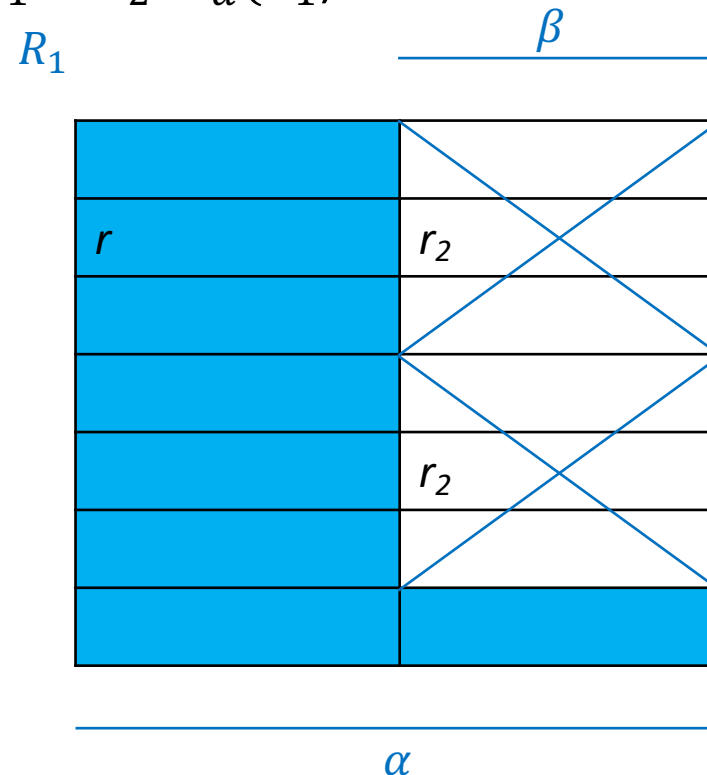
## **Division ( $\div$ )**

- Notation:  $R_1 \div R_2$
- It is not a base operator, but it can be useful, because can simplify the query
- 2 relations are combined
- $R_1, R_2$  – the relations considered
- $R_1[\alpha], R_2[\beta], \beta \subset \alpha$ , where  $\alpha, \beta$  are attributes of  $R_1$  and  $\beta$  is an attribute of  $R_2$
- $R_1 \div R_2 = \{\alpha \mid \exists(\alpha, \beta) \in R_1 \forall \beta \in R_2\}$ , i.e.  $R_1 \div R_2$  contains all the tuples  $\alpha$  such that for each tuple  $\beta \in R_2$ , exists a  $\alpha\beta$  tuple from  $R_1$  (or, if the set of  $\beta$  values associated with a value  $\alpha$  from  $R_1$  contains all the values  $\beta \in R_2$  then  $\alpha$  will be returned as a result for  $R_1 \div R_2$ )
- Result set: a relation with the attributes that are in  $R_1 \div R_2$  combined
  - schema:  $\alpha - \beta$
  - tuples: a record  $r \in R_1 \div R_2$  if  $\forall r_2 \in R_2, \exists r_1 \in R_1$  such that
    - $\pi_{\alpha-\beta}(r_1) = r$
    - $\pi_{\beta}(r_1) = r_2$
    - i.e. a record  $r$  belongs to the result if in  $R_1$   $r$  is concatenated with every record in  $R_2$

# Relational Algebra

## **Division ( $\div$ )**

- Generalization:  $\alpha$  and  $\beta$  can represent any set of attributes;  $\beta$  is the set of attributes from  $R_1$  and  $\alpha \cup \beta$  represent the attributes of  $R_1$
- Division operator is not essential; it is just a shortcut (like join, that is used more often)
- For  $R_1 \div R_2$  will be determined the  $\alpha$  values that are not connected with some  $\beta$  values from  $R_2$  (the  $\alpha$  value is not connected if attaching to it a  $\beta$  value from  $R_2$  is obtained a tuple  $\alpha\beta$  that is not in  $R_1$ ) - not connected  $\alpha$  values:  $R_1 \div R_2 = \pi_\alpha(R_1)$



# Relational Algebra

## **Rename**

- Notation:  $\rho(R'(A_1 \rightarrow A'_1, A_2 \rightarrow A'_2), R)$  or  $\rho_{R'(A'_1, A'_2)}(R)$
- The new relation  $R'$  has the same instance as  $R$ , and its structure contains the attribute  $A'_i$  instead of the attribute  $A_i$
- Used when the attributes and relations have the same name (e.g. *join* between 1 table, *join* for 2 tables on the same attributes); one must be renamed

Example:  $\rho(Student2(StudentId \rightarrow SId, StudentName \rightarrow SName), Student) \Leftrightarrow$

*SELECT StudentId as SId, StudentName as SName FROM Student, Student2*

Student

StudentId	StudentName	Age
4	Maria	19
5	Dan	19
6	Florin	21
7	Lara	20

Student2

SId	SName	Age
4	Maria	19
5	Dan	19
6	Florin	21
7	Lara	20



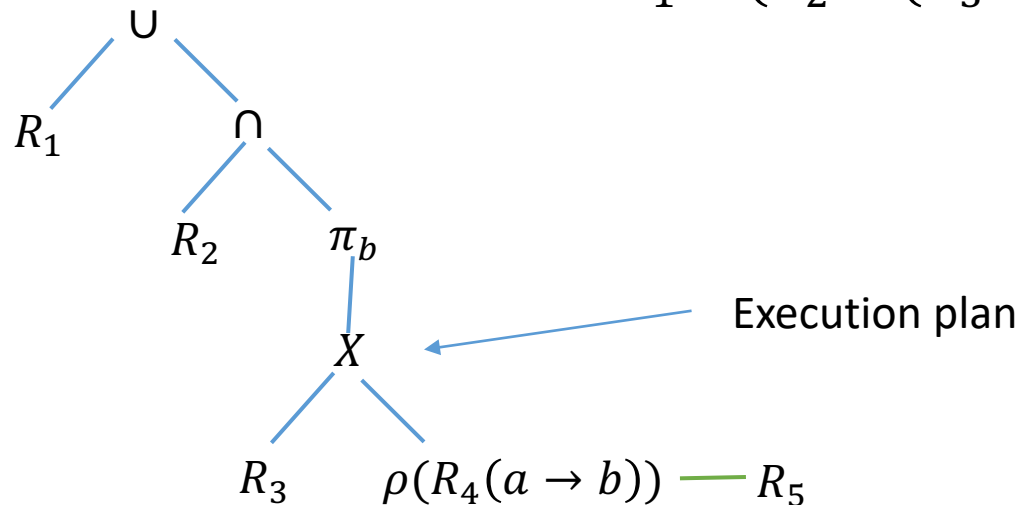
# Relational Algebra

## **Assignment ( $\leftarrow$ )**

- Notation:  $Temporary\_variable \leftarrow \pi_X(R_1 \times R_2)$  or  $R[list] := expression$
- Offers a simple way in which the complex queries can be treated
- The result is saved in a temporary variable
- The result of the expression from the right side of  $\leftarrow$  is assigned to the attribute that is in the left side of the operator  $\leftarrow$
- The variables can be used, after, in other expressions (e.g.  $result \leftarrow Temporary\_variable - R_2$ )

## **Complex expressions**

$$R_1 \cup (R_2 \cap (R_3 \times \rho(R_4(a \rightarrow b), R_5)))$$



# Relational Algebra

**Example:** Display the name of the students that have grades on the course 11.

○ Solution 1

$$\pi_{StudentName} \left( \left( \sigma_{CourseId=11}(Exam) \right) * Student \right)$$

○ Solution 2

$$\begin{aligned} &\rho(Temp1, \sigma_{CourseId=11}(Exam)) \\ &\rho(Temp2, Temp1 * Student) \\ &\pi_{StudentName}(Temp2) \end{aligned}$$

○ Solution 3

$$\pi_{StudentName} \left( \sigma_{CourseId=11}(Exam * Student) \right)$$

# Relational Algebra

**Example:** Display the name of the students that have grades on the courses with 6 credits.

- the number of credits is in the table *Course*, so, a natural join will be added.

- Solution 1

$$\pi_{StudentName} \left( \left( \sigma_{NoOfCredits=6}(Course) \right) * Exam * Student \right)$$

- Solution 2

$$\pi_{StudentName} \left( \pi_{StudentId} \left( \pi_{CourseId} \left( \sigma_{NoOfCredits=6}(Course) \right) * Exam \right) * Students \right)$$

- Solution 2 is more efficient than Solution 1

- The query optimization model allows the transformation of the Solution 1 in Solution 2

# Relational Algebra

**Example:** Display the name of the students that have grades on the courses with 5 or 6 credits.

- Are identified the courses with 5 or 6 credits and then are displayed the students that have grades on these courses.
- Solution

$$\rho (TempCourse, (\sigma_{NoOfCredits=5 \vee NoOfCredits=6}(Course))) \\ \pi_{StudentName}(TempCourse * Exam * Student)$$

- *TempCourse* can be defined also as a union.

**Example:** Display the name of the students that have grades on the courses with 5 AND 6 credits.

- First are identified the students that have grades in a 5 credit course; then are identified the students that have grades in a 6 credits course; then are intersected (StudentId is used)
- Solution

$$\rho (Temp5, (\sigma_{NoOfCredits=5}(Course) * Exam)) \\ \rho (Temp6, (\sigma_{NoOfCredits=6}(Course) * Exam)) \\ \pi_{StudentName}((Temp5 \cap Temp6) * Student)$$

# Relational Algebra

**Example:** Display the name of the students that have grades on all the courses

- The *division* is used; must be prepared the relation structures before use the division operator
- Solution

$$\rho \left( TempStudentId, \pi_{StudentId, CourseId}(Exam) \right) \div \pi_{CourseId}(Course) \\ \pi_{StudentName}(TempStudentId * Student)$$

**Other notations and operations:** let  $R$  be a relation

- **Eliminating duplicates from a relation:**  $\delta(R)$
- **Sorting records in a relation:**  $S_{\{list\_attributes\}}(R)$
- **Grouping:**  $\gamma_{\{list1\_attributes\} GROUP BY \{list2\_attributes\}}(R)$ 
  - *list1\_attributes* (can contain aggregate functions) is evaluated for each group of records
  - the records from  $R$  are grouped by the columns in *list2\_attributes*

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the name of the students from the group 821.

$$R := \pi_{StudentName} \left( \sigma_{SGroup=821}(Student) \right)$$

equivalent with

*SELECT StudentName FROM Student WHERE SGroup = 821*

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display for each student group from *Computer Science*, the students alphabetically.

$$A := \pi_{GroupId} \left( \sigma_{Specialization = 'Computer Science'}(Group) \right)$$

$$R := S_{SGroup, StudentName} \left( \sigma_{SGroup \in A}(Student) \right)$$

equivalent with

```
SELECT * FROM Student WHERE SGroup IN
      (SELECT GroupId FROM Group
       WHERE Specialization = 'Computer Science')
ORDER BY SGroup, StudentName
```

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the number of students from every group for the specialization *Computer Science*.

$$A := \sigma_{SGroup \text{ IN } \left( \pi_{GroupId} \left( \sigma_{Specialization = 'Computer Science'}(Group) \right) \right)}(Student)$$

$$R := \gamma_{\{SGroup, COUNT(*)\} \text{ GROUP BY } \{SGroup\}}(A)$$

equivalent with

```
SELECT SGroup, COUNT(*) FROM
( SELECT * FROM Student WHERE SGroup IN
  (SELECT GroupId FROM Group
   WHERE Specialization = 'Computer Science') ) B
GROUP BY SGroup
```



# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the program for the student *Popescu*.

$$R := \sigma_{SGroup \text{ IN } \left( \pi_{GroupId} \left( \sigma_{StudentName='Popescu'}(Student) \right) \right)}(Program)$$

equivalent with

```
SELECT * FROM Program WHERE SGroup IN
(SELECT GroupId FROM Student
WHERE StudentName = 'Popescu')
```

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the number of hours per week for each group.

$$A(\text{Number}, \text{SGroup}) := \pi_{\text{EndHour} - \text{StartHour}, \text{SGroup}}(\text{Program})$$
$$R(\text{SGroup}, \text{NoOfHours}) := \gamma_{\{\text{SGroup}, \text{SUM}(\text{Number})\}} \text{ GROUP BY } \{\text{SGroup}\}(A)$$

equivalent with

$$\begin{aligned} & \text{SELECT } \text{SGroup}, \quad \text{SUM}(\text{EndHour} - \text{StartHour}) \\ & \quad \text{FROM Program} \\ & \quad \text{GROUP BY SGroup} \end{aligned}$$

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the name of the professors that have been teach the student with the name *Popescu*.

$$A := (\sigma_{StudentName='Popescu'}(Student)) \otimes_{Student.SGroup=Program.SGroup} Program$$
$$B := \pi_{ProfessorId}(A)$$
$$C := Professor \otimes_{Professor.ProfessorId=B.ProfessorId} B$$
$$R := \pi_{ProfessorName}(C)$$

equivalent with *SELECT Student.StudentName*

*FROM Student INNER JOIN Program ON Student.SGroup = Program.SGroup  
INNER JOIN Professor ON Professor.ProfessorId = Program.ProfessorId  
WHERE Student.StudentName = 'Popescu'*

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the name of the professors that have not teach (with no program).

$$R := \pi_{ProfessorName}(Professor) - \\ -\pi_{ProfessorName}(Professor \otimes_{Professor.ProfessorId=Program.ProfessorId} Program)$$

equivalent with

*SELECT P1.ProfessorName FROM Professor P1 EXCEPT  
SELECT P2.ProfessorName  
FROM Professor P2 INNER JOIN Program P ON P2.Professorid = P.ProfessorId*

- Is it ok if there are professors with the same name? (ProfessorId, ProfessorName)

# Relational Algebra

**Example:** Consider the following relations

Group [GroupId, Year, Specialization]

Student [StudentId, StudentName, Age, SGroup]

Professor [ProfessorId, ProfessorName, Dob]

Program [Class, Day, StartHour, EndHour, ProgramType, ProfessorId, SGroup]

- Display the students that have program in every day of the week (all the days in which are programs).

$$A := \delta \left( \pi_{Day}(Program) \right)$$

$$B := Student \otimes_{Student.SGroup=Program.SGroup} Program$$

$$C := \delta \left( \pi_{StudentName, Day}(B) \right)$$

- Is it ok if there are students with the same name? (StudentId, StudentName, Day)

# References:

- C.J. Date, *An Introduction to Database Systems (8th Edition)*, Addison-Wesley, 2003.
- H. Garcia-Molina, J. Ullman, J. Widom, *Database Systems: The Complete Book*, Prentice Hall Press, 2008.
- G. Hansen, J. Hansen, *Database Management And Design (2nd Edition)*, Prentice Hall, 1996.
- R. Ramakrishnan, J. Gehrke, *Database Management Systems*, McGraw- Hill, 2007.  
<http://pages.cs.wisc.edu/~dbbook/openAccess/thirdEdition/slides/slides3ed.html>
- R. Ramakrishnan, J. Gehrke, *Database Management Systems (2nd Edition)*, McGraw-Hill, 2000.
- A. Silberschatz, H. Korth, S. Sudarshan, *Database System Concepts*, McGraw-Hill, 2010.  
<http://codex.cs.yale.edu/avi/db-book/>
- L. Țâmbulea, *Curs Baze de date*, Facultatea de Matematică și Informatică, UBB, 2013-2014.
- J. Ullman, J. Widom, *A First Course in Database Systems*,  
<http://infolab.stanford.edu/~ullman/fcdb.html>