Lab 14

Methods for solving nonlinear systems. Methods for soving differential equations

1. Solve the system

$$\begin{cases} x_1^3 + 3x_2^2 - 21 = 0 \\ x_1^2 + 2x_2 + 2 = 0 \end{cases}$$

using Newton's method with $x^{(0)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\varepsilon = 10^{-6}$.

2. Find the approximative solution of the following Cauchy problem:

$$\begin{aligned} y'(x) &= 2x - y \\ y(0) &= -1 \end{aligned}$$

for the equidistant nodes $x_i = a + ih$, $i = 0, ..., N$; $h = \frac{b-a}{N}$, with $a = 0$, $b = 1$, $N = 10$, using Euler's method and Runge-Kutta method of 4th order. Plot the obtained approximations together with the exact solution $y(x) = e^{-x} + 2x - 2$, in the same figure.