Lab 2

Orthogonal and Taylor polynomials. Finite and divided differences

1. The first 4 Legendre polynomials are given by:

$$l_1(x) = x$$

$$l_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$$

$$l_3(x) = \frac{5}{2}x^3 - \frac{3}{2}x$$

$$l_4(x) = \frac{35}{8}x^4 - \frac{15}{4}x^2 + \frac{3}{8}, \quad x \in [0, 1].$$

Divide the display in 4 parts and plot in each part the Legendre polynomial l_i , i = 1, ..., 4. (Use the *subplot* command).

2. a) Chebyshev polynomials of the first kind are defined by

$$T_n(t) = \cos(n \arccos t), \quad t \in [-1, 1].$$

Plot, in the same figure, the polynomials T_1, T_2, T_3 .

b) Plot, in the same figure, the first n Chebyshev polynomials of the first kind, using the following recurrence formula:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \qquad x \in [-1, 1],$$

with $T_0(x) = 1$ and $T_1(x) = x$.

- 3. Taylor polynomial of n-th degree, associated to the function f and the point x_0 , is given by $P_n(x) = \sum_{k=0}^n \frac{(x-x_0)^k}{k!} f^{(k)}(x_0)$. Plot, in the same figure, the first six Taylor polynomials for $f(x) = e^x$ and $x_0 = 0$, on the interval [-1,3].
- 4. Considering h = 0.25, $x_i = 1 + ih$, $i = \overline{0,6}$, and $f(x) = \sqrt{5x^2 + 1}$, construct the finite differences table.
- 5. For $x_0 = 2$, $x_1 = 4$, $x_2 = 6$, $x_3 = 8$ and $f_0 = 4$, $f_1 = 8$, $f_2 = 14$, $f_3 = 16$ construct the divided differences table.

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