DATA STRUCTURES (AND ALGORITHMS)

Dynamic Array. Amortized complexity. Iterator.

Lect. PhD. Diana-Lucia Miholca

2023 - 2024



Babeş - Bolyai University, Faculty of Mathematics and Computer Science

In Lecture 1...

- Abstract Data Types
- Data Structures
- Pseudocode conventions
- Algorithm Complexity Analysis
 - O, Ω and Θ notations
 - Best case, worst case, average case

Today

Dynamic Array

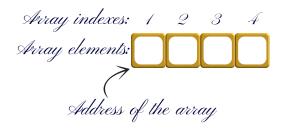
Amortized complexity analysis

Iterator

Arrays

The array is one of the simplest and most basic data structures.

An array can hold a fixed number of elements of the same type and these elements occupy a contiguous memory block.



Arrays



- 1 The type of the elements in the array
- The maximum number of elements that can be stored in the array (*capacity* of the array)
- The memory occupied by the array is the capacity times the size of one element.
- The array itself is referenced by the address of its first element.

Arrays - C++ Example 1



An array of *boolean* values (which occupy one byte)

Size of bool: 1 Address of the array: 0x781a60 7871072 Address of the element from index 0: 0x781a60 7871072 Address of the element from index 1: 0x781a61 7871073 Address of the element from index 2: 0x781a62 7871074 Address of the element from index 3: 0x781a63 7871075 Address of the element from index 4: 0x781a64 7871076 Address of the element from index 5: 0x781a65 7871077



Can you guess the address of the element from index 6?

Arrays - C++ Example 2



An array of *integer* values (which occupy 4 bytes)

```
Size of int: 4
Address of the array: 0xf31a60 15932000
Address of the element from index 0: 0xf31a60 15932000
Address of the element from index 1: 0xf31a64 15932004
Address of the element from index 2: 0xf31a68 15932008
Address of the element from index 3: 0xf31a6c 15932012
Address of the element from index 4: 0xf31a70 15932016
Address of the element from index 5: 0xf31a74 15932020
```



Can you guess the address of the element from index 6?

Arrays - advantage

The main **advantage** of arrays is that any element of the array can be accessed in constant time $(\Theta(1))$.



Computing the address of an array's element:

Address of i^{th} element = address of array + (i - 1) * size of an element

Obs: If array indexing starts from 0, the above formula is still valid, but with i instead of i - 1.

Arrays - disadvantage

- An array is a **static** data structure: its capacity is fixed.
- **Disadvantage**: we need to know the number of elements from the beginning:
 - if the capacity is too small: we cannot store every element we want to
 - if the capacity is too large: we waste memory

Dynamic Array

A dynamic array is an array with dynamic capacity. Its capacity can grow or shrink, depending on the number of elements that need to be stored in the array.

Dynamic arrays still have the advantage of accessing any element in $\Theta(1)$ time.

Dynamic Array - Representation

- For representing a Dynamic Array we need the following fields:
 - capacity the number of slots allocated for the array
 - length the actual number of elements stored in the array
 - elems the actual array with capacity slots allocated



Representation of a Dynamic Array:

Dynamic Array:

capacity: Integer length: Integer elems: TElem[]

Dynamic Array - Resize

When the value of *length* equals the value of *capacity*, the array is full. If more elements need to be added, the *capacity* of the array is increased, the array being *resized*.

When *resizing* the dynamic array, a new, bigger array is allocated and the existing elements are copied from the old array to the new one.

Optionally, *resize* can be performed after deletion as well: if the dynamic array becomes "too empty", a resize operation can be performed to shrink its size.

Dynamic Array - DS vs. ADT

- Dynamic Array is a data structure:
 - It describes how data is actually stored in a the computer's memory and how it can be accessed and processed
 - It can be used to implement different container ADTs
- Being so frequently used, in most programming languages it exists as a container ADT as well.
 - The Dynamic Array is not really an ADT, but we still can treat it as an ADT and discuss its domain and interface.

Domain of ADT Dynamic Array:

```
\mathcal{DA} = \{ \mathbf{da} | \ da = (capacity, length, e_1e_2e_3...e_{length}), capacity, length \in N, length \leq capacity, e_i \ \text{is of type TElem} \ \forall i \in \{1, 2, 3, ..., length\} \}
```

- init(da, cp)
 - description: creates a new, empty Dynamic Array with initial capacity cp (constructor)
 - pre: cp ∈ N*
 - post: $da \in \mathcal{DA}$, da.capacity = cp, da.length = 0
 - throws: an exception if cp is negative or zero

- destroy(da)
 - description: destroys a dynamic array
 - pre: $da \in \mathcal{DA}$
 - **post:** *da* was destroyed (the memory occupied by the dynamic array *da* has been freed)

- size(da)
 - description: returns the size (number of elements) of the Dynamic Array da
 - pre: $da \in \mathcal{DA}$
 - **post:** size = the size of *da* (the number of elements in *da*)

- getElement(da, i)
 - description: returns the element from index i in the Dynamic Array
 da
 - pre: $da \in \mathcal{DA}$, $1 \le i \le da.length$
 - post: getElement = e, e ∈ TElem, e = da.e_i (the element from index i)
 - throws: an exception if i is not a valid index for the Dynamic Array da

- setElement(da, i, e)
 - description: changes the element from a position to another value
 - **pre:** $da \in \mathcal{DA}$, $1 \le i \le da.length$, $e \in TElem$
 - **post**: $da' \in \mathcal{DA}$, $da'.e_i = e$ (the i^{th} element from da' becomes e), setElement = $da.e_i$ (returns the old value from index i)
 - throws: an exception if i is not a valid index for the Dinamyc Array da

- addToEnd(da, e)
 - description: adds an element to the end of a Dynamic Array. If the array is full, its capacity will be increased
 - pre: $da \in \mathcal{DA}$, $e \in TElem$
 - post: da' ∈ DA, da'.length = da.length + 1; da'.e_{da'.length} = e (da.capacity = da.length ⇒ da'.capacity = da.capacity * 2)

- addToPosition(da, i, e)
 - description: adds an element to a given position in the Dynamic Array. If the array is full, its capacity will be increased.
 - pre: $da \in \mathcal{DA}$, $1 \le i \le da.length + 1$, $e \in TElem$
 - **post:** $da' \in \mathcal{DA}$, da'.length = da.length + 1, $da'.e_i = da.e_{i-1} \forall j = da'.length$, da'.length 1, ..., i+1, $da'.e_i = e$, $da'.e_i = da.e_i \ \forall j = i-1$, ..., 1 $(da.capacity = da.length \Rightarrow da'.capacity = da.capacity * 2)$
 - **throws:** an exception if *i* is not a valid position (da.length+1 is a valid position when adding a new element)

- deleteFromPosition(da, i)
 - description: deletes an element from a given position (index) from the Dynamic Array. Returns the deleted element.
 - **pre:** $da \in \mathcal{DA}$, $1 \leq i \leq da.length$
 - **post:** deleteFromPosition = e, $e \in TElem$, $e = da.e_i$, $da' \in \mathcal{DA}$, da'.length = da.length 1, $da'.e_j = da.e_{j+1} \forall i \leq j \leq da'.length$, $da'.e_j = da.e_j$, $\forall j = 1, ..., i 1$
 - throws: an exception if i is not a valid index

- iterator(da, i)
 - description: returns an iterator for a Dynamic Array
 - pre: $da \in \mathcal{DA}$
 - **post:** $i \in \mathcal{I}$, i is an iterator over da

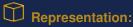
- Other possible operations:
 - Check if the Dynamic Array is empty or not
 - Delete an element (giving the element itself and not its index)
 - Check if an element appears in the Dynamic Array or not
 - Return the index of a given element
 - etc.

Dynamic Array - Implementation

Most operations from the interface of the Dynamic Array are very easy to implement.

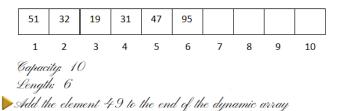
In the following we will discuss the implementation of three operations: addToEnd, addToPosition and deleteFromPosition.

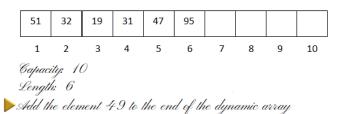
We are going to use the representation discussed earlier.

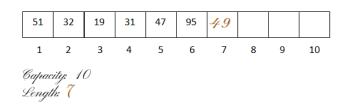


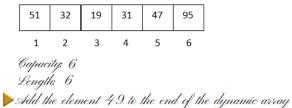
Dynamic Array:

capacity: Integer length: Integer elems: TElem[]



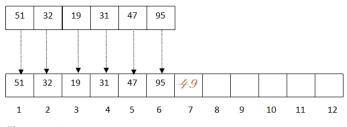








▶Add the element 49 to the end of the dynamic array



Capacity: **12** Length: 7

26

Dynamic Array - addToEnd



Adding to the end of a Dynamic Array:

 $\textbf{subalgorithm} \ \text{addToEnd} \ (\text{da}, \ e) \ \textbf{is:}$

Dynamic Array - addToEnd

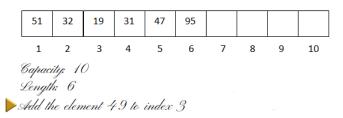


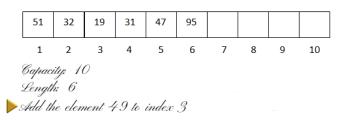
Adding to the end of a Dynamic Array:

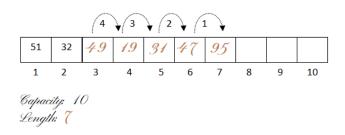
```
subalgorithm addToEnd (da, e) is:
   if da.length = da.capacity then
  //The dynamic array is full. We need to resize it first
      da.capacity ← da.capacity * 2
      newElems ← @ an array with da.capacity empty slots
      //We need to copy all the existing elements into newElems
      for index ← 1, da.length execute
         newElems[index] ← da.elems[index]
      end-for
      //Depending on the prog. lang., we may need to free the old elems array
      free(da.elems)
      //We need to replace the old elements array with the new one
      da.elems ← newElems
  end-if
  //Now we certainly have space for the element e
  da.length ← da.length + 1
  da.elems[da.length] \leftarrow e
end-subalgorithm
```



What is the complexity of addToEnd?









Adding to a given position in a Dynamic Array:

```
subalgorithm addToPosition (da, i, e) is:

if i > 0 and i ≤ da.length+1 then

if da.length= da.capacity then //The dynamic array is full. We need to resize it da.capacity ← da.capacity * 2

newElems ← @ an array with da.capacity empty slots

for index ← 1, da.length execute

newElems[index] ← da.elems[index]

end-for

free(da.elems)

da.elems ← newElems

end-if //Now we certainly have space for the element e
```



Adding to a given position in a Dynamic Array:

```
subalgorithm addToPosition (da, i, e) is:
   if i > 0 and i \le da.length+1 then
      if da.length= da.capacity then //The dynamic array is full. We need to resize it
         da.capacity ← da.capacity * 2
         newElems ← @ an array with da.capacity empty slots
         for index ← 1, da.length execute
            newElems[index] ← da.elems[index]
         end-for
         free(da.elems)
         da.elems ← newFlems
      end-if //Now we certainly have space for the element e
      da.length ← da.length+1
      for index ← da.length, i+1, -1 execute //Move the elements to the right
         da.elems[index] ← da.elems[index-1]
      end-for
      da.elems[i] ← e
  else
      @throw exception
  end-if
end-subalgorithm
```

Dynamic Array - addToPosition



What is the complexity of addToPosition?

Dynamic Array



- While it is not mandatory to double the capacity, it is important to compute the new capacity by multiplying the old one with a constant number greater than 1.
- After resizing a dynamic array, its elements will still occupy a contiguous memory block, but a different one.

Dynamic Array - resize

E What are the resize factors in different programming languages?

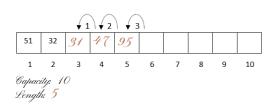
- C++ 1.5
- Java 1.5
- Python \approx 1.125
- C# 2

Dynamic Array - Delete From Position



Dynamic Array - Delete From Position





Dynamic Array - deleteFromPosition



Deleting the element at a given index in a Dynamic Array:

```
subalgorithm deleteFromPosition (da, i) is:
  if i > 0 and i \le da.length then
     e ← da.elems[i]
     for index ← i, da.length-1 execute
        da.elems[i] \leftarrow da.elems[i+1]
     end-for
     da.length ← da.length - 1
     deleteFromPosition \leftarrow e
  else
     @throw exception
  end-if
end-subalgorithm
```



What is the complexity of deleteFromPosition?



- Size $\Theta(1)$ GetElement -

- Size Θ(1)
- getElement Θ(1)
- setElement -

- **⊘** size Θ(1)
- getElement Θ(1)
- SetElement Θ(1)
- iterator ⊖(1)
- addToPosition -

- **⊘** size Θ(1)
- getElement Θ(1)
- SetElement Θ(1)
- iterator ⊖(1)
- addToPosition O(n)
- deleteFromEnd -

- **Size** Θ(1)
- getElement Θ(1)
- SetElement Θ(1)
- iterator $\Theta(1)$
- addToPosition O(n)
- O deleteFromEnd Θ(1)
- deleteFromPosition -

- Size Θ(1)
- getElement Θ(1)
- SetElement Θ(1)
- iterator $\Theta(1)$
- addToPosition O(n)
- O deleteFromEnd Θ(1)
- deleteFromPosition O(n)
- addToEnd -

- Size Θ(1)
- getElement ⊖(1)
- SetElement Θ(1)
- iterator $\Theta(1)$
- \bigcirc addToPosition O(n)
- O deleteFromEnd Θ(1)
- deleteFromPosition O(n)
- addToEnd Θ(1) amortized

The amortized analysis is a method of analyzing algorithms that considers an entire series of operations, computes the cost of the entire series and "charges" each individual operation with a share of the total cost.

We can, at times, obtain tighter bounds using amortized complexity rather than worst-case complexity.

While certain operations may be costly, they cannot occur at a high-enough frequency to weigh down the entire program.

addToEnd has the complexity the O(n). Consequently, n calls to the addToEnd would have complexity the $O(n^2)$.

But we rarely have to resize a Dynamic Array if we consider a sequence of *n* operations.

- Consider c_i the cost for the i^{th} call to addToEnd.
- Supposing that we start with an initial capacity of 1 and we double the capacity at each resizing, at the *i*th operation we perform a resize iff *i-1* is a power of 2. So, the cost of the *i*th operation *i*, c_i, is:

$$c_i =$$

$$\begin{cases} i, & \text{if i-1 is an exact power of 2} \\ 1, & \text{otherwise} \end{cases}$$



The cost of n operations is:

$$\sum_{i=1}^{n} c_i \leq n + \sum_{j=0}^{\lceil log_2 n \rceil} 2^j \leq n + 2^{\lceil log_2 n \rceil + 1} - 1 < n + 2n = 3n$$

Since the total cost of n operations is 3n, we can say that the cost of one operation is 3, which is constant.

While the worst case time complexity of addToEnd is still $\Theta(n)$, the amortized complexity is $\Theta(1)$.

The amortized complexity is no longer valid if resizing means increasing the capacity by a constant.

 \bigcirc In case of *addToPosition*, the complexity remains O(n) - even if resizing is performed rarely, we need to shift elements.

When do we have amortized complexity?

The reason why in case of *addToEnd* we can talk about amortized complexity is that the worst case situation happens rarely.

Whenever you want to determine whether amortized complexity computation is applicable, ask the following question:

Can we have worst case complexity for two consecutive calls? If the answer is YES, than you do **not** have a situation of amortized complexity.

Resizing after deletion

Obs: In order to avoid having a Dynamic Array with too many empty slots, we can resize the array after deletion as well, if the array becomes "too empty".

How empty should the array become before resizing? Which of the following two strategies do you think is better? Why?

- Wait until the array is only half full (da.length≈ da.capacity/2) and resize it to the half of its capacity
- Wait until the array is only a quarter full (da.length≈ da.capacity/4) and resize it to the half of its capacity

Iterator

An **iterator** is a data type that is used to iterate through the elements of a container.

It offers a generic way of traversing a container, irrespective to its representation.

Every iterable container should include in its interface an operation called *iterator* that will return an iterator over it.

Iterator



An iterator usually contains:

- a reference to the container it iterates over
- a cursor, which is reference to a current element from the container

Iterating through the elements of the container means moving the *cursor* from one element to the next until the iterator becomes *invalid*

The exact way of representing the *cursor* of the iterator depends on the data structure used for representing the container.

Iterator - domain

Domain of the Iterator ADT:

 $\mathcal{I} = \{ i \mid i \text{ is an iterator over a container with elements of type TElem } \}$

What operations should we have in the **interface** of the Iterator ADT?

- init(i, c)
 - description: creates a new iterator for a container
 - **pre:** c is a container
 - post: i ∈ I and i refers the first element in c if c is not empty or i is not valid

- getCurrent(i)
 - description: returns the current element refered by the iterator
 - **pre:** $i \in \mathcal{I}$, i is valid
 - post: getCurrent = e, e ∈ TElem, e is the current element referred by i
 - throws: an exception if i is not valid

- next(i)
 - description: moves the cursor of the iterator to the next element or makes the iterator invalid if no elements are left
 - **pre:** $i \in \mathcal{I}$, i is valid
 - post: i' ∈ I, the cursor of i' points to the next element from the container or, if no more elements are left, i' is invalid
 - throws: an exception if i is not valid

- valid(i)
 - description: verifies if the iterator is valid
 - pre: $i \in \mathcal{I}$
 - post:

$$valid = \begin{cases} \textit{True}, & \text{if } i \text{ refers a valid element from the container} \\ \textit{False}, & \text{otherwise} \end{cases}$$

- first(i)
 - description: sets the cursor of an iterator to point the first element of the iterated container
 - pre: $i \in \mathcal{I}$
 - **post:** $i' \in \mathcal{I}$, the cursor of i' points to the first element of the container, if it is not empty, or i' is invalid, otherwise

Types of iterators

The interface presented presented previously describes the simplest iterator: *unidirectional* and *read-only* .

A *unidirectional* iterator can be used to iterate through a container in one direction only (usually *forward*).

A *read-only* iterator can be used to iterate through the container, but without changing it.

Types of iterators

A **bidirectional** iterator can be used to iterate in both directions. It has a *previous* operation in addition to the *next* operation.

A **random access** iterator allows performing multiple steps at once (not just one step forward or one step backward).

A **read-write** iterator can be used to add/delete elements to/from the container.

Using the iterator



Printing the content of a container using the iterator:

```
subalgorithm printContainer(c) is:
//pre: c is a container
//post: the elements of c were printed
//we create an iterator using the iterator method of the container
   iterator(c, i)
   while valid(i) execute
      //get the current element from the iterator
      elem ← getCurrent(i)
      print elem
      //go to the next element
      next(i)
   end-while
end-subalgorithm
```

Obs: This sub-algorithm is general and can be used to print the elements of any iterable container.

Iterator for a Dynamic Array

What will be the type of the iterator cursor?

Iterator for a Dynamic Array

- What will be the type of the iterator cursor?
- In case of a Dynamic Array, the simplest way to represent the iterator is to retain the index of the current element.



Representation:

IteratorDA:

da: Dynamic Array current: Integer

Iterator for a Dynamic Array - init



Creating a new iterator:

subalgorithm init(i, da) is:

//i is an IteratorDA, da is a Dynamic Array

i.da ← da

i.current \leftarrow 1

end-subalgorithm



Complexity:

Iterator for a Dynamic Array - init



Creating a new iterator:

subalgorithm init(i, da) is: //i is an IteratorDA, da is a Dynamic Array

i.da ← da

i.current \leftarrow 1

end-subalgorithm



Complexity: ⊖(1)

Iterator for a Dynamic Array - getCurrent



Getting the current element:

```
function getCurrent(i) is:
  if not valid(i) then
    @throw an exception
  end-if
  getCurrent ← i.da.elems[i.current]
end-function
```



Iterator for a Dynamic Array - getCurrent

```
Getting the current element:
```

```
function getCurrent(i) is:
   if not valid(i) then
     @throw an exception
   end-if
   getCurrent ← i.da.elems[i.current]
end-function
```



Iterator for a Dynamic Array - next



Moving to the next element in the traversal:

subalgorithm next(i) is:
if not valid(i) then
@throw exception
end-if
i.current ← i.current + 1
end-subalgorithm



Iterator for a Dynamic Array - next



Moving to the next element in the traversal:

subalgorithm next(i) is: if not valid(i) then @throw exception end-if i.current ← i.current + 1 end-subalgorithm



59

Iterator for a Dynamic Array - valid



Checking if the iterator is valid:

```
function valid(i) is:

if i.current <= i.da.length then

valid ← True

else

valid ← False

end-if

end-function
```



Iterator for a Dynamic Array - first



Resetting the iterator:

subalgorithm first(i) is:

i.current \leftarrow 1

end-subalgorithm



O Complexity: ⊖(1)

Iterator for a Dynamic Array



We can print the content of a Dynamic Array in two ways:

- Using an iterator (as presented above for a container)
- 2 Using the indexes of elements

Print with Iterator



Printing all the elements of an array using the iterator:

```
subalgorithm printDAWithIterator(da) is:
//pre: da is a DynamicArray
//we create an iterator using the iterator method of DA
iterator(da, it)
while valid(it) execute
//get the current element from the iterator
elem ← getCurrent(it)
print elem
//go to the next element
next(it)
end-while
end-subalgorithm
```



What is the complexity of printDAWithIterator?

Print with indexes



Printing all the elements of an array using indexes:

```
subalgorithm printDAWithIndexes(da) is:
//pre: da is a Dynamic Array
for i ← 1, size(da) execute
elem ← getElement(da, i)
print elem
end-for
end-subalgorithm
```



What is the complexity of printDAWithIndexes?

Iterator for a Dynamic Array

In case of a Dynamic Array both printing algorithms have $\Theta(n)$ complexity

For other data structures/containers we need iterators because there are no indexes or using them violates the abstraction principle.

Dynamic Array - review

The elements of a dynamic array occupy a contiguous memory block.



Advantages:

- accessing any element in Θ(1)
- adding at the end in Θ(1) amortized and removing from the end in Θ(1) (amortized)
- space efficiency
- Disadvantages:
 - adding and removing from the beginning of the array in $\Theta(n)$

Containers represented using dynamic arrays

Dynamic arrays are used for representing the following containers:

- ADT List
 ArrayList in Java, Vector in C++ STL, Python lists ([])
- ADT Stack
 Stack in Java
- Sorted List
- Bag
- Matrix

Arrays - Applications



Applications of arrays:



Machine Learning / Data Mining

Representing the data instances as an array of characteristics



Speech processing

• Each speech signal is an array. The filters that are used to remove noise in a recording are also arrays.



Multi-player games

 To store the scores and sort them in descending order to clearly make out the rank of each player in the game



= Representing strings

· Every string is an array of characters



- David M. Mount, Lecture notes for the course Data Structures (CMSC 420), at the Dept. of Computer Science, University of Maryland, College Park, 2001
- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, and Clifford Stein, Introduction to Algorithms, Third Edition, The MIT Press. 2009
- Narasimha Karumanchi, Data Structures and Algorithms Made Easy: Data Structures and Algorithmic Puzzles, Fifth Edition, 2016
- Clifford A. Shaffer, A Practical Introduction to Data Structures and Algorithm Analysis, Third Edition, 2010

Thank you

