Methods for solving nonlinear equations

A

1. Approximate $\sqrt[4]{2}$ using 2 iterations of Newton's method and the starting point $x_0 = 1$. (Hint: You may use the equation $x^4 = 2$).
2. Compute the next two iterates of the secant, false position and bisection methods to solve the equation $x^3 - 2x^2 = 2$, using $x_0 = 1$, $x_1 = 4$.
3. Show that the function $g(x) = 2 + \frac{5}{x^2}$ has a unique fixed point in the interval $[\frac{5}{2}, 3]$. How many iterations are necessary to find a fixed point that is accurate to within 10^{-3}? Starting with $x_0 = 3$, compute x_1 and x_2.

Facultative:

- Consider Newton's method for finding the positive square root of $A > 0$. Assuming that $x_0 > 0$, $x_0 \neq \sqrt{A}$, show that the sequence of iterates can be written as
$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{A}{x_n}\right).$$
- Suppose that a is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval that contains a. Show that the following fixed-point method has $g'(a) = 0$:
$$g(x) = x - m\frac{f(x)}{f'(x)}.$$

B

1. Approximate $\sqrt[4]{3}$ using 2 iterations of Newton's method and the starting point $x_0 = 1$. (Hint: You may use the equation $x^4 = 3$).
2. Compute the next two iterates of the secant, false position and bisection methods to solve the equation $x^3 + 3x^2 = 1$, using $x_0 = -3$, $x_1 = -2$.
3. Show that the function $g(x) = 2 + \frac{5}{x^2}$ has a unique fixed point in the interval $[\frac{5}{2}, 3]$. How many iterations are necessary to find a fixed point that is accurate to within 10^{-3}? Starting with $x_0 = 3$, compute x_1 and x_2.

Facultative:

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