Probability Theory - Coursework
Spring 2025

Provide the solutions to the following 4 problems. Submit them via Teams, in pdf form.
Deadline: 30 May 2025.

Problem 1. Compute the expectation and the variance for a random variable X with the following continuous distributions, using, respectively, the density functions provided in the lecture notes:

a. Uniform distribution

b. Exponential distribution

c. Gamma distribution

d. Normal distribution

Problem 2. Compute the expectation and the variance for a random variable X with the following discrete distributions, using, respectively, the probability mass functions provided in the lecture notes:

a. Bernoulli distribution

b. Binomial distribution

c. Geometric distribution

d. Poisson distribution

Problem 3. A robotic system consists of two main sensors. The first sensor has a 35% chance of malfunctioning. The second sensor is independent from the first one and has a 45% chance of malfunctioning. A malfunction in the first sensor alone causes the system to fail with a 60% probability. If the second sensor malfunctions alone, the probability of system failure is 85%. If both sensors malfunction, the system fails with a probability of 92%. Given that the system has failed, what is the probability that both sensors are malfunctioning?

Problem 4. A medical test consists of 5 independent chemical analysis steps, performed one after the other. Each step involves a reagent reaction whose completion time follows an exponential distribution with a mean of 7 minutes.

a. Find the expected total duration of the full testing process and its variance.

b. What is the probability that the entire test is completed in under 20 minutes?