UCB Math 128B, Spring 2016: Programming Assignment 1

Due February 25

In this assignment, we will study the solution of a linear system using iterative methods. The system comes from the discretization of a two-dimensional PDE, Poisson's equation. For an integer size n, it can be created with the MATLAB code

```
A = delsq(numgrid('S', n+2)) * (n+1)^2;
b = ones(n^2, 1);
```

which produces a matrix A of size $N \times N$ where $N = n^2$.

1. Solve the system $Ax = b$ for $n = 50$ using the following 6 methods, all starting from the zero initial vector.

   (a) Jacobi's method

   (b) The Gauss-Seidel method

   (c) The SOR method, with $\omega = 2/(1 + \sqrt{8}/(n+1))$

   (d) The Conjugate Gradient (CG) method

   (e) The Preconditioned CG method, using $M =$ the tridiagonal part of A

   (f) The Preconditioned CG method, using $M = R^t R$ where R is computed by the MATLAB command `R = ichol(A);`. Note: Do not form the matrix M explicitly, but use two backslashes involving R^t and R.

   Perform 1000 iterations with each method and compute the ∞-norm of the error at each iteration. Plot the convergence (errors vs. iteration) of the methods in a semi-log plot.

2. Solve the system again using the 6 methods above, but for the values $n = 5, 10, 20, 50, 100$. Iterate until the error is less than 10^{-6} times the initial error, but not more than 1,000 iterations. Plot the number of iterations vs. the system size N in a log-log plot.

3. Using the plot in **2.** and assuming that the computational time for each iteration is proportional to N, estimate the exponent p in the *total solution time* $T = \text{constant} \cdot N^p$ for each of the methods.