UCB Math 128B, Spring 2016: Programming Assignment 3

Due April 14

1. Solve problem 10.2.13 in the textbook as it is written. Note that the expressions are considerably simplified by the fact that $\mu = 0$. Also note that if the array `th` contains the angles θ_n, for $n = 1, \ldots, N$, the shape of the chute can be plotted using the commands

```
x = [0; cumsum(dy*tan(th))];
y = -dy * (0:N)';
plot(x,y,'.-'), axis equal, drawnow
```

2. Write a Newton solver for the general case, not assuming $\mu = 0$. The Jacobian matrix is quite complicated, so compute it using *numerical differentiation* instead of deriving it. That is, for a given function $\boldsymbol{F}(\boldsymbol{x})$, set

$$J = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}} \approx \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \cdots & \boldsymbol{a}_n \end{bmatrix} \quad \text{with} \quad \boldsymbol{a}_j \approx \frac{\boldsymbol{F}(\boldsymbol{x} + \delta \boldsymbol{e}_j) - \boldsymbol{F}(\boldsymbol{x} - \delta \boldsymbol{e}_j)}{2\delta}$$

Implement the function and the Jacobian matrix in MATLAB functions of the form

```
function F = p13F(th, mu, X, dy, v0)
function J = p13J(th, mu, X, dy, v0)
```

Use the functions in Newton's method, and solve using the parameters

$$N = 50, \qquad \mu = 0.2, \qquad X = 2, \qquad \Delta y = 0.05, \qquad v_0 = 0.2$$

with zero initial condition. Iterate until $\|\theta^{(k)} - \theta^{(k-1)}\|_\infty < 10^{-8}$. Output $\|\theta^{(k)} - \theta^{(k-1)}\|_\infty$ for each iteration k, and plot the final chute shape.

3. Try solving the problem in **2** again, with the parameter $\Delta y = 0.02$. Observe the convergence problems. To address this, implement a Runge-Kutta based continuation algorithm as described in Section 10.5 of the textbook, using $N = N_C$ steps (note: not the same N as before). Try your solver with $N_C = 1$, 2, and 4, and if it succeeds, perform a final solve for full accuracy using Newton's method. Report your convergence results and plot the final chute shape (for the correct solution).