

Ejemplo 1. Construya un spline cúbico natural que pase por los puntos (1, 2), (2, 3), (3, 5).

$$\begin{aligned}
 S_0(x) &= a_0 + b_0(x-1) + c_0(x-1)^2 + d_0(x-1)^3 \\
 S_1(x) &= a_1 + b_1(x-2) + c_1(x-2)^2 + d_1(x-2)^3 \\
 S_2(x) &= a_2 + b_2(x-3) + c_2(x-3)^2 + d_2(x-3)^3
 \end{aligned}$$

$$\begin{aligned}
 1 \Rightarrow S_0(1) &= a_0 = 2 \\
 3 \Rightarrow S_1(2) &= 2 + b_0 + c_0 + d_0 \Rightarrow 1 = b_0 + c_0 + d_0 \quad (1) \\
 5 \Rightarrow S_1(3) &= 3 + b_1 + c_1 + d_1 \Rightarrow 2 = b_1 + c_1 + d_1 \quad (2)
 \end{aligned}$$

$$\begin{aligned}
 S_0'(x) &= b_0 + 2c_0(x-1) + 3d_0(x-1)^2 \\
 S_1'(x) &= b_1 + 2c_1(x-2) + 3d_1(x-2)^2 \\
 b_0 + 2c_0 + 3d_0 &= S_0'(2) = S_1'(2) = b_1 \quad (3)
 \end{aligned}$$

$$\begin{aligned}
 S_0''(x) &= 2c_0 + 6d_0(x-1) \\
 S_1''(x) &= 2c_1 + 6d_1(x-2) \\
 2c_0 + 6d_0 &= S_0''(2) = S_1''(2) = 2c_1 \\
 c_0 + 3d_0 &= c_1 \quad (4)
 \end{aligned}$$

$$\begin{aligned}
 0 &= S_0'(1) = 2c_0 \Rightarrow c_0 = 0 \\
 0 &= S_1'(3) = 2c_1 + 6d_1 \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 b_0, d_0, b_1, c_1, d_1 \\
 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{Gauss} \begin{bmatrix} I_{5 \times 5} \\ 0 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 S_0(x) &= a_0 + b_0(x-1) + c_0(x-1)^2 + d_0(x-1)^3 \\
 \text{y el otro para } [2, 3], \text{ que se denota} \\
 S_1(x) &= a_1 + b_1(x-2) + c_1(x-2)^2 + d_1(x-2)^3
 \end{aligned}$$

$$\begin{aligned}
 S_0(x) &= 2 + \frac{3}{4}(x-1) + 0(x-1)^2 + \frac{1}{4}(x-1)^3 \\
 S_1(x) &= 3 + \frac{3}{2}(x-2) + \frac{3}{4}(x-2)^2 + \frac{1}{4}(x-2)^3 \\
 S(x) &= \begin{cases} 2 + \frac{3}{4}(x-1) + \frac{1}{4}(x-1)^3, & \text{para } x \in [1, 2] \\ 3 + \frac{3}{2}(x-2) + \frac{3}{4}(x-2)^2 + \frac{1}{4}(x-2)^3, & \text{para } x \in [2, 3]. \end{cases}
 \end{aligned}$$

### 3. Construya el trazador cúbico libre de los siguientes datos.

$$\begin{array}{c|c|c}
 c_i & x_i & f(x_i) \\
 \hline
 x_0 = -0.5 & -0.0247500 & = 0_0 \\
 x_1 = -0.25 & 0.3349375 & = a_1 \\
 x_2 = 0 & 1.1010000 & = a_2
 \end{array}$$

$$\begin{aligned}
 h_0 &= -0.25 + 0.5 = 0.25 = \frac{1}{4} \\
 h_1 &= 0 - 0.25 = 0.25 = \frac{1}{4}
 \end{aligned}$$

$$A = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 12(1.101 - 0.3349375) - 12(0.3349375 - 0.02475) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.8765 \\ 0 \end{bmatrix}$$

$$\begin{aligned}
 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 4.8765 \\ 0 \end{bmatrix} &\xrightarrow{-\frac{1}{2}F_2 + F_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 4.8765 \\ 0 \end{bmatrix} \\
 &\xrightarrow{-\frac{1}{2}F_3 + F_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 4.8765 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} c_0 &= 0 \\ c_1 &= 3/4 \\ c_2 &= 0 \end{aligned}
 \end{aligned}$$

$$b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1}), \quad (3.20)$$

$$\begin{aligned}
 j=0 \Rightarrow b_0 &= \frac{1}{\frac{1}{4}}(3 - 2) - \frac{1}{3}(2 \cdot 0 + \frac{3}{4}) = \frac{3}{4} \\
 j=1 \Rightarrow b_1 &= \frac{1}{\frac{1}{3}}(5 - 3) - \frac{1}{3}(2 \cdot \frac{3}{4} + 0) = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 c_{j+1} &= c_j + 3d_j/h_j. \quad (3.17) \\
 j=0 \Rightarrow \frac{3}{4} &= 0 + 3d_0 \cdot \frac{1}{4} \Rightarrow d_0 = \frac{1}{4} \\
 j=1 \Rightarrow 0 &= \frac{3}{4} + 3d_1 \cdot 1 \Rightarrow d_1 = -\frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 S_0(x) &= -0.02475 + 1.032375(x + 0.5) + 0(x + 0.5)^2 + 6.502(x + 0.5)^3 \\
 S_1(x) &= 0.3349375 + 2.2515(x + 0.25) + 4.8765(x + 0.25)^2 + 6.502(x + 0.5)^3
 \end{aligned}$$

$c_i$	$i$	$a_i$	$b_i$	$c_i$	$d_i$
0	0	-0.02475000	1.03237500	0.00000000	6.50200000
1	1	0.33493750	2.25150000	4.87650000	-6.50200000

$$b_j = \frac{1}{h_j}(a_{j+1} - a_j) - \frac{h_j}{3}(2c_j + c_{j+1}), \quad (3.20)$$

$$b_0 = \frac{1}{\frac{1}{4}}(3 - 2) - \frac{1}{3}(2 \cdot 0 + \frac{3}{4}) = \frac{3}{4}$$

$$b_1 = \frac{1}{\frac{1}{3}}(5 - 3) - \frac{1}{3}(2 \cdot \frac{3}{4} + 0) = \frac{3}{2}$$

$$c_{j+1} = c_j + 3d_j/h_j. \quad (3.17)$$

$$j=0 \Rightarrow 4.8765 = 0 + 3d_0 \cdot \frac{1}{4} \Rightarrow d_0 = 6.502$$

$$j=1 \Rightarrow 0 = 4.8765 + 3d_1 \cdot \frac{1}{4} \Rightarrow d_1 = -6.502$$

$$S_0(x) = -0.02475 + 1.032375(x + 0.5) + 0(x + 0.5)^2 + 6.502(x + 0.5)^3$$

$$S_1(x) = 0.3349375 + 2.2515(x + 0.25) + 4.8765(x + 0.25)^2 + 6.502(x + 0.5)^3$$