Escola Tècnica Superior d'Enginyeria
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Approximate Reasoning

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Fuzzy Expert Systems

MESIIA – MIA

Logic: Inference rules

- Modus Ponens

$$\begin{array}{l} A \rightarrow B \\ \underline{A \qquad} \\ \therefore B \end{array}$$

If (p and q) or (not s) then c

- Expert Systems are composed by a set of rules and an inference engine that determines how the rules are fired to obtain the final conclusion

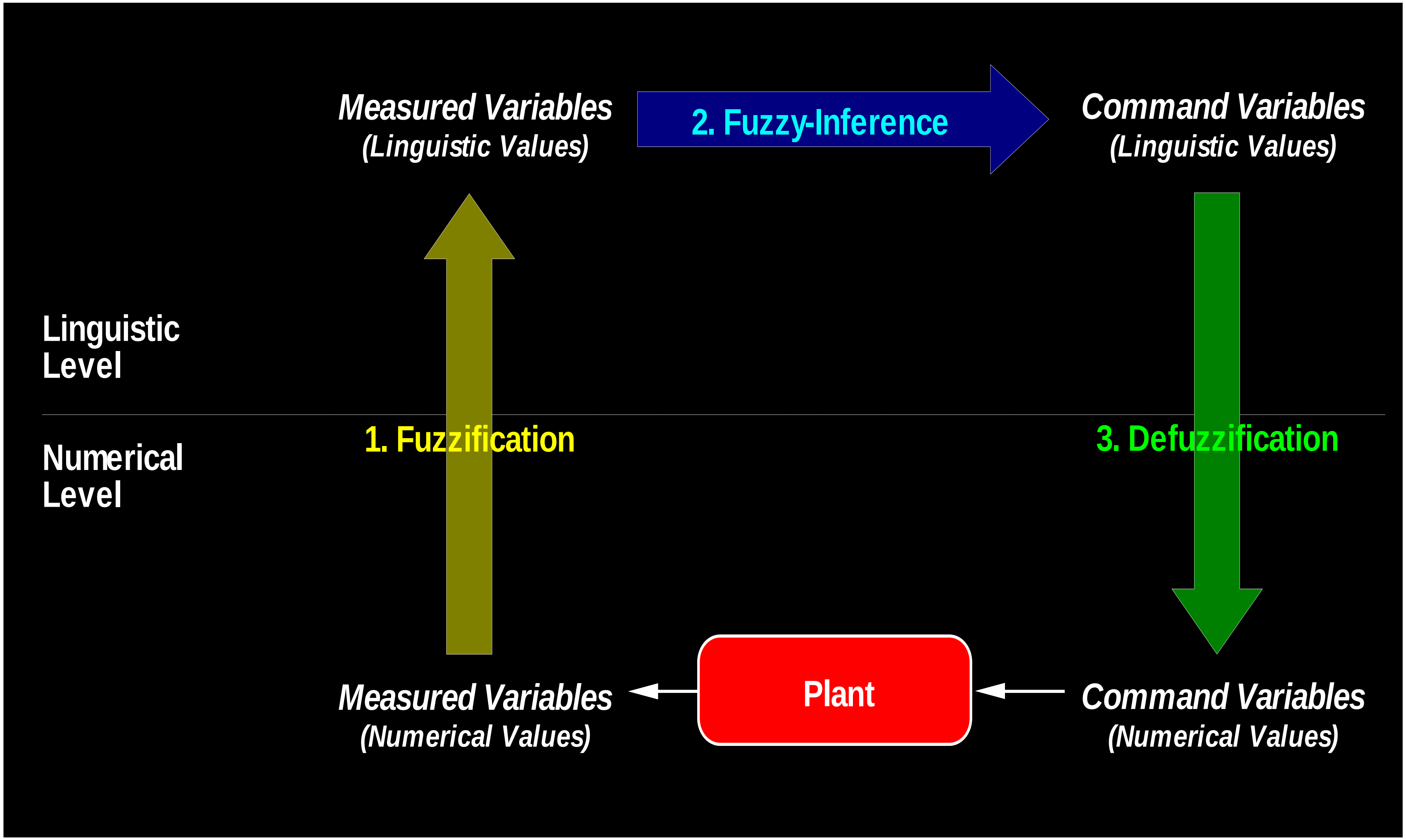
A fuzzy expert system approach

- Rule-based system

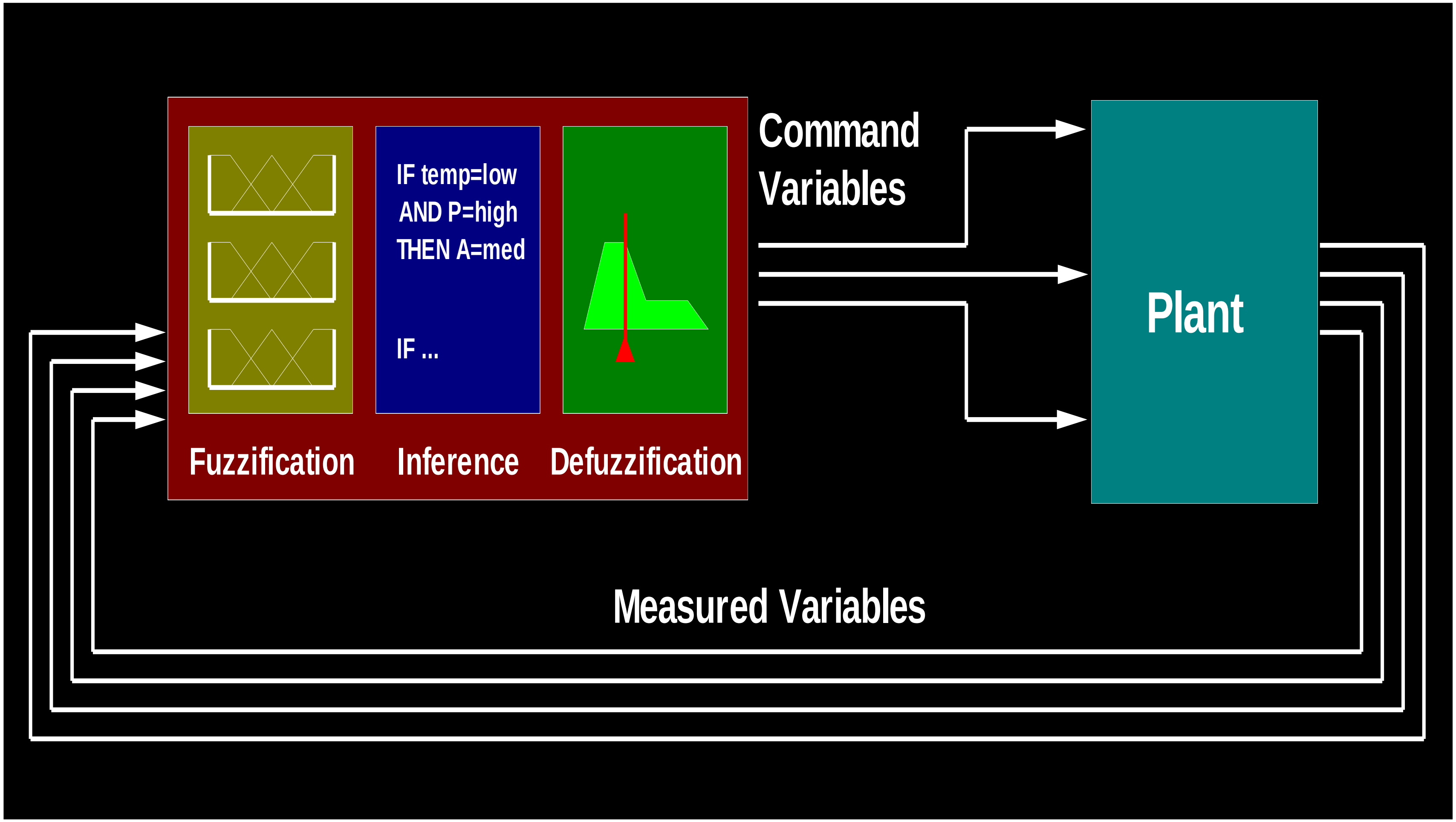
$$p_1 \wedge p_2 \wedge \ldots \wedge p_n \rightarrow q$$

- Fuzzy sets to define linguistic variables
- Each premisse is satisfied at a certain degree
- The conclusion is derived according to the degree of satisfaction of the premises
- Many rules can be activated simultaneously, their conclusions are aggregated
- The conclusion is a fuzzy set that can be defuzzified into a numerical value

Fuzzy Expert System
Architecture of a control system



Fuzzy Expert System Architecture



Fuzzy Rules

- Linguistic variables are used in the conditions and conclusion of the rules
- Examples of fuzzy rules to decide the degree of risk of clients in a bank:
  - If salary is high and the history is good then the risk is nul
  - If salary is medium and the history is good then the risk is low
  - If salary is medium and history is bad then the risk is high

Fuzzy Rules

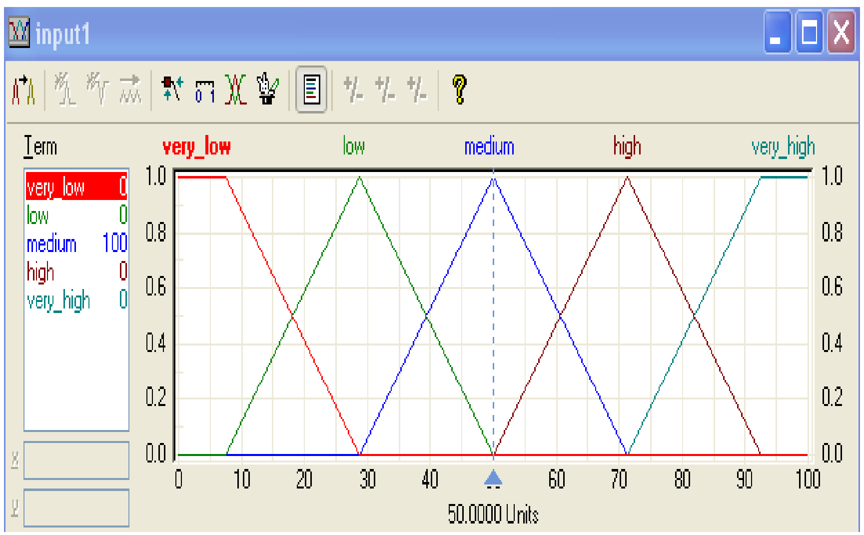
- For each possible combination of values in the conditions, a rule can be written. This is represented in a matrix.

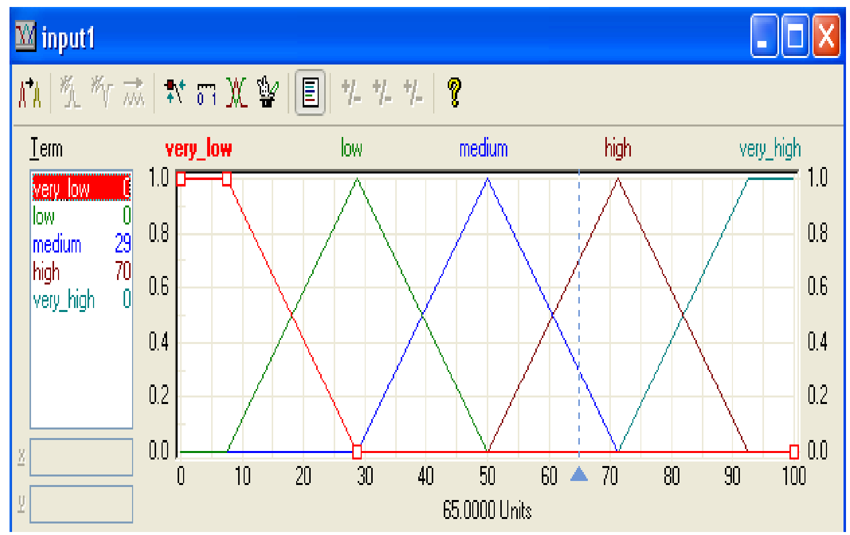
	Good	Bad
Low	Medium	Extreme
Medium	Low	High
High	Nul	Medium

- We have here a variable with 3 values and another with 2, in total 3x2=6 rules.
- The decision variable has 5 values (5 possible different conclusions)

Fuzzification

- The goal is to convert a numerical value into one or two linguistic values.
- Each linguistic value has a fuzzy set that explains its meaning.
- We find the membership degree to all the possible linguistic values.





Inference

The most usual is the Mamdami inference method:

1. Evaluate the antecedent for each rule
2. Obtain each rule's conclusion
3. Aggregate conclusions

Evaluation of the antecedent

- The degree of fulfillment of the conditions is calculated.
- The membership degree of each of the input values is calculated for each linguistic variable.
- The membership values are aggregated.

Evaluation of the antecedent
Aggregation Operators

- Rules are usually conjunctive.
- For conjunctive rules the aggregation operator must be a T-norm: min or prod
- In case of modeling a disjunctive rule, a T-conorm must be used: max or bounded sum (=probabilistic or).
- Other aggregation operators could be used, such as averages (but be careful with properties like compensation).

Activation of the conclusion

- If the satisfaction degree is c, the conclusion of the rule is truncated at the level c.
- If many rules are activated, each one is truncated at the corresponding level. Then the results are joined together.

Activation of the conclusion
Union Operators

- The union operators for fuzzy sets are T-conorms.

Max	$\mu(x) = \max_{r=1..m} \mu_r(x)$
Bounded Sum	$\mu(x) = \min\left(1, \sum_{r=1}^{m} \mu_r(x)\right)$

- La combination of T-norm min with T-conorm max, is known as Mamdani inference method.
- *There is another well-known inference method, known as Sugeno or Takagi-Sugeno.*

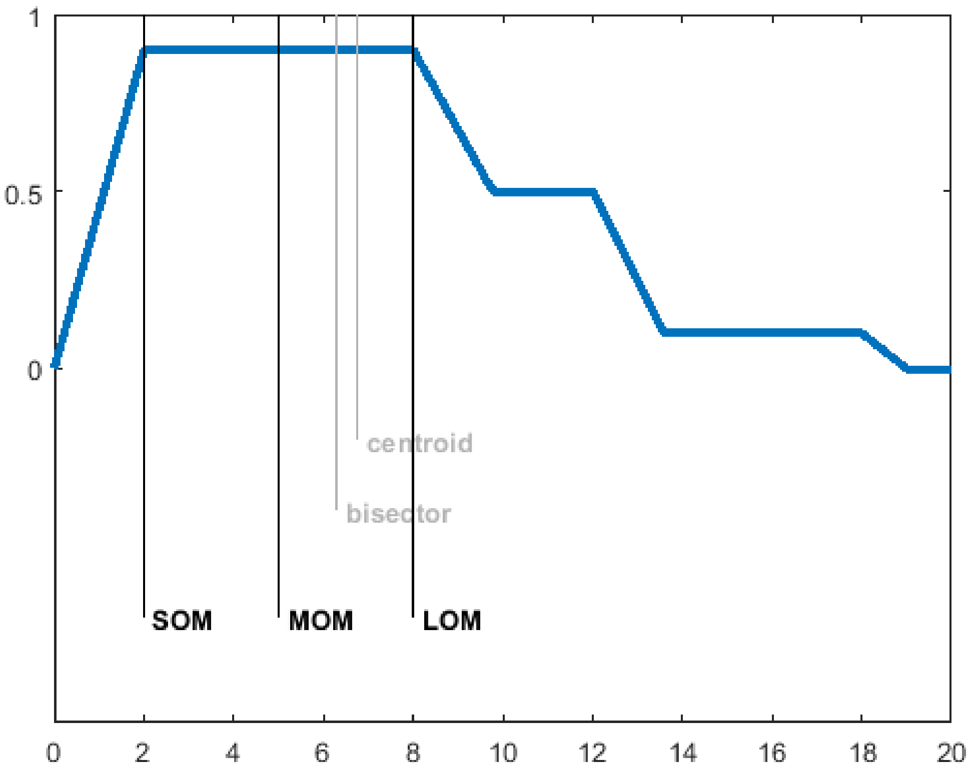
Defuzzification

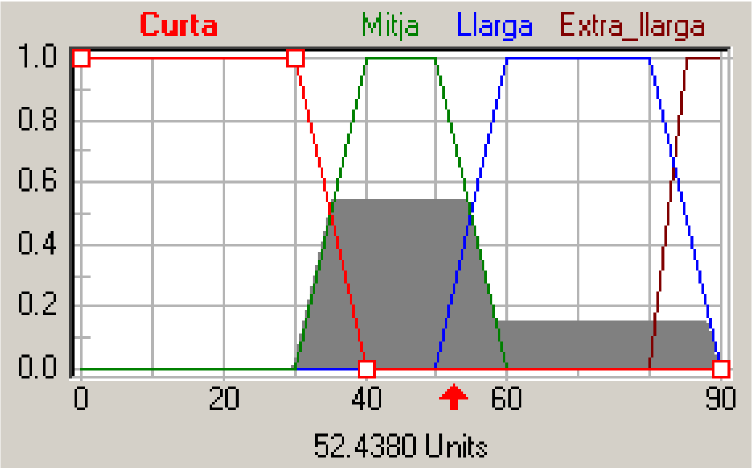
- The goal is to reduce a fuzzy set into a single value in the reference domain.
- There are many defuzzification methods.
- They basically consider the value of the domain with the highest membership value.
- Sometimes it is also interesting to consider the membership values of the rest of points.

Defuzzification methods

- Centre of Area: average of all the values in the referece domain, which are weighted by the corresponding membership degree.
- Median of Maximum (MoM): median of the values that correspond to the linguistic term with maximum degree of memberhip.
- Others:
  - Largest of Maximum (Lom): maximum x of the values that correspond to the term with maximum degree of membership
  - Smallest of Maximums (Som): minimum x of the values that correspond to the term with maximum degree of membership

Defuzzification



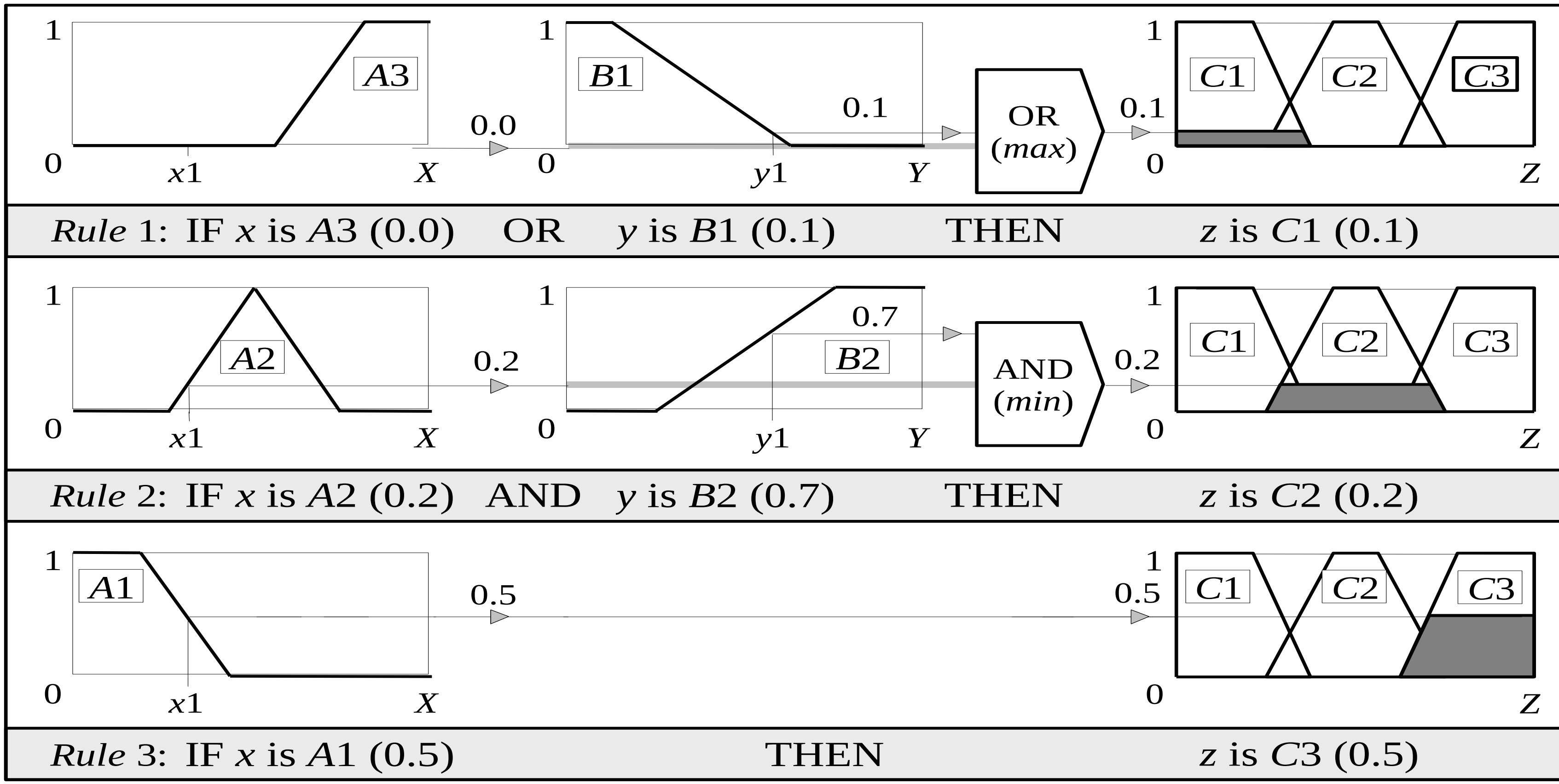


$$CoA_{x=1..90} = \frac{\sum x \cdot \mu(x)}{\sum \mu(x)}$$

Centroid = Center of Are

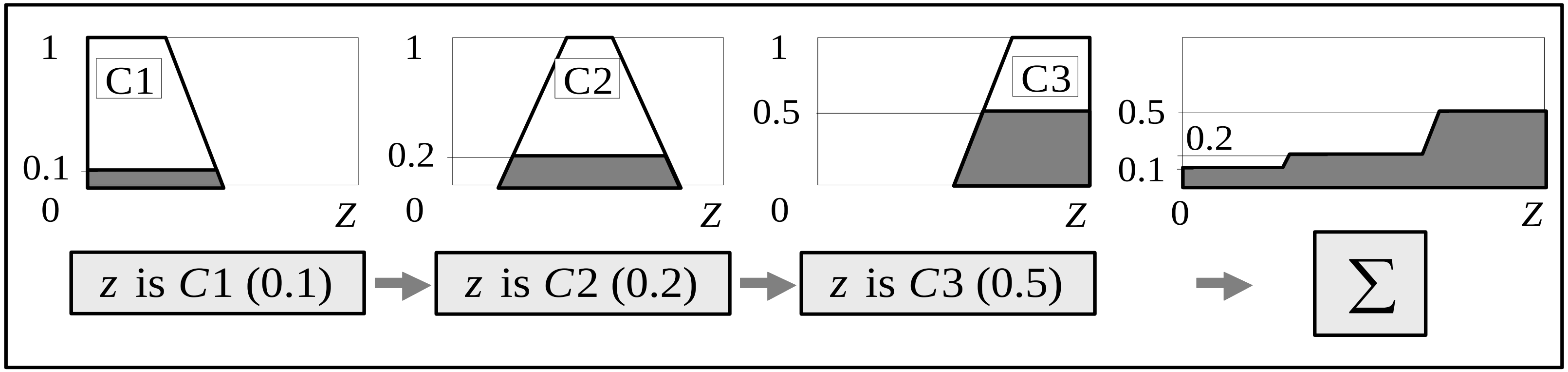
Inference: Mamdani method

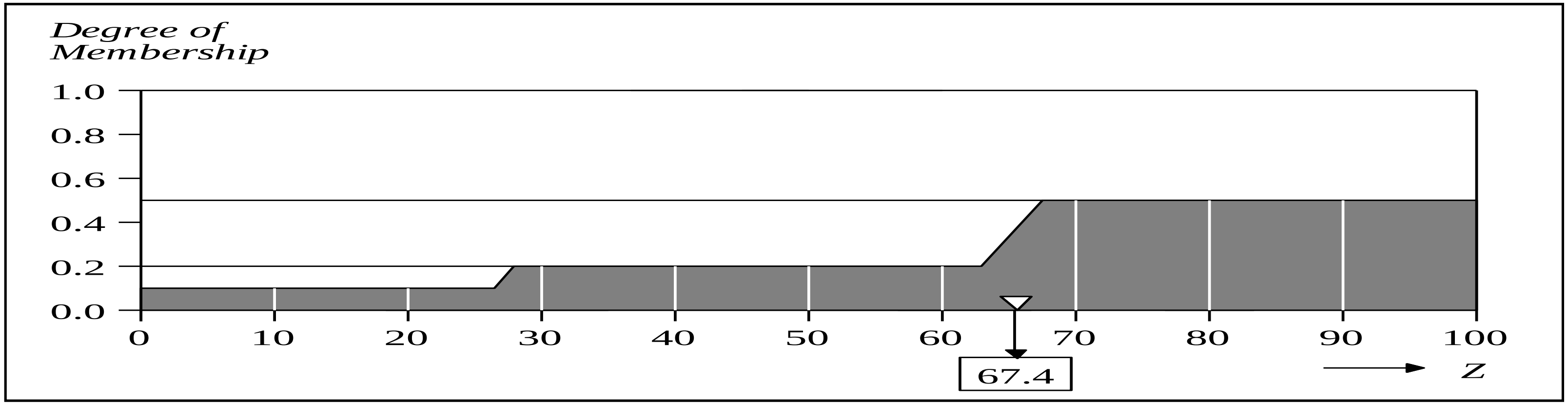
1. Evaluate the antecedent for each rule
2. Obtain each rule's conclusion



Inference: Mamdani method

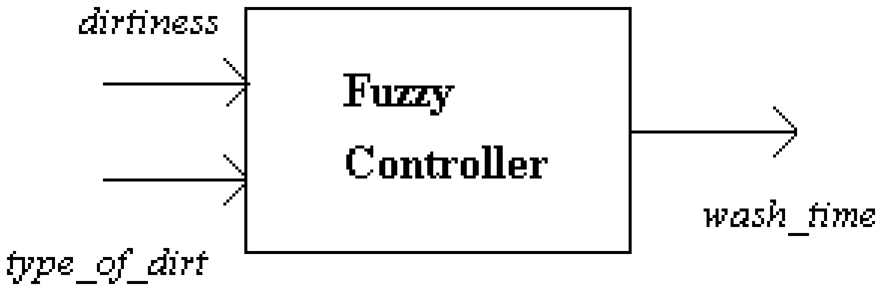
3. Aggregate conclusions
4. Defuzzification





Inference example

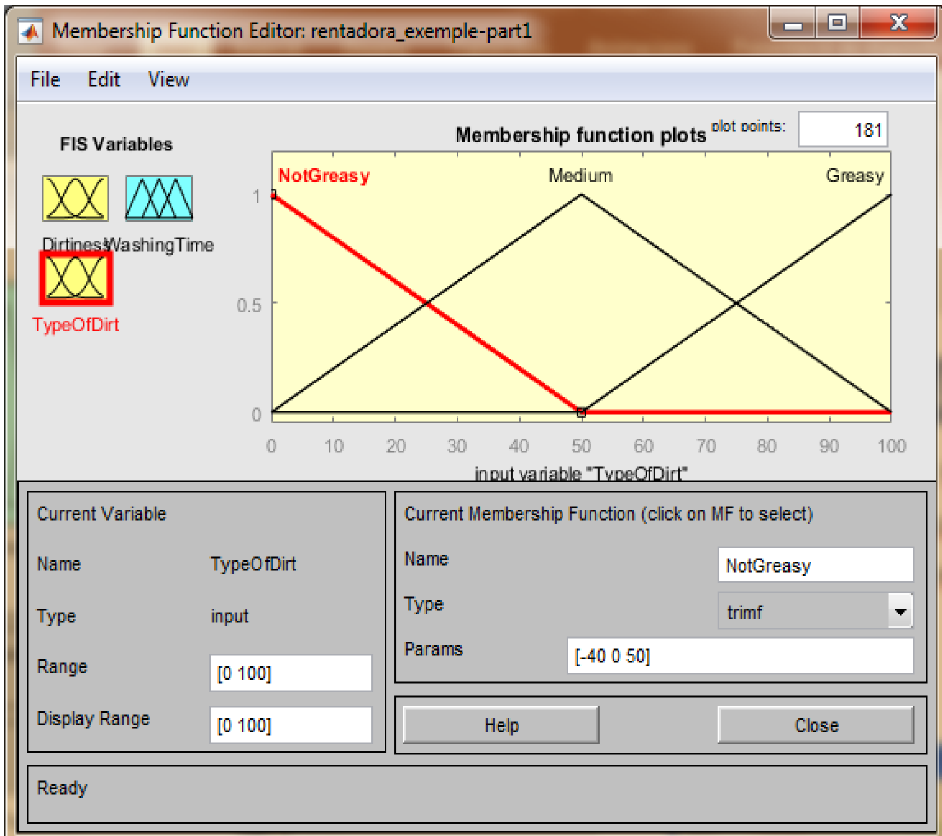
- Many washing machines include fuzzy controllers to adjust the water needed, as well as, the washing time.
- Example with Mamdami system and defuzzification with CoA.

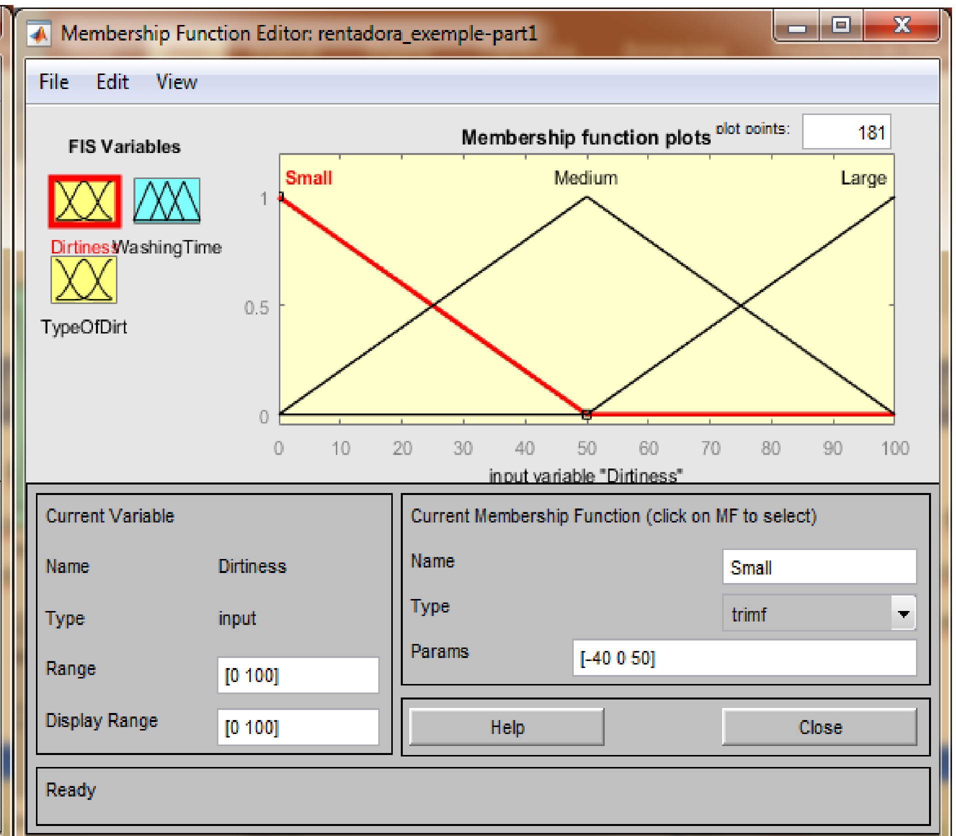


The following screenshots have been taken with the Fuzzy plugin of Matlab software.

EXAMPLE: washing machine

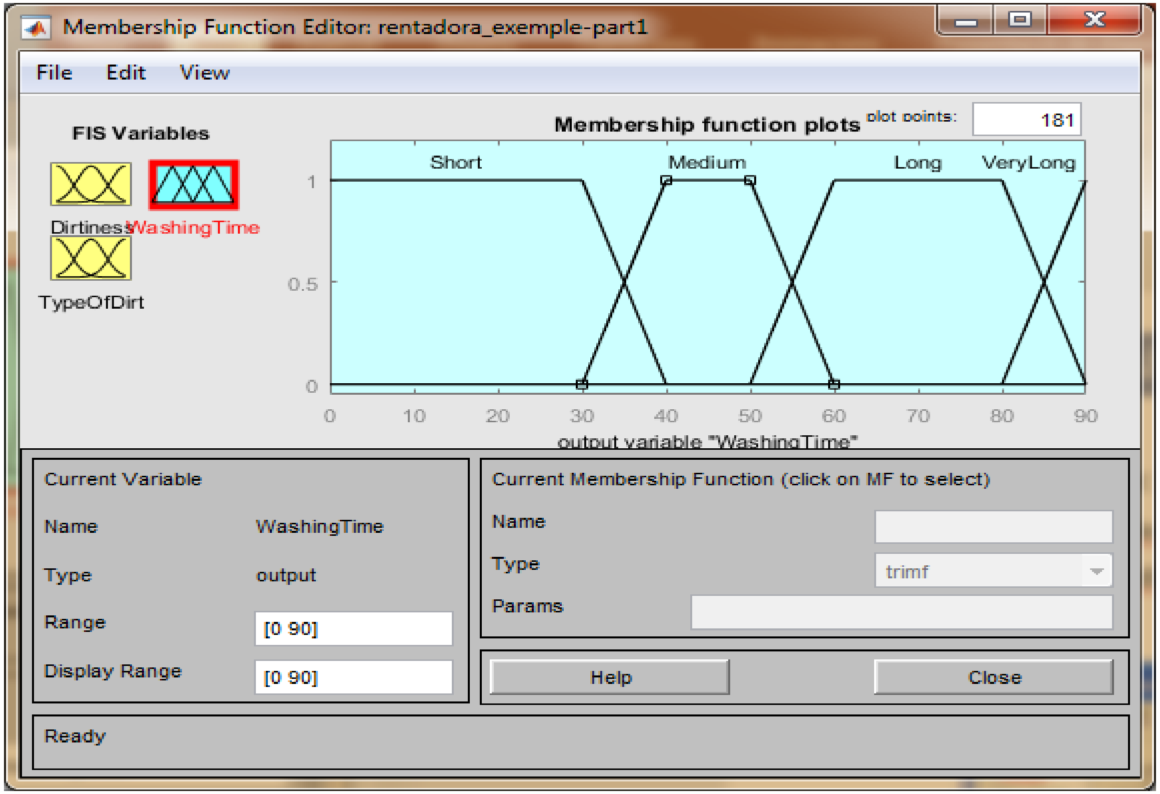
- Input variables: dirtiness and type of dirt
  - Reference scale in percentages (horizontal axis)
  - 3 values for each variable, with different labels





EXAMPLE: washing machine

- Output variable: washing time
  - Reference scale in minutes, from 0 to 90 min.
  - 4 values with different distribution (not homogeneous)



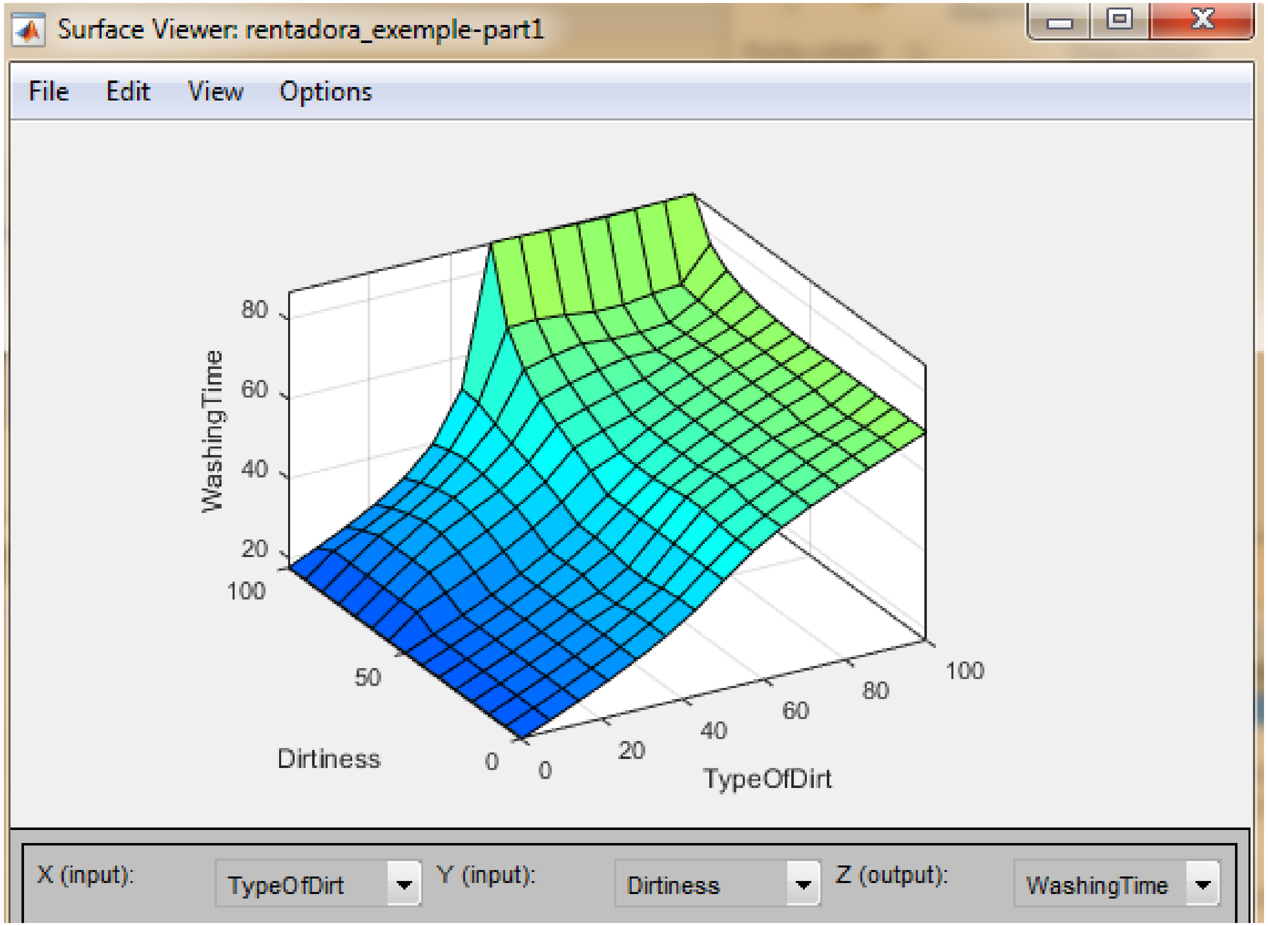
EXAMPLE: washing machine

- Set of rules
  - With 3 values of 2 variables we have 3x3=9 combinations.
  - With these 7 rules we can model all the posible input combinations.
  - The first rules only has 1 premise, it is not using typeOfDirt (so it is covering the 3 posible values of typeOfDirt).

```
1. (TypeOfDirt==NotGreasy) => (WashingTime=Short) (1)
2. (Dirtiness==Small) & (TypeOfDirt==Medium) => (WashingTime=Medium) (1)
3. (Dirtiness==Medium) & (TypeOfDirt==Medium) => (WashingTime=Medium) (1)
4. (Dirtiness==Large) & (TypeOfDirt==Medium) => (WashingTime=VeryLong) (1)
5. (Dirtiness==Small) & (TypeOfDirt==Greasy) => (WashingTime=Long) (1)
6. (Dirtiness==Medium) & (TypeOfDirt==Greasy) => (WashingTime=Long) (1)
7. (Dirtiness==Large) & (TypeOfDirt==Greasy) => (WashingTime=VeryLong) (1)
```

EXAMPLE: washing machine

- Graphical representation of the output given 2 inputs, according to the rules in the expert system



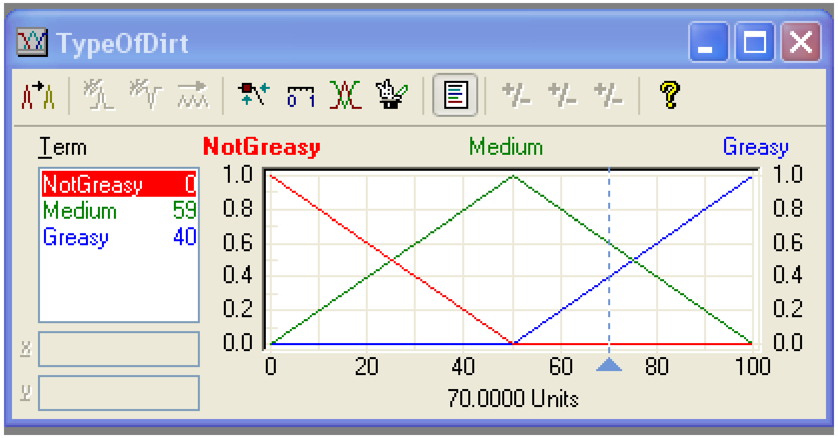
EXAMPLE: washing machine

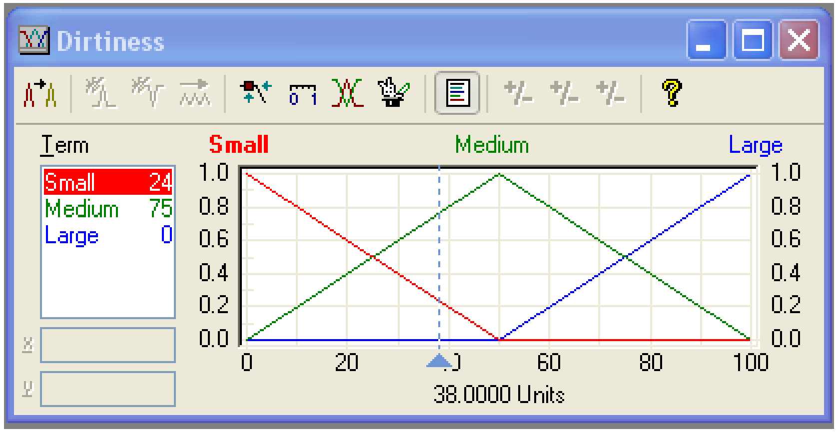
On Friday, mum has to clean the dirty clothes of his son. She puts all the clothes inside the washing machine.

The evaluation of the machine gives:

TypeOfDirt=70 & Dirtiness=38

$\mu_{medium}(70)=0.6$ $\mu_{greasy}(70)=0.4$ $\mu_{small}(38)=0.25$ $\mu_{medium}(38)=0.75$



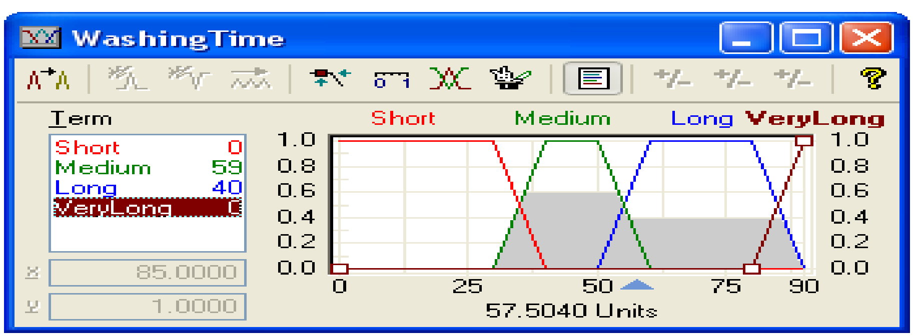


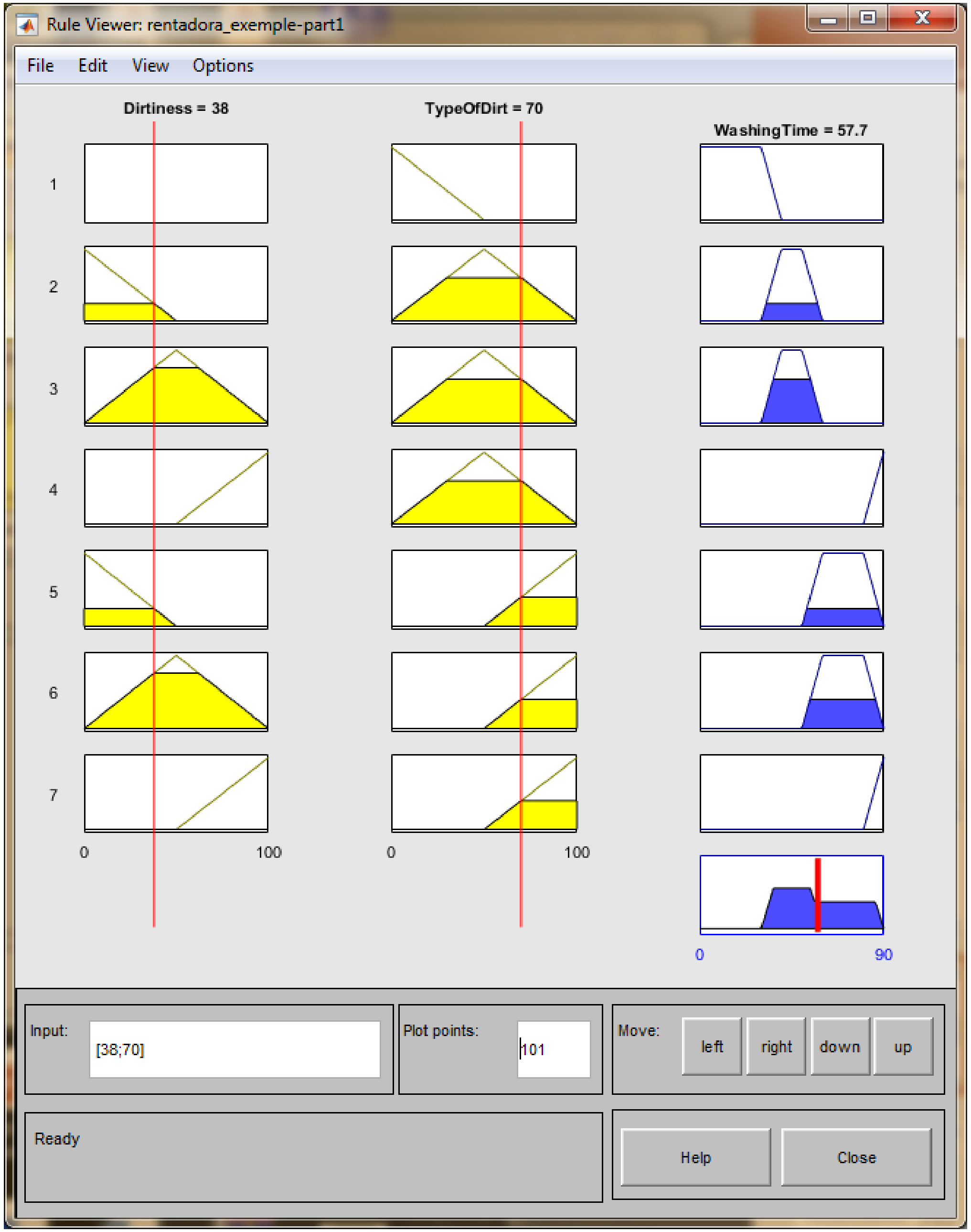
EXAMPLE: washing machine

$\mu_{medium}(70)=0.6$ $\mu_{greasy}(70)=0.4$ $\mu_{small}(38)=0.25$ $\mu_{medium}(38)=0.75$

Rules that are applicable?

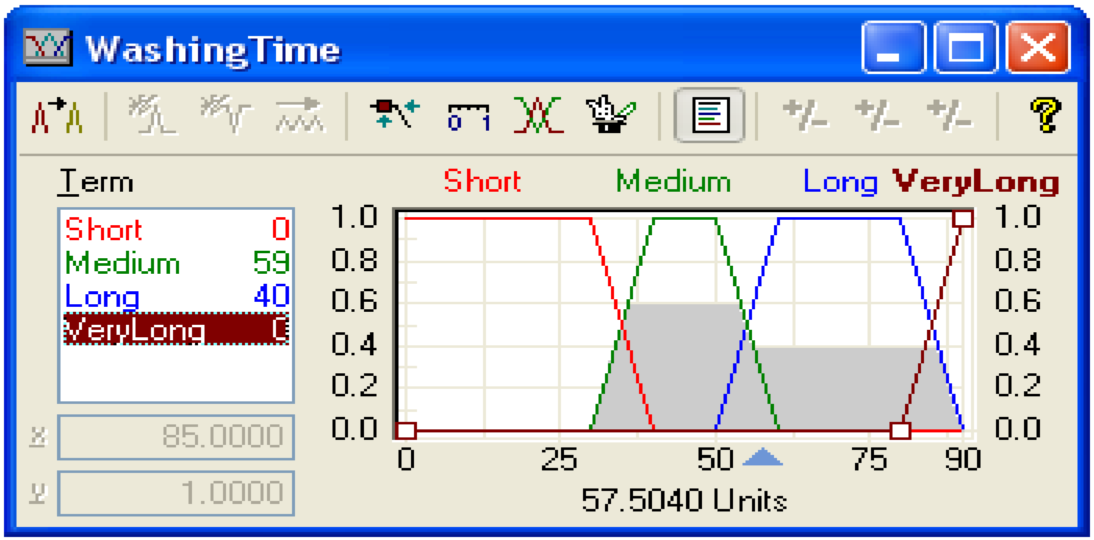
If	And	Operators	Then	With
1	2	Min / Max	1	
TypeOfDirt	Dirtiness	Min	WashingTime	Max [%]
TypeOfDirt.*NotGreasy*		=>	WashingTime.*Short*	100
0.6 eOfDirt.*Medium*	0.25 iness.*Small*	0.25	WashingTime.*Mediu*	0.6 100
0.6 eOfDirt.*Medium*	0.75 iness.*Medium*	0.6	WashingTime.*Medium*	100
TypeOfDirt.*Medium*	Dirtiness.*Large*	=>	WashingTime.*VeryLong*	100
0.4 eOfDirt.*Greasy*	0.25 ness.*Small*	0.25	WashingTime.*Long*	0.4 100
0.4 eOfDirt.*Greasy*	0.75 ness.*Medium*	0.4	WashingTime.*Long*	100
TypeOfDirt.*Greasy*	Dirtiness.*Large*	=>	WashingTime.*VeryLong*	100




Rule Viewer: rentadora_exemple-part1
File
Edit
View
Options
Dirtiness = 38
TypeOfDirt = 70
WashingTime = 57.7
1
2
3
4
5
6
7
0
100
0
100
0
90
Input:
[38;70]
Plot points:
101
Move:
left
right
down
up
Ready
Help
Close

EXAMPLE: washing machine

- The defuzzification gives us the number of minutes of wash.
- We can calculate it in a discrete approximation using intervals (e.g 2,5 minutes) with CoA.



- The time needed to clean the clothes is 57,7 min.

$$CoA_{x=1..90} = \frac{\sum x \cdot \mu(x)}{\sum \mu(x)}$$

30	0	0,0
32,5	0,25	8,1
35	0,5	17,5
37,5	0,6	22,5
40	0,6	24,0
42,5	0,6	25,5
45	0,6	27,0
47,5	0,6	28,5
50	0,6	30,0
52,5	0,6	31,5
55	0,5	27,5
57,5	0,4	23,0
60	0,4	24,0
62,5	0,4	25,0
65	0,4	26,0
67,5	0,4	27,0
70	0,4	28,0
72,5	0,4	29,0
75	0,4	30,0
77,5	0,4	31,0
80	0,4	32,0
82,5	0,4	33,0
85	0,4	34,0
87,5	0,25	21,9
90	0	0,0
sum:	10,5	606,0
CoA		57,7

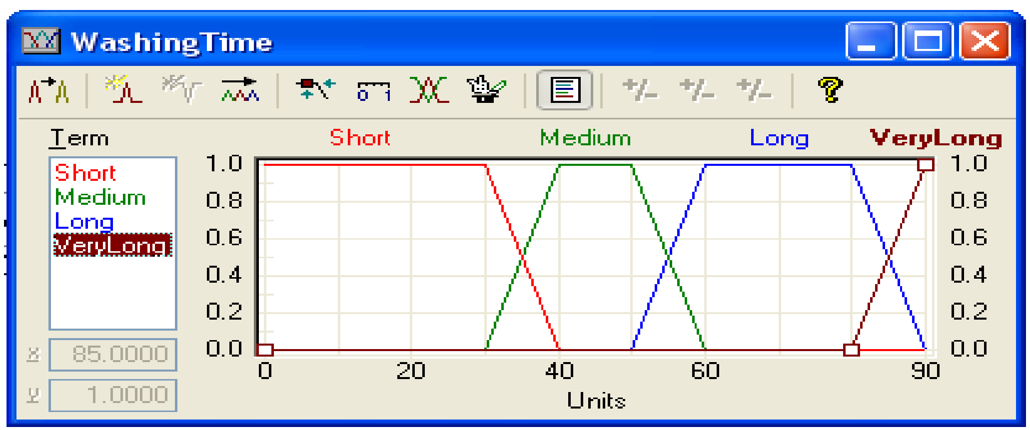
Degree of support of the rules

- Degree of support (DoS) of a rule: indicates the level of importance of the rule.
- By default the degree is maximum (=1).
- Other degrees can be used (from 0 to 1).
- The degree of support must be multiplied by the degree of satisfaction of the antecedents in order to calculate the final degree of satisfaction of the rule (truncating level).

EXAMPLE: washing machine

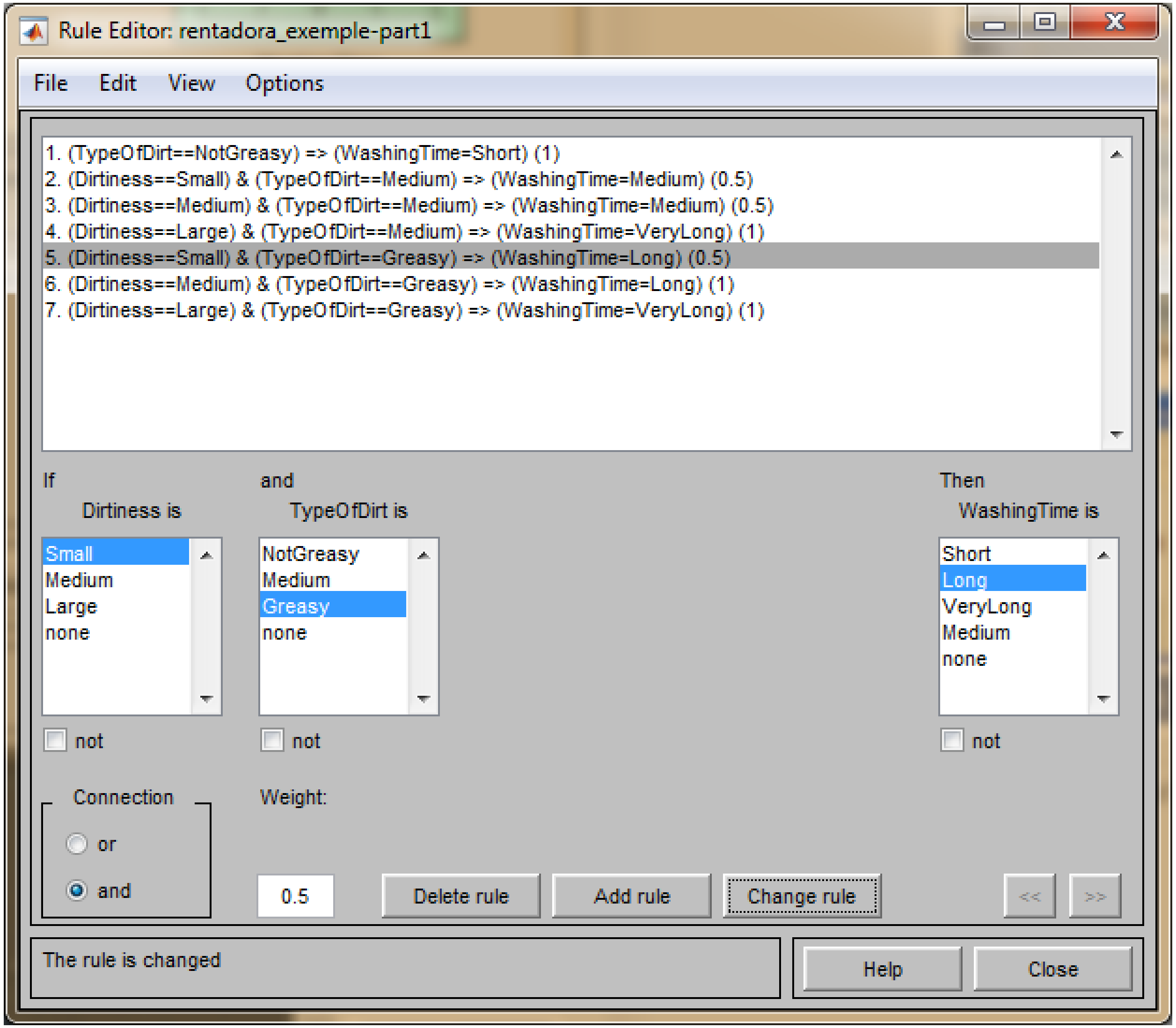
$\mu_{medium}(70)=0.6$ $\mu_{greasy}(70)=0.4$ $\mu_{small}(38)=0.25$ $\mu_{medium}(38)=0.75$

Name	If	And	Operators	Then	With	
SE1	1	2	Min / Max	1		
	TypeOfDirt: 70	Dirtiness: 38	Min	WashingTime: 61,672	DoS [%]	Max
	TypeOfDirt.*NotGreasy*		=>	WashingTime.*Short*	100	
	0.6 TypeOfDirt.*Medium*	0.25 irtiness.*Small*	0.25	WashingTime.*Medium*	50	
	0.6 TypeOfDirt.*Medium*	0.75 irtiness.*Medium*	0.6	WashingTime.*Medium*	50	
	TypeOfDirt.*Medium*	Dirtiness.*Large*	=>	WashingTime.*VeryLong*	100	
	0.4 ypeOfDirt.*Greasy*	0.25 rtiness.*Small*	0.25	WashingTime.*Long*	50	
	0.4 ypeOfDirt.*Greasy*	0.75 rtiness.*Medium*	0.4	WashingTime.*Long*	100	
	TypeOfDirt.*Greasy*	Dirtiness.*Large*	=>	WashingTime.*VeryLong*	100	



Calculate the result

EXAMPLE: washing machine



Additional comments (1)

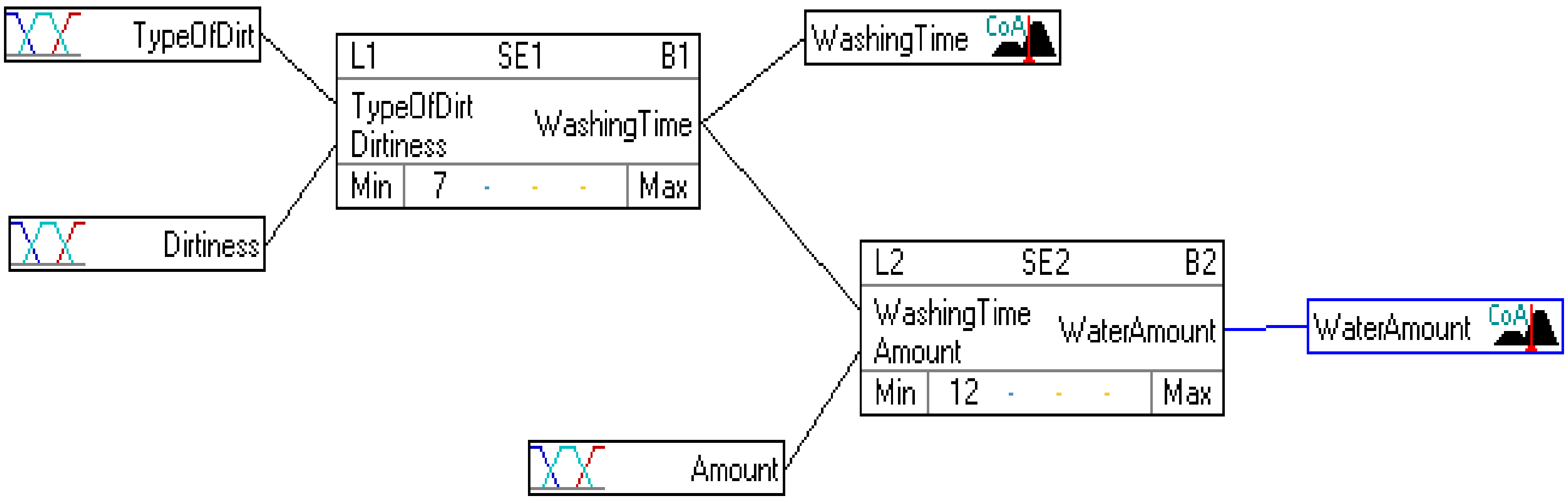
- Drawbacks of expert systems:
  - Variable selection is not trivial
  - A large number of rules must be defined
  - Changes in the domain must invalidate some rules
  - The systems are not easily adaptable
  - Experts spend many time in defining the system

Additional comments (2)

- Hierarchical fuzzy systems
  - The conclusions of one level are used in the following one
  - In this case, the total number of rules decreases, and also the difficulty of defining the rule blocks.
- Difficulties in Hierarchical fuzzy systems:
  - Define the hierarchy of rules
  - Defuzzify or not the conclusion before being used in the following rule block

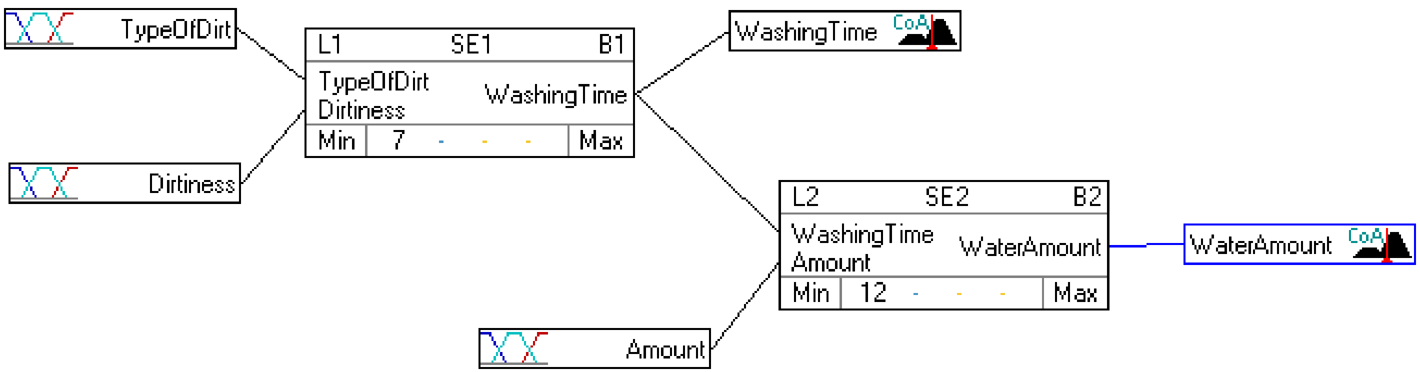
Example of hierarchical ES

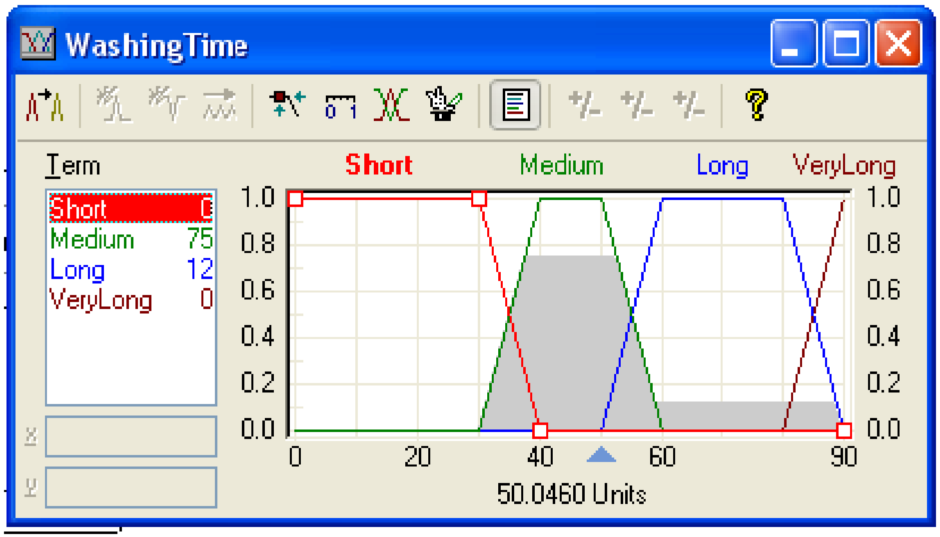
Washing Machine with two levels



Example of hierarchical ES

Washing Machine with two levels





- Defuzzification: 50 min
  + fuzzification: $\mu_{medium}(50)=1.0$

- Direct Propagation:
  $\mu_{medium}(.)=0.78$ and $\mu_{long}(.)=0.16$