Approximate Reasoning

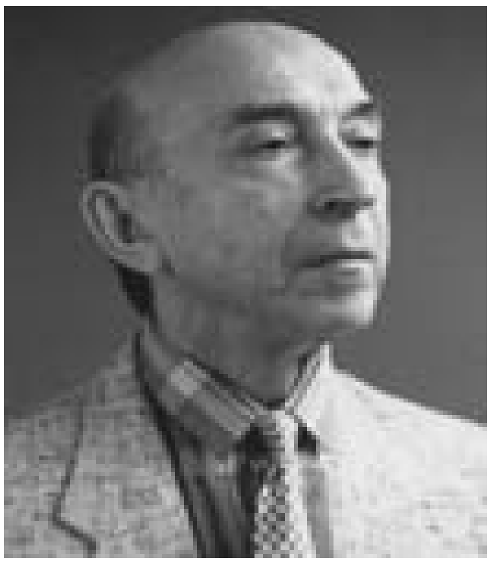
Aida Valls

Fuzzy Logic

MESIIA – MIA

Fuzzy Logic

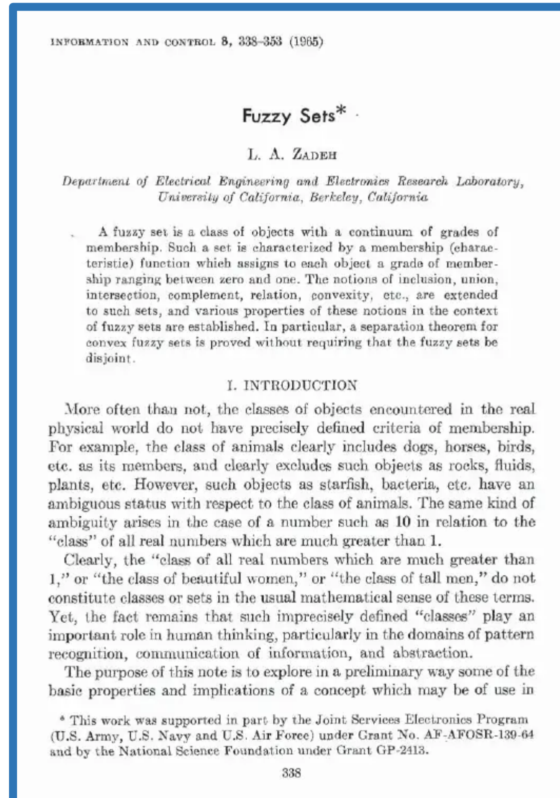
- This model of approximate reasoning was invented by Prof. Lotfi Zadeh in 1965



Lotfi Zadeh (1965 - 2017)

New York Times: [obituary Sept 2017]

Video: https://youtu.be/2ScTwFCcXGo



INFORMATION AND CONTROL 8, 338-353 (1965)

Fuzzy Sets*

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A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

I. INTRODUCTION

More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes such objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers which are much greater than 1," or "the class of beautiful women," or "the class of tall men," do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction.

The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in

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Fuzzy Logic

- Boolean or crisp sets: any object belongs to the set (1) or not (0). The membership to a set is strict, without doubt.
- Fuzzy sets: any object belongs to a set up to a certain degree, between 0 and 1. The membership function takes values in the real domain.

$$\mu_C: X \rightarrow [0,1]$$

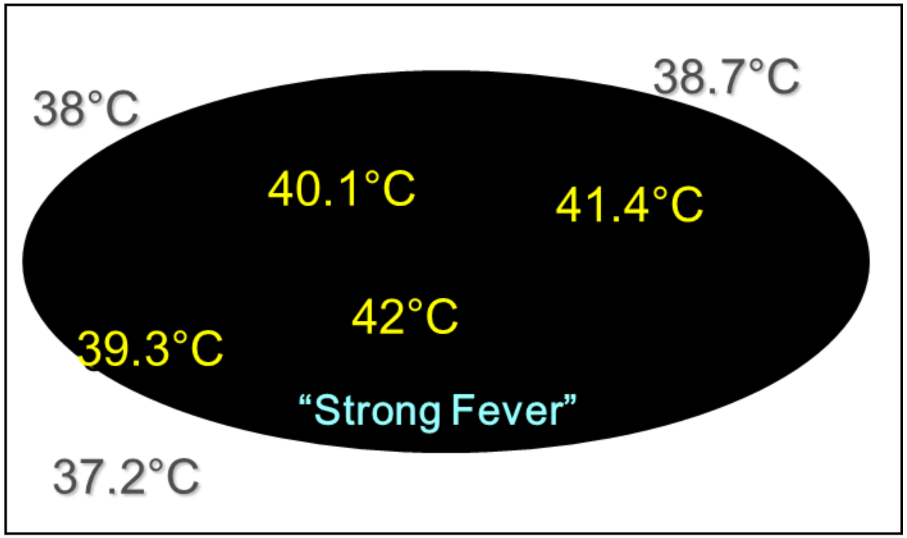
X is the range of possible values of the object, C is the fuzzy set

Remember that Sets and Logic are dual concepts

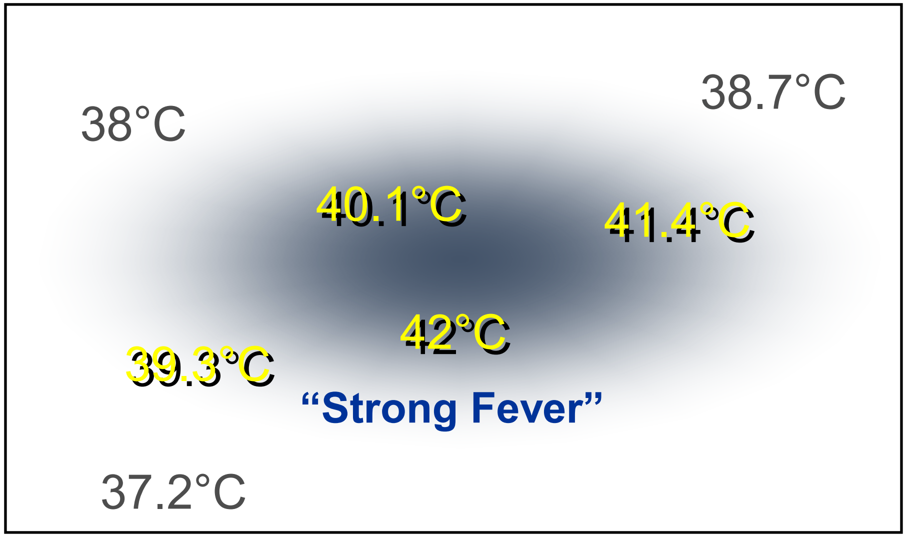
Fuzzy Logic

- Boolean (true/false) logic is extended to Fuzzy (from true to false)

Boolean Set

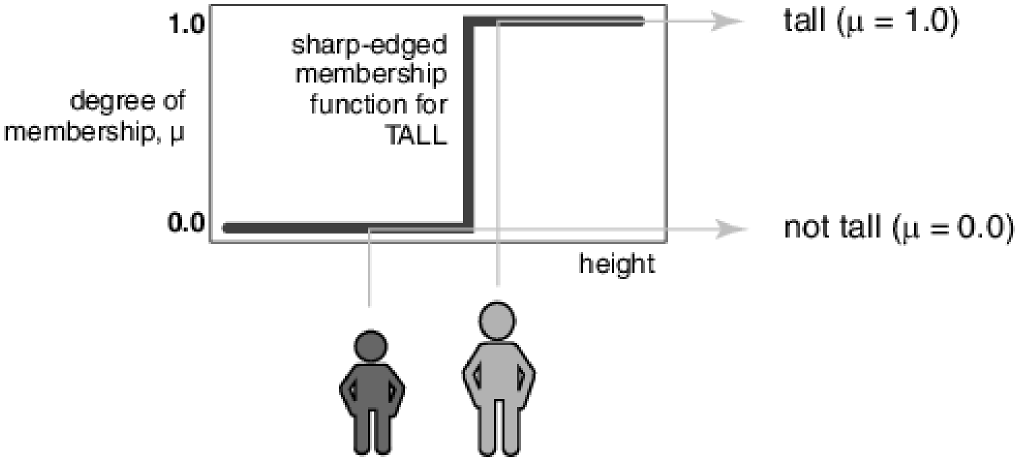


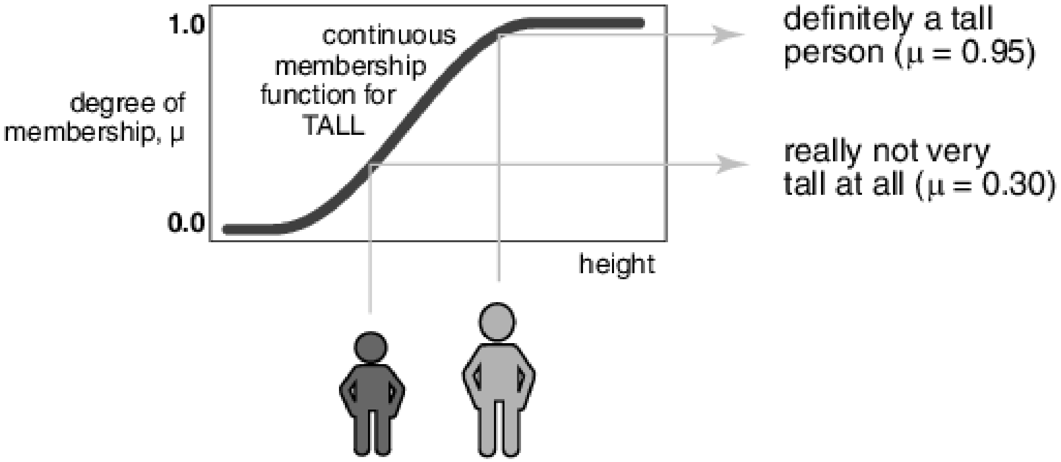
Fuzzy Set



Membership Functions

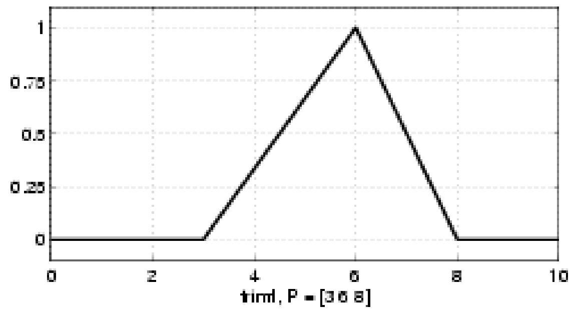
- **Membership function** to define the degree of fulfillment of a predicate



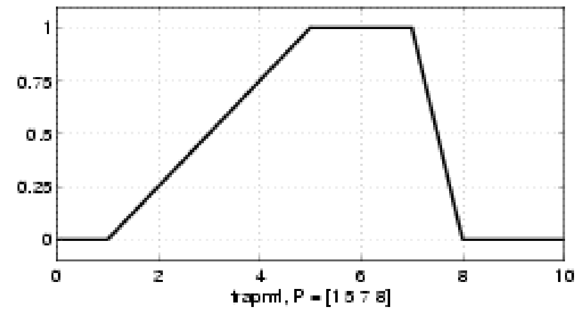


Membership Functions

- The membership function can take different forms but it must be convex

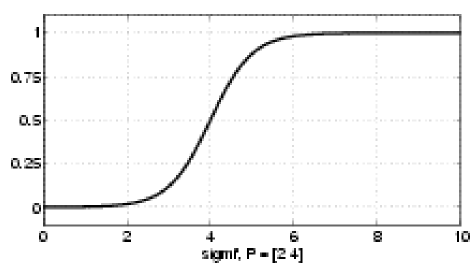


trimf

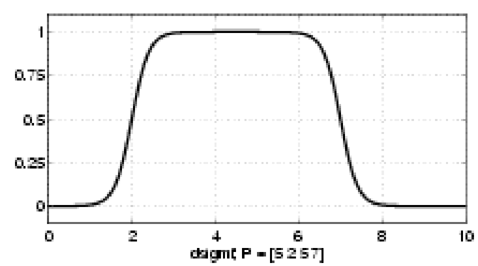


trapmf

Triangular and trapezoidal (the most usual ones)



sigmf



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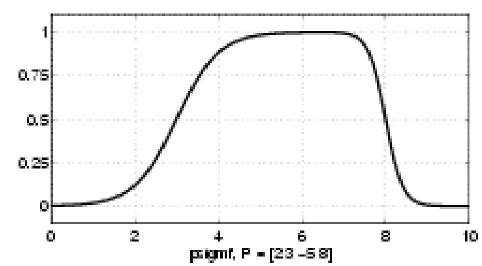


psigmf

Sigmoidal functions

Membership Functions

- Definition of the membership function

Define the membership degree of each point in the reference domain (discrete):

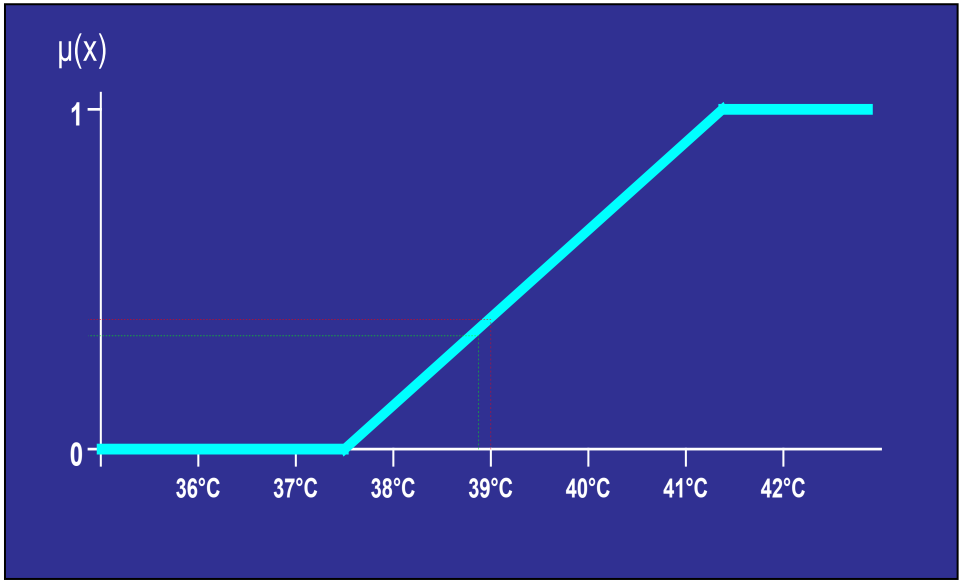
$\mu_{SF}(35°C) = 0$ $\mu_{SF}(38°C) = 0.1$ $\mu_{SF}(41°C) = 0.9$

$\mu_{SF}(36°C) = 0$ $\mu_{SF}(39°C) = 0.35$ $\mu_{SF}(42°C) = 1$

$\mu_{SF}(37°C) = 0$ $\mu_{SF}(40°C) = 0.65$ $\mu_{SF}(43°C) = 1$

Continuous (functional) definition:

$$\mu(x) = \begin{cases} 0 & x < 37.5 \\ 0.25x - 9.375 & 37.5 \le x \le 41.5 \\ 1 & x > 41.5 \end{cases}$$

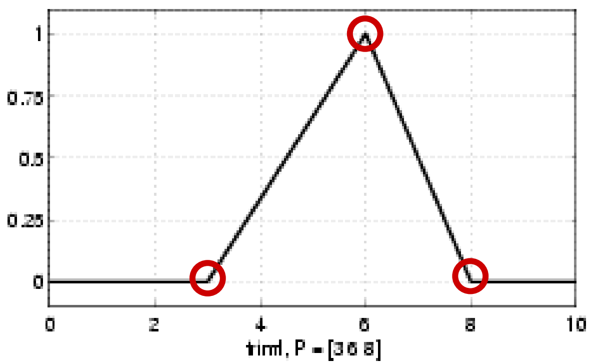


Membership Functions

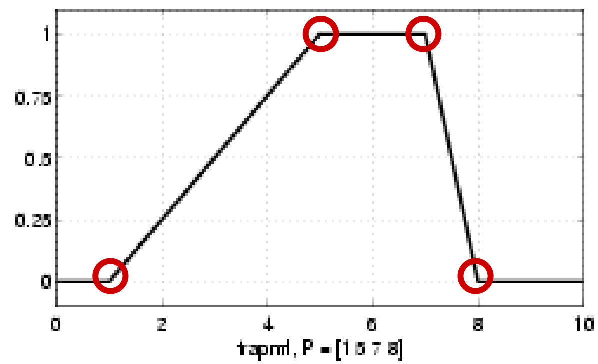
- Triangular and trapezoidal functions can be easily defined with three/four points.

  Tuple (a, b, c, d)

  - a is the value where the membership starts to increase from 0
  - b is the value where the membership arrives to 1
  - c is the value where the membership starts to decrease from 1
  - d is the value where the membership arrives to 0 again



trimf



trapmf

trimf: (3, 6, 6, 8)

trapmf: (1, 5, 7, 8)

Operators

- Negation (complement): $N(x)$

  Properties:

  - Boundaries: $N(0)=1$ i $N(1)=0$
  - Monotonicity: if $x<y$ then $N(x)>N(y)$
  - Involution: $N(N(x))=x$

- Several operators fulfill these conditions. The most known negation operator is

$$N(x)=1-x$$

Operators

- Conjunction (intersection): P(x) and Q(x)

  This operator is called T-norm. Properties:

  - Commutativity: T(x,y) = T(y,x)
  - Associativity: T(x,T(y,z)) = T(T(x,y),z)
  - Monotonicity: if $u<v$ and $x<z$ then $T(u,x)<T(v,z)$
  - Neutrality: T(x,1)=x

- Several operators can be used as T-norms. The best known is the minimum:

$$T(x,y) = \min(x,y)$$

Operators

- Disjunction (union): P(x) or Q(x)

  This operator is called T-conorm. Properties:

  - Commutativity: $S(x,y) = S(y,x)$
  - Associativity: $S(x,S(y,z)) = S(S(x,y),z)$
  - Monotonicity: if $u<v$ and $x<z$ then $S(u,x)<S(v,z)$
  - Neutrality: $S(x,0)=x$

- Several operators can be uses as T-conorms. The most known is the maximum:

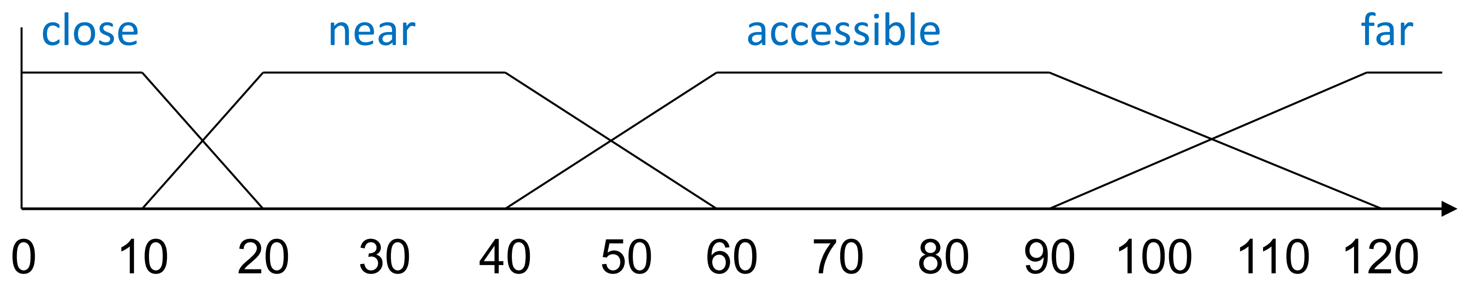
$$S(x,y) = \max(x,y)$$

Linguistic Variable

- There is a fixed set of linguistic values for the variable. {short, medium, tall}
- Each term has an implicit semantics that must be made explicit
- In fuzzy systems, each term has an associated fuzzy membership function on a reference domain (ºC)
- Some conditions can be added to the membership functions of the terms (e.g. add up 1 in each point)

Linguistic Variable

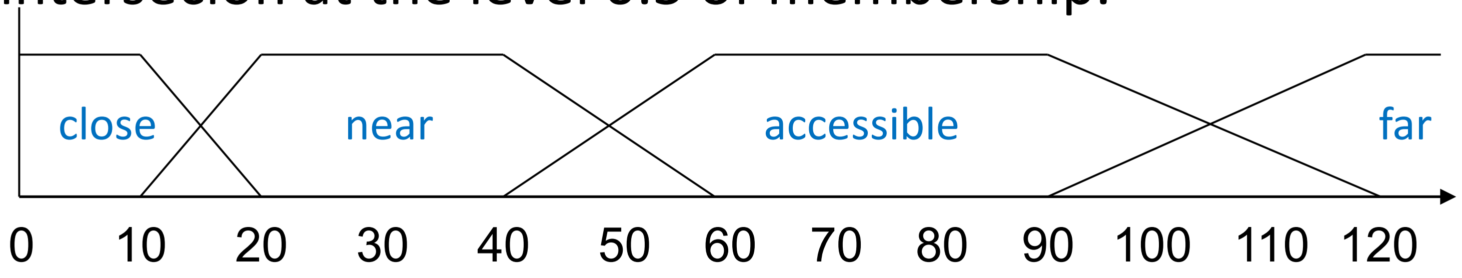
- We will use triangular and trapezoidal fuzzy membership functions
- Example: DISTANCE
  - Reference domain: from 0 to 150 Km.
  - Set of linguistic labels: {close, near, accessible, far}



close = (0,0,10,20)
near = (10,20,40,60)
accessible = (40,60,90,120)
far = (90,120,1000,1000)

Linguistic Variable => Fuzzy partition

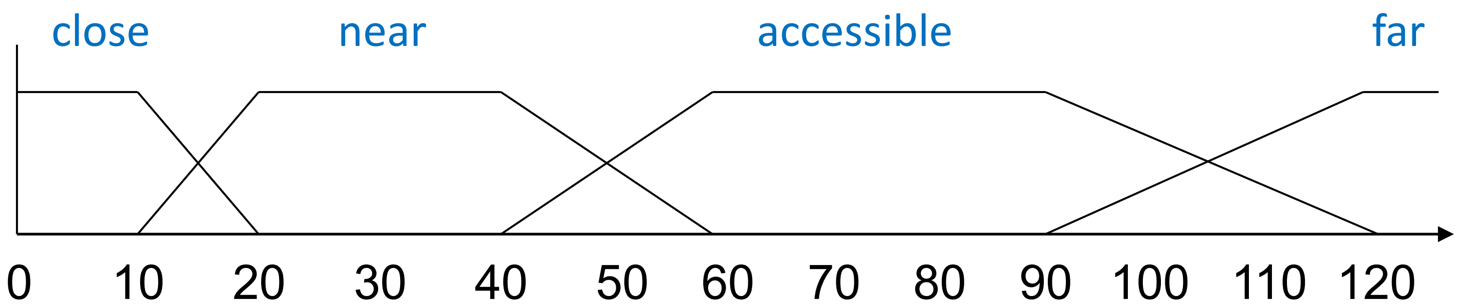
- It is recommended that the membership values of each point in the reference domain add up to 1. Then it satisfies the property of Fuzzy Partition.
- With this restriction, two consecutive terms must have the intersecion at the level 0.5 of membership.



x	$\mu_{close}(x)$	$\mu_{near}(x)$	$\mu_{acces}(x)$	$\mu_{far}(x)$
5	1	0	0	0
45	0	0,75	0,25	0
105	0	0	0,5	0,5

From numerical value to fuzzy value

- Fuzzyfication: procedure that transforms a numerical input value into a fuzzy value (label, membership).

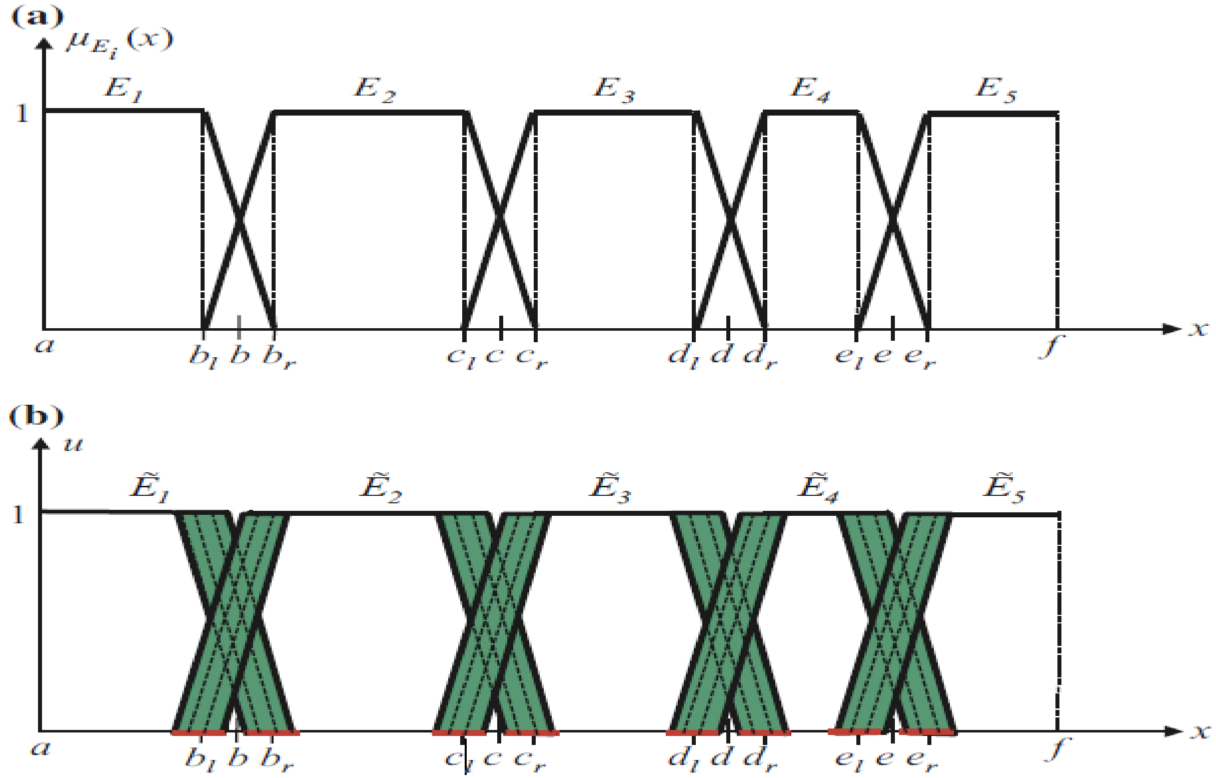


Exercise:
Fill in the table with the membership value for each label, for the X values.

x	$\mu_{close}(x)$	$\mu_{near}(x)$	$\mu_{acces}(x)$	$\mu_{far}(x)$
55	0	0,25	0,75	0
8				
95				
105				

Fuzzy sets types

- We have seen the most common fuzzy sets: type-1
- Type-2 fuzzy sets consider that the definition of the membership function is also fuzzy.



Operators

- Book: Fuzzy sets and fuzzy logic : theory and aplications / George J. Klir and Bo Yuan
- Book: https://link.springer.com/book/10.1007/978-3-319-51370-6

  Accessible through URV credentials (SABIDI tool)

  Study Chapter 2 (2.1-2.4) & Chapter 3 (all)

- European association (students are welcomed by an small fee):
  - EUSFLAT