Escola Tècnica Superior d'Enginyeria
Universitat Rovira i Virgili

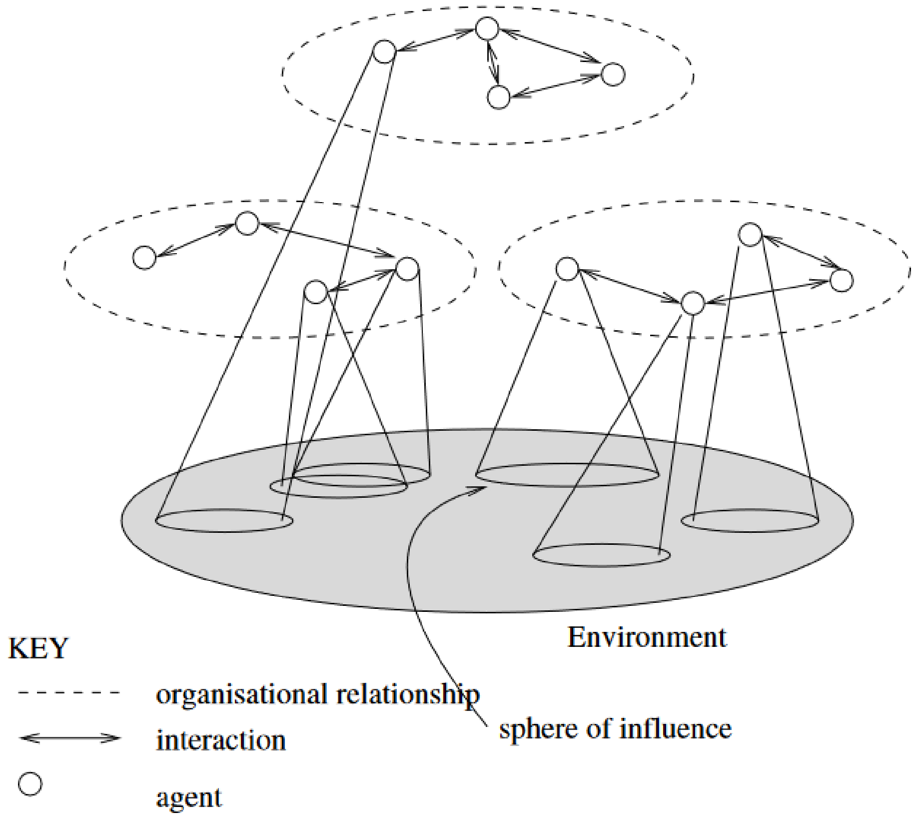
# LECTURE 7: Multiagent Decision Making (I)

Introduction to Multi-Agent Systems (MESIIA, MIA)

URV

# What are Multi-Agent Systems?

- A multiagent system contains a number of agents that:
  - interact through communication;
  - are able to act in an environment;
  - have different "spheres of influence" (which may coincide); and
  - will be linked by other (organisational) relationships.

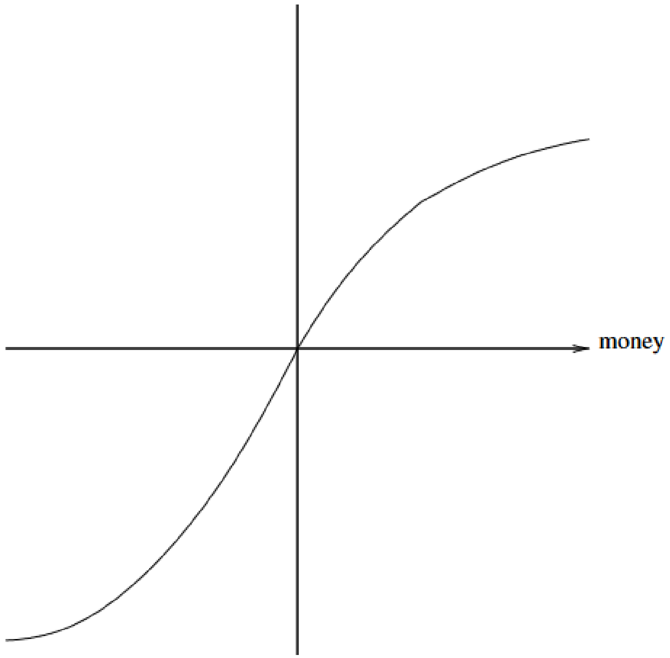


# Types of Agreement

- **Multiagent encounters (game-like character)**
- **Voting.**
- **Coalition forming.**
- **Allocating resources (Auctions)**

# Utilities and Preferences

- Our Assumptions:
  - Assume we have just two agents:
  - Agents are assumed to be self-interested i.e. they have preferences over how the environment is.
  - Assume $\Omega = \{\omega 1, \omega 2, \ldots\}$ is the set of "outcomes" that agents have preferences over.
- We capture preferences by utility functions, represented as real numbers ($\mathbb{R}$):
- Utility functions lead to preference orderings over outcomes, e.g.:




**Utility is not money. Just a way to encode preferences.**

# Multiagent Encounters

- We need a model of the environment in which these agents will act…
  - *agents simultaneously choose an action to perform, and as a result of the actions they select, an outcome in $\Omega$ will result*
  - *the actual outcome depends on the combination of actions*
  - *assume each agent has just two possible actions that it can perform:*
    - *i.e. $Ac = \{C,D\}$, where*
    - *C ("cooperate") and*
    - *D ("defect")*
- *Environment behaviour given by state transformer function $\tau$*

$$\tau : \underbrace{Ac}_{\text{agent } i\text{'s action}} \times \underbrace{Ac}_{\text{agent } j\text{'s action}} \to \Omega$$

# Multiagent Encounters

- Here is a state transformer function
  - This environment is sensitive to actions of both
  - agents.

$$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2$$
$$\tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$$

- With this state transformer, neither agent has any influence in this environment.

$$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_1$$
$$\tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_1$$

- With this one, the environment is controlled by j

$$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2$$
$$\tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_2$$

# Rational Action

- Suppose we have the case where both agents can influence the outcome, and they have the following utility functions:

$$u_i(\omega_1)=1 \quad u_i(\omega_2)=1 \quad u_i(\omega_3)=4 \quad u_i(\omega_4)=4$$
$$u_j(\omega_1)=1 \quad u_j(\omega_2)=4 \quad u_j(\omega_3)=1 \quad u_j(\omega_4)=4$$

  - With a bit of abuse of notation:

$$u_i(D,D) = 1 \quad u_i(D,C) = 1 \quad u_i(C,D) = 4 \quad u_i(C,C) = 4$$
$$u_j(D,D) = 1 \quad u_j(D,C) = 4 \quad u_j(C,D) = 1 \quad u_j(C,C) = 4$$

  - Then agent $i$'s preferences are $(C, C) \succcurlyeq_i (C, D) \succ_i (D, C) \succcurlyeq_i (D, D)$
  - In this case, what should $i$ do?
- $i$ prefers all outcomes that arise through C over all outcomes that arise through D.
  - Thus, C is the *rational choice* for $i$.

# Payoff Matrices

- We can characterise the previous scenario in a payoff matrix shown opposite
  - Agent i is the column player and gets the upper reward in a cell.
  - Agent j is the row player and gets the lower reward in a cell.
- Actually there are two matrices here, one (call it A) that specifies the payoff to i and another B that specifies the payoff to j.
- Sometimes we'll write the game as (A, B) in recognition of this.

In this case, *i* **cooperates** and gains a **utility of 4**; whereas *j* **defects** and gains a **utility of only 1**.

| | | *i* | |
|---|---|---|---|
| | | defect | coop |
| *j* | defect | 1<br>1 | 4<br>1 |
| | coop | 1<br>4 | 4<br>4 |

$$(C, C) \succeq_i (C, D) \succ_i (D, C) \succeq_i (D, D)$$

# Solution Concepts

- How will a rational agent will behave in any given scenario?

- *Play. . .*
  - *Dominant strategy;*
  - *Nash equilibrium strategy;*
  - *Pareto optimal strategies;*
  - *Strategies that maximise social welfare.*

# Dominant Strategies

- Given any particular strategy s (either C or D) that agent i can play, there will be a number of possible outcomes.
  - *We say s1 dominates s2 if every outcome possible by i playing s1 is preferred over every outcome possible by i playing s2.*

- *Thus in the game opposite, C dominates D for both players.*

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 1<br>1 | 4<br>1 |
| | coop | 1<br>4 | 4<br>4 |

# Dominant Strategies

- A rational agent will never play a *dominated strategy*.
  - i.e, a strategy that is dominated (and thus inferior) by another.
- So in deciding what to do, we can delete dominated strategies.
  - *Unfortunately*, there isn't always a unique undominated strategy.

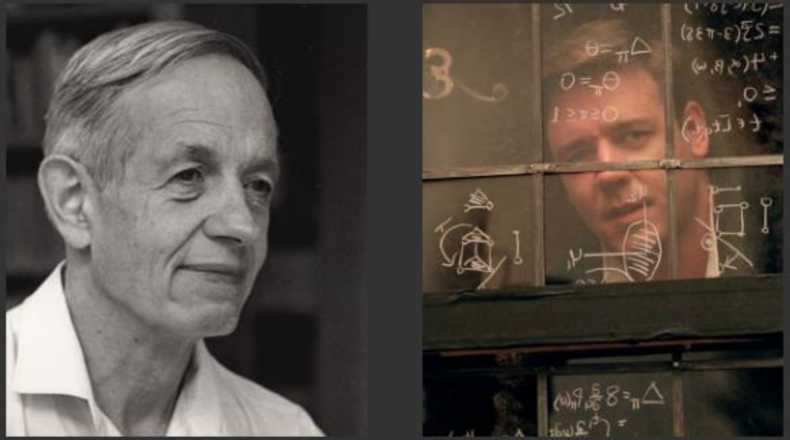
| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 1<br>1 | 4<br>1 |
| | coop | 1<br>4 | 4<br>4 |

# Nash Equilibrium

- In general, we will say that two strategies s1 and s2 are in Nash equilibrium (NE) if:
  - under the assumption that agent i plays s1, agent j can do no better than play s2;
    - I.e. if ***I*** drive on the right side of the road, ***you*** can do no better than also driving on the right!
  - under the assumption that agent j plays s2, agent I can do no better than play s1.
    - I.e. if ***you*** drive on the right side of the road, ***I*** can do no better than also driving on the right!
- Neither agent has any incentive to deviate from a Nash Equilibrium (NE).

***John Forbes Nash***

*(Nobel Laureate in Economics)*



*Portrayed by Russel Crowe in the film "A Beautiful Mind"*

# Nash Equilibrium

- Consider the payoff matrix opposite:
  - Here the Nash equilibrium (NE) is (D, D).
  - In a game like this you can find the NE by cycling through the outcomes, asking if either agent can improve its payoff by switching its strategy.

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 5<br>3 | 1<br>2 |
| | coop | 0<br>2 | 0<br>1 |

- Thus, for example, (C, D) is not a NE because i can switch its payoff from 1 to 5 by switching from C to D.

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 5<br>3 | 1<br>2 |
| | coop | 0<br>2 | 0<br>1 |

# Nash Equilibrium

- Not every interaction scenario has a pure strategy Nash Equilibrium (NE).
  - The game opposite (upper) has two pure strategy NEs, (C, C) and (D, D)

| $j$ \ $i$ | defect | coop |
|---|---|---|
| defect | 5<br>3 | 1<br>2 |
| coop | 0<br>2 | 3<br>3 |

- Some interaction scenarios have more than one pure strategy Nash Equilibrium (NE).
  - The game opposite has no pure strategy NE
  - For every outcome, one of the agents will improve its utility by switching its strategy.

| $j$ \ $i$ | defect | coop |
|---|---|---|
| defect | 2<br>1 | 1<br>2 |
| coop | 0<br>2 | 1<br>1 |

# Mixed Strategy Nash equilibrium

- Matching Pennies
  - Players i and j simultaneously choose the face of a coin, either “heads” or “tails”.
  - If they show the same face, then i wins, while if they show different faces, then j wins.
- NO pair of strategies forms a pure strategy NE:
  - whatever pair of strategies is chosen, somebody will wish they had done something else

| | | $i$ | |
|---|---|---|---|
| | | heads | tails |
| $j$ | heads | 1<br>-1 | -1<br>1 |
| | tails | -1<br>1 | 1<br>-1 |


- The solution is to allow mixed strategies:
  - play “heads” with probability 0.5
  - play “tails” with probability 0.5.
- This is a Mixed Nash Equilibrium strategy.

# Mixed Strategy Nash equilibrium

- Consider the Game Rock/Paper/Scissors
  - Paper covers rock
  - Scissors cut paper
  - Rock blunts scissors
- This has the following payoff matrix

| | | $i$ | | |
|---|---|---|---|---|
| | | rock | paper | scissors |
| | rock | 0<br>0 | 1<br>0 | 0<br>1 |
| $j$ | paper | 0<br>1 | 0<br>0 | 1<br>0 |
| | scissors | 1<br>0 | 0<br>1 | 0<br>0 |

- What should you do?
  - Choose a strategy at random!

# Mixed Strategies

- A mixed strategy has the form
  - play α1 with probability p1
  - play α2 with probability p2
  - …
  - play αk with probability pk.
  - such that p1+p2+⋯ +pk =1.

***Nash's Theorem***

*Nash proved that* ***every finite game has a Nash equilibrium in mixed strategies****. (Unlike the case for pure strategies.)*

*So this result overcomes the lack of solutions; but there still may be more than one Nash equilibrium. . .*

# Pareto Optimality

- An outcome is said to be Pareto optimal (or Pareto efficient) if:
  - *there is no other outcome that makes one agent better off without making another agent worse off.*
  - *If an outcome is Pareto optimal, then at least one agent will be reluctant to move away from it (because this agent will be worse off).*
- *"Reasonable" agents would agree to move to ω' in this case.*
  - *Even if I don't directly benefit from ω', you can benefit without me suffering.*

**This game has one Pareto efficient outcome:** $(D, D)$

| | | $i$: defect | $i$: coop |
|---|---|---|---|
| $j$ | defect | 5<br>3 | 1<br>2 |
| | coop | 0<br>2 | 0<br>1 |

**There is no solution in which either agent does better**

# Social Welfare

- The social welfare of an outcome ω is the sum of the utilities that each agent gets from ω:

$$\sum_{i \in Ag} u_i(\omega)$$

- Think of it as the "total amount of money in the system".
- As a solution concept:
  - may be appropriate when the whole system (all agents) has a single owner (then overall benefit of the system is important, not individuals)
  - It doesn't consider the benefits to individuals.
  - A very skewed outcome can maximise social welfare.

**In both these games, $(C, C)$ maximises social welfare**

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 2, 2 | 1, 1 |
| | coop | 3, 3 | 4, 4 |

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 2, 2 | 1, 1 |
| | coop | 3, 3 | 7, 0 |

# The Prisoner's Dilemma

- Payoff matrix for prisoner's dilemma:

| | | *i* | |
|---|---|---|---|
| | | defect | coop |
| *j* | defect | 2<br>2 | 1<br>4 |
| | coop | 4<br>1 | 3<br>3 |

- Top left: If both defect, then both get punishment for mutual defection.
- Top right: If i cooperates and j defects, i gets sucker's payoff of 1, while j gets 4.
- Bottom left: If j cooperates and i defects, j gets sucker's payoff of 1, while i gets 4.
- Bottom right: Reward for mutual cooperation (i.e. neither confess)

**The Prisoner's Dilemma**

Two men are collectively charged with a crime and held in separate cells, with
no way of meeting or communicating. They are told that:

- if one confesses and the other does not (C,D) or (D,C), the confessor will be freed, and the other will be jailed for three years;
- if both confess (D,D), then each will be jailed for two years.

Both prisoners know that if neither confesses (C,C), then they will each be jailed for one year.

# What should you do?

- The *individual rational action* is defect.
  - This guarantees a payoff of no worse than 2, whereas cooperating guarantees a payoff of at most 1.
  - So defection is the best response to all possible strategies: both agents defect, and get payoff = 2.
- But intuition says this is not the best outcome:
  - Surely they should both cooperate and each get payoff of 3!
- This is why the Prisoners Dilemma game is Interesting
  - The analysis seems to give us a *contradictory* answer.

**Solution Concepts**

- The dominant strategy here is to defect.
- $(D, D)$ is the only Nash equilibrium.
- All outcomes **except** $(D, D)$ are Pareto optimal.
- $(C, C)$ maximises social welfare.

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 2<br>2 | 1<br>4 |
| | coop | 4<br>1 | 3<br>3 |

# The Prisoner's Dilemma

- This apparent contradiction is the fundamental problem of multi-agent interactions.
  - It appears to imply that cooperation will not occur in societies of self-interested agents.

- The prisoner's dilemma is ubiquitous.
  - Can we recover cooperation?

**Solution Concepts**

- The dominant strategy here is to defect.
- *(D, D)* is the only Nash equilibrium.
- All outcomes **except** *(D, D)* are Pareto optimal.
- *(C, C)* maximises social welfare.

| | | $i$ | |
|---|---|---|---|
| | | defect | coop |
| $j$ | defect | 2<br>2 | 1<br>4 |
| | coop | 4<br>1 | 3<br>3 |

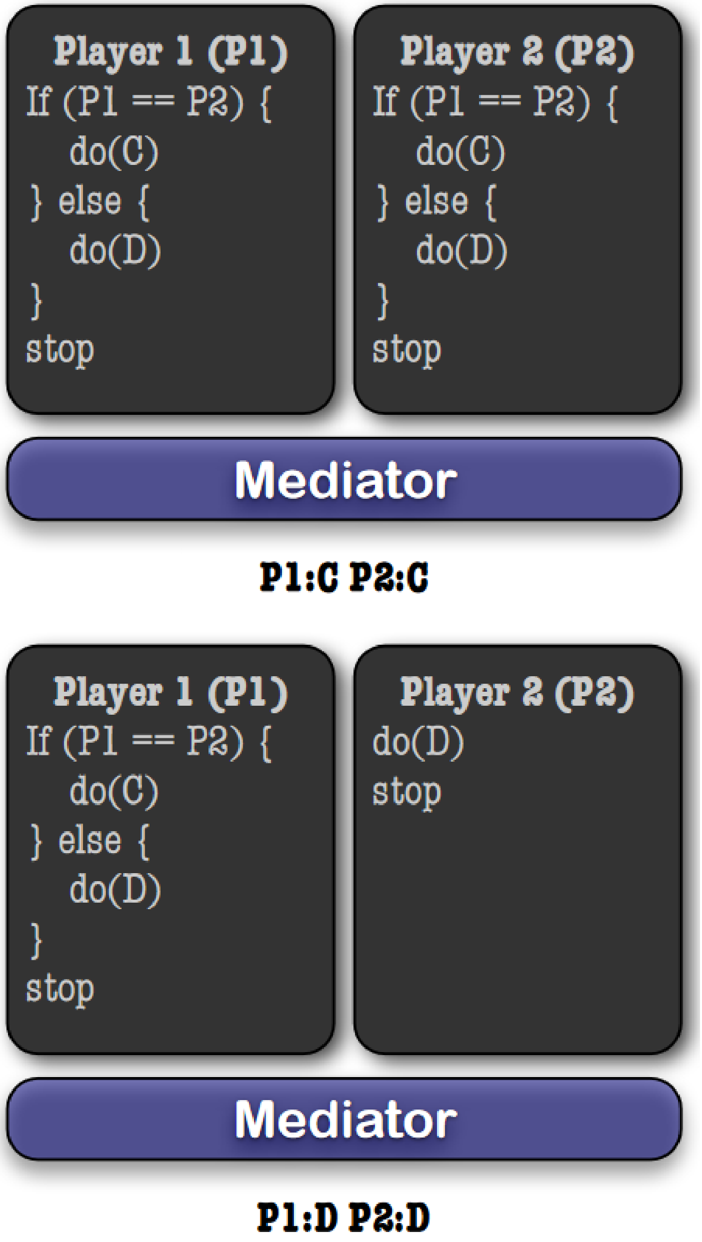
# Arguments for Recovering Cooperation

- Conclusions that some have drawn from this analysis:
  - the game theory notion of rational action is wrong!
  - somehow the dilemma is being formulated wrongly

- Arguments to recover cooperation:
  - We are not all machiavelli!
  - The other prisoner is my twin!
  - ***Program equilibria and mediators***
  - The shadow of the future. . .

# Program Equilibria

- The strategy you really want to play in the prisoner's dilemma is: *I'll cooperate if he will*
  - Program equilibria provide one way of enabling this.
- Each agent submits a *program strategy* to a *mediator* which *jointly executes* the strategies.
  - Crucially, strategies can be conditioned on the strategies of the others.
- The best response to this program:
  - *submit the same program*, giving an outcome of (C, C)!



# Social Choice

- *Social choice theory* is concerned with group decision making.
  - Agents make decisions based on their preferences, but they are aware of other agents' preferences as well.
- Classic example of social choice theory: *voting*
  - Formally, the issue is combining preferences to *derive a social outcome*.

# Components of a Social Choice Model

- Assume a set Ag = {1,…,n} of *voters*.
  - These are entities who express preferences.
  - Voters make group decisions with respect to a set Ω = {ω1,ω2,…} of *outcomes*.
    - Think of these as the *candidates*.
  - If |Ω| = 2, we have a pairwise election.

- Each voter has preferences over Ω
  - An ordering over the set of possible outcomes Ω.
    - Sometimes we will want to pick one, most preferred candidate.
    - More generally, we may want to rank, or order these candidates.

> ***Preference Order Example***
>
> Suppose
>
> $\Omega = \{pear, plum, banana, orange\}$
>
> then we might have agent $i$ with preference order:
>
> $(banana, plum, pear, orange)$
>
> meaning
>
> $banana \succ_i plum \succ_i pear \succ_i orange$

# Preference Aggregation

- The fundamental problem of social choice theory is that…
  - ***…different voters typically have different preference orders!***

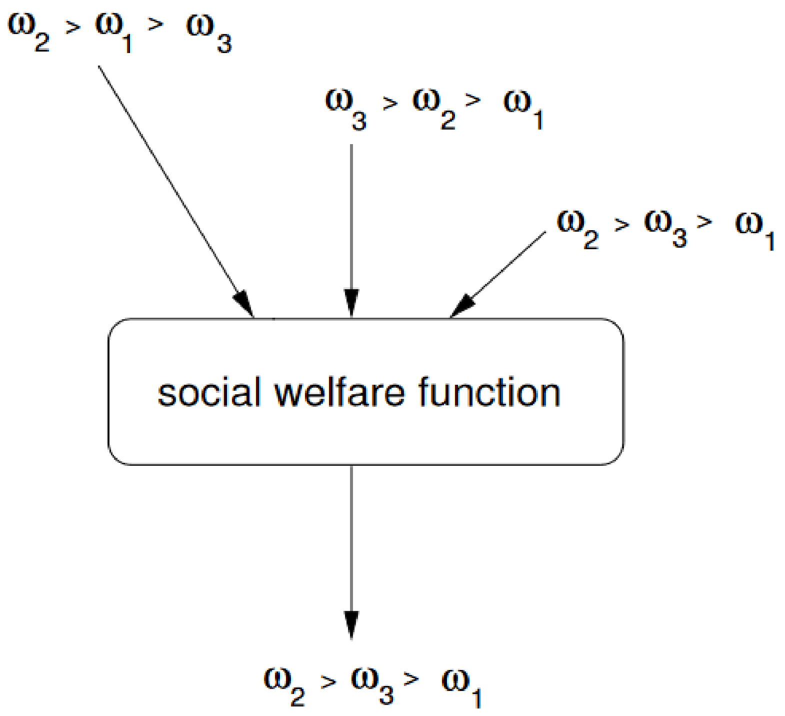
> "… given a collection of preference orders, one for each voter, how do we combine these to derive a group decision, that reflects as ***closely as possible*** the preferences of voters? ***…"***

- We need a way to combine these opinions into on overall decision.
  - What social choice theory is about is finding a way to do this.
  - Two variants of preference aggregation:
    - social welfare functions
    - social choice functions

# Social Welfare Function

- Let Π(Ω) be a set of preference orderings over Ω
  - A *social welfare function* takes voter preferences and produces a *social preference order*.
    - That is, it merges voter opinions and comes up with an order over the candidates.
- We let $\succ *$ denote to the outcome of a social welfare function: $\omega \succ * \omega'$
  - which indicates that ω is ranked above ω' in the social ordering
    - Example: combining search engine results, collaborative filtering, collaborative planning, etc.

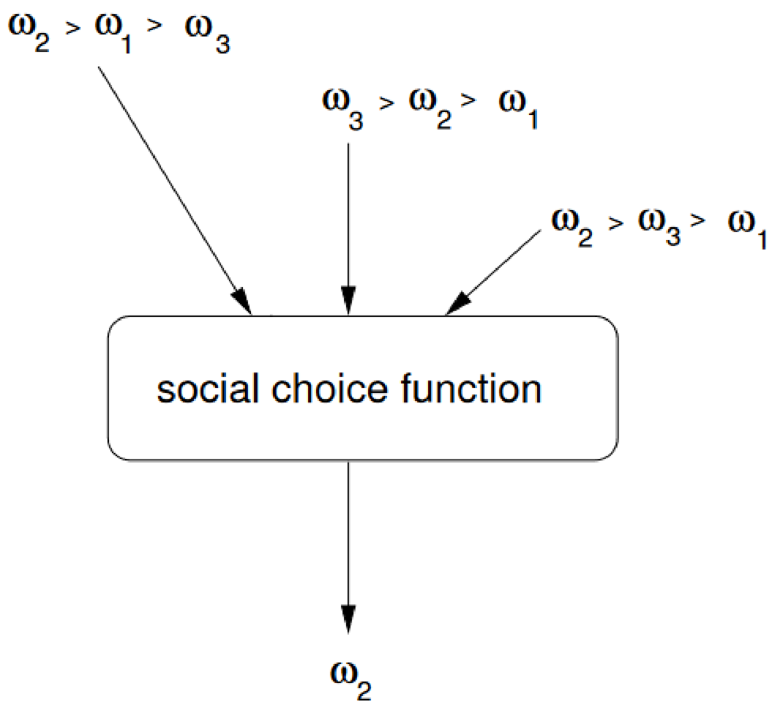
$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Pi(\Omega)$$



# Social Choice Function

- Sometimes, we just one to select *one* of the possible candidates, rather than a social order.
  - This gives a *social choice function* (see opposite)
- In other words, we don't get an ordering out of a social choice function but, as its name suggests, we get a *single choice*.
  - Of course, if we have a social welfare function, we also have a social choice function.
- For the rest of this lecture...
  - ...we'll refer to both social choice and social welfare functions as *voting procedures*.

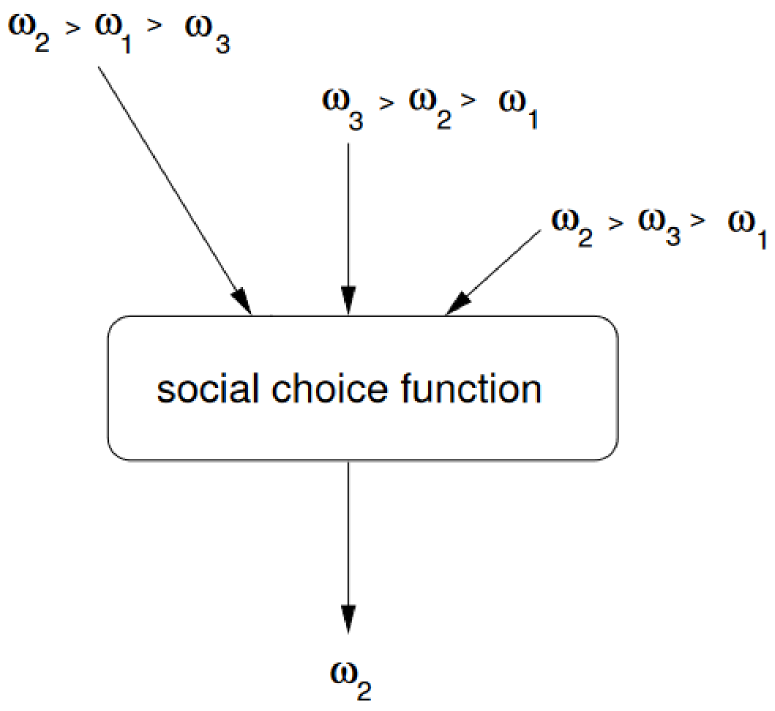
$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Omega$$



# Desirable Properties of the Social Choice Function

- Calculability
  - A social preference ordering <* should exist for all possible inputs.
- Completeness
  - <* should be defined for every pair of alternatives (ω, ω ‘) ∈ Ω
- Linearity
  - <* should be antisymmetric and transitive over Ω
- Anonimity / No dictatorship
  - The outcome of the social choice rule depends on the set of opinions, but not on which agents have these opinions.

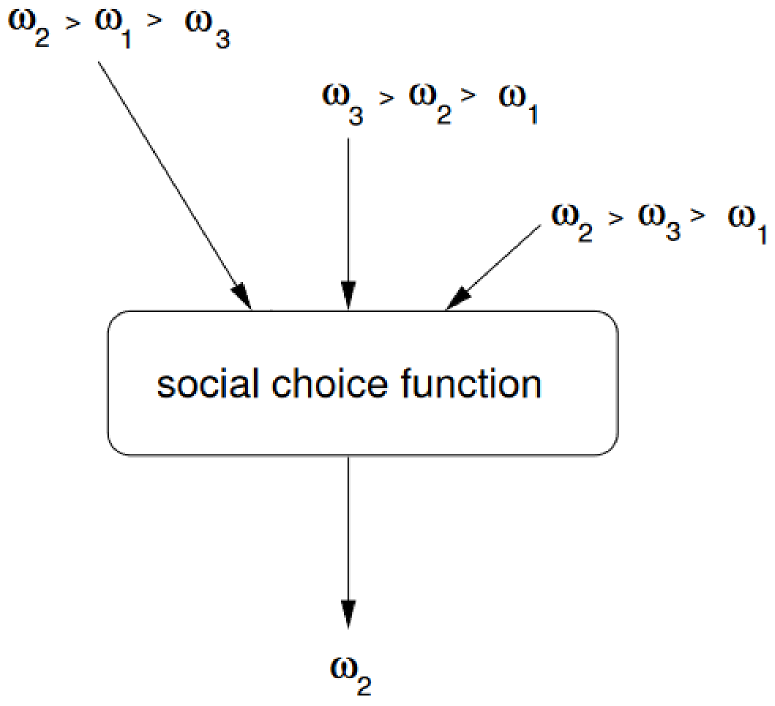
$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Omega$$



# Desirable Properties of the Social Choice Function

- Unanimity / Pareto efficiency
  - Do not misorder the options if all agents agree.
  - If everybody thinks that A is better than B, A should be preferred to B in the aggregated order.
- Neutrality
  - The outcome of the social choice function should not depend on how alternatives are named or ordered.
- Independence of irrelevant alternatives
  - Removing / Adding an irrelevant alternative should not affect the winner of the vote

$$f : \underbrace{\Pi(\Omega) \times \cdots \times \Pi(\Omega)}_{n \text{ times}} \mapsto \Omega$$



# Simple Voting Procedures

- Some basic voting mechanisms
  - Plurality / Anti-plurality / Best-Worst / Approval
- Protocols based on total orders
  - Binary protocol (series of votes of 2 options each)
  - Borda protocol (sum of all the preferences of the agents)
  - Condorcet protocol (pairwise comparison of options, given full preference ordering of each agent)
- All the procedures are problematic in one sense or another.

# Plurality Voting Procedure

- Social choice function: selects a single outcome.
  - Each agent (i.e., voter) can give 1 vote to 1 of the alternatives
  - The alternative with the highest number of votes wins
- If we have only two candidates, then plurality is a *simple majority election*

# Anomalies with Plurality

- Suppose:
  - $|Ag| = 100$ and $\Omega = \{\omega 1, \omega 2, \omega 3\}$
- with:
  - 40% voters voting for $\omega 1$
  - 30% of voters voting for $\omega 2$
  - 30% of voters voting for $\omega 3$
- With plurality, $\omega 1$ gets elected even though a clear majority (60%) prefer another candidate

# Advantages of plurality voting

- Most simple voting mechanism
- Very efficient from the computational point of view
- Equality principle, as it preserves the idea of *1 agent = 1 vote*

# Another version: anti-plurality

- Each voter gives a *negative* vote to the alternative they consider the worst
- The option with *less* votes *wins*
- Example:
  - 30% CBDA
  - 30% CADB
  - 20% ABDC
  - 20% BADC
    - C gets 40% negative votes but also first option for 60%
    - A and B get 30% negative votes
    - D is the winner with 0 negative votes – but it was not the first or second option for anyone

# Best-worst voting systems

- Each agent gives a *positive* vote to his best alternative and a *negative* vote to his worst alternative
- Each alternative receives α>0 points for each positive vote and -δ<0 points for each negative vote
- The option with more points wins

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| C | C | B | B | A | A | A |
| B | B | C | C | B | C | C |
| A | A | A | A | C | B | B |

A $\to 3\alpha - 4\delta$ points  B $\to 2\alpha - 2\delta$ points  C $\to 2\alpha - \delta$ points

| | |
|---|---|
| Plurality $(\delta = 0)$ | A $\succ$ B $\sim$ C |
| Antiplurality $(\alpha = 0)$ | C $\succ$ B $\succ$ A |
| $\alpha = \delta = 1$ | C $\succ$ B $\succ$ A |
| $\alpha = 2$ and $\delta = 1$ | C $\succ$ A $\sim$ B |
| $\alpha = 4$ and $\delta = 1$ | A $\succ$ C $\succ$ B |

# Approval voting

- Each voter selects a *subset* of the candidates
- The candidate with *most votes wins*
- *k-approval* voting
  - Each voter selects a subset of k candidates
    - k=1: plurality
    - k= n-1: anti-plurality

# Voting Procedures based on Linear Orders

- Each voter gives a full list of the options, ordered according to his preferences (from best to worst)

- A voter prefers option i to option j if option i apears before option j in his list

# Binary Procedure

- All the options are ordered and then evaluated in pairs (options 1 and 2, the winner with option 3, the winner with option 4, etc.)
- Simple majority: option A is better than option B if and only if the number of voters that prefer A to B is greater than the number of voters that prefer B to A
- The option that wins the last evaluation is the overall winner
  - win(a5, win (a4, win (a23, win(a2, a1))))

# The Ordering Problem

- Example:
  - x > z > y (35%)
  - y > x > z (33%)
  - z > y > x (32%)
- Note that y is preferred to x (65-35), x is preferred to z (68-32), and z is preferred to y (67-33)
  - win(x, win(y,z))=x
  - win(y,win(x,z))=y
  - win(z, win(x,y))=z
- The order of the pairings affects the outcome !
  - The voter organiser may influence the result
  - The last options have more chances of winning
    - *No Neutrality*

# Another problematic example

35% of agents have preferences $c > d > b > a$

33% of agents have preferences $a > c > d > b$

32% of agents have preferences $b > a > c > d$

- Evaluation in the order abcd:
  - Win(a,c)=a Win(a,b)=b Win(b,d)=d => d Wins
    - d was the worst alternative for 32%
    - d was not the best alternative for anyone
    - Everybody prefers c to d (!) – No Unanimity

# Summary of problems of binary voting

- Decisive role of the ordering of the alternatives

- An alternative x may win even if there is another alternative x' which is preferred to x by all agents
    - Alternatives may be misordered

- Temporal cost of the voting process
    - sequence of pairwise eliminative votes

# Borda Procedure

- One reason plurality has so many anomalies is that it ignores most of a voter's preference orders: it only looks at the top ranked candidate.
  - The Borda count takes whole preference order into account.
- Suppose we have k candidates - i.e. k = |Ω|
  - For each candidate, we have a variable, counting the strength of opinion in favour of this candidate.
  - If ωi appears first in a preference order, then we increment the count for ωi by k − 1;
  - we then increment the count for the next outcome in the preference order by k − 2,
  - . . . , until the final candidate in the preference order has its total incremented by 0.
- After we have done this for all voters, then the totals give the ranking.

***Example of Borda Count***

Assume we have three voters with preferences:

$$\omega_2 >_1 \omega_1 >_1 \omega_3$$
$$\omega_3 >_2 \omega_2 >_2 \omega_1$$
$$\omega_1 >_3 \omega_2 >_3 \omega_3$$

The Borda count of $\omega_2$ is 4:

*2 from the first place vote of voter 1.*

*1 each from the second place votes of voters 2 and 3.*

What are the Borda counts of the other candidates?

# Borda Inconsistency

- a > b > c > d
- b > c > d > a
- c > d > a > b
- a > b > c > d
- b > c > d > a
- c > d > a > b
- a > b > c > d
- a = 11, b = 12, **c=13**,d=6

- If the worst alternative –d- is removed
- a > b > c
- b > c  > a
- c  > a > b
- a > b > c
- b > c > a
- c > a > b
- a > b > c
- **a = 8**, b = 7, c=6

**Even if we keep the relative preferences between a, b and c, the final result changes completely**

# Problems of the Borda Procedure

- Most computationally expensive

- Eliminating (or adding) one irrelevant alternative may totally change the outcome of the voting
  - Winner => Last
  - Second worst => Winner

- Total order changes if options are removed one by one

# Borda Procedure with Weak Orders

- The Borda protocol has been extended to manage weak orders in different ways
- A simple one: an option o receives from a voter v as many points as the number of options that are considered worst than o by v.

| $R_1$ | $R_2$ | $R_3$ |
|---|---|---|
| $x_1$ | $x_2 \quad x_4$ | $x_3$ |
| $x_2 \quad x_3$ | $x_3$ | $x_1$ |
| $x_4$ | $x_1$ | $x_2$ |
| | | $x_4$ |

$$x_1 \rightarrow 3+0+2=5 \qquad x_2 \rightarrow 1+2+1=4$$

$$x_3 \rightarrow 1+1+3=5 \qquad x_4 \rightarrow 0+2+0=2$$

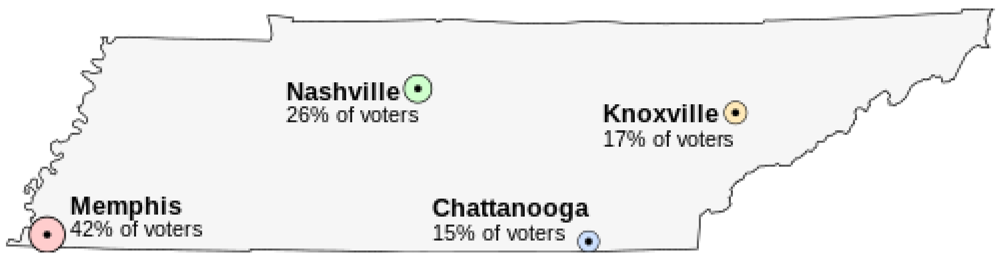
$$x_1 \sim x_3 \succ x_2 \succ x_4$$

# Condorcet Procedure

- Each voter ranks the candidates in order of preference
- Each candidate is compared to each other
- If a candidate wins all the comparisons, it is the winner of the election
- In the event of a tie, use another resolution method (e.g. Borda count)

# Condorcet Procedure

- Example: Voting on the location of Tennessee's capital
  - Election of the capital city of Tennessee
  - Everybody prefers to have the capital as close as possible
  - The candidates for the capital are:
    - Memphis, the state's largest city, with 42% of the voters, but located far from the other cities
    - Nashville, with 26% of the voters, near the center of the state
    - Knoxville, with 17% of the voters
    - Chattanooga, with 15% of the voters




| 42% of voters (close to Memphis) | 26% of voters (close to Nashville) | 15% of voters (close to Chattanooga) | 17% of voters (close to Knoxville) |
|---|---|---|---|
| 1. **Memphis** | 1. **Nashville** | 1. **Chattanooga** | 1. **Knoxville** |
| 2. Nashville | 2. Chattanooga | 2. Knoxville | 2. Chattanooga |
| 3. Chattanooga | 3. Knoxville | 3. Nashville | 3. Nashville |
| 4. Knoxville | 4. Memphis | 4. Memphis | 4. Memphis |

The preferences of the voters

# Condorcet Procedure

- Example: Voting on the location of Tennessee's capital
  - Election of the capital city of Tennessee
  - Everybody prefers to have the capital as close as possible
  - The candidates for the capital are:
    - Memphis, the state's largest city, with 42% of the voters, but located far from the other cities
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| Pair | Winner |
|---|---|
| Memphis (42%) vs. Nashville (58%) | Nashville |
| Memphis (42%) vs. Chattanooga (58%) | Chattanooga |
| Memphis (42%) vs. Knoxville (58%) | Knoxville |
| Nashville (68%) vs. Chattanooga (32%) | Nashville |
| Nashville (68%) vs. Knoxville (32%) | Nashville |
| Chattanooga (83%) vs. Knoxville (17%) | Chattanooga |

<table>
<tr><td>1st</td><td colspan="4">Nashville [N]</td><td>3 Wins ↓</td></tr>
<tr><td>2nd</td><td colspan="3">Chattanooga [C]</td><td>1 Loss →<br>↓ 2 Wins</td><td>[N] 68%<br>[C] 32%</td></tr>
<tr><td>3rd</td><td colspan="2">Knoxville [K]</td><td>2 Losses →<br>↓ 1 Win</td><td>[C] 83%<br>[K] 17%</td><td>[N] 68%<br>[K] 32%</td></tr>
<tr><td>4th</td><td>Memphis [M]</td><td>3 Losses →</td><td>[K] 58%<br>[M] 42%</td><td>[C] 58%<br>[M] 42%</td><td>[N] 58%<br>[M] 42%</td></tr>
</table>

**Nashville wins**

# Problem of Condorcet method

- Possibility of circular ambiguities
  - No alternative wins to all the other alternatives
  - There are many ways to resolve them
    - Keep the candidate that wins more matches (Copeland)
    - Take into account the relative strengths of defeats (Minimax, Ranked Pairs, Schulze, …)
    - You can look at the winning votes or at the winning margin

# Use in practical exercise

- The collector agents could vote the order in which to pick up the discovered treasures.
- The vote of each collector could depend on its current position, the position of the treasures, its current state (idle, already moving towards a treasure, with a list of assigned treasures pending to be collected, etc.).
- The collector assigned to a treasure could then be somehow chosen between its voters.

# Readings for this week

- Chapters 11, 12 of the book by M.Wooldridge “An introduction to Multi-Agent Systems” (2nd edition).