QUANT MACRO: PROBLEM SET 3

Mario Serrano

Question 1. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \ (z\ h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1-\delta)\ k_t \tag{4}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

a) Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of 0.25.

Setting initial parameters according with the statement we obtain:

z1 = 1.6296760964691483

Computing also depreciation, imposing SS in the investment function:

δ = 0.0625

And finally, we compute Beta, from the Euler equation. We have obtained this through solving the maximization problem.

1/β = (1-θ)*(h*z)^θ*k^(- θ) + 1 – δ → β = 0.9803921568627457

b) Double permanently the productivity parameter z and solve for the new steady state.

Setting z at its new value:

z2 = 3.2593521929382967

We obtain a value of k for the new SS:

k = 8.000000000000027

c) Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labour and output.

In order to plot the transition of the economy, we need to get an expression of tomorrow k as a function of todays. Euler equation can be written as

u'(c_t) = u'(c_t+1)*f'(k_t+1)

u'(f(k)-k_t+1) = u'(f(k_t+1) - k_t+2)*f'(k_t+1)

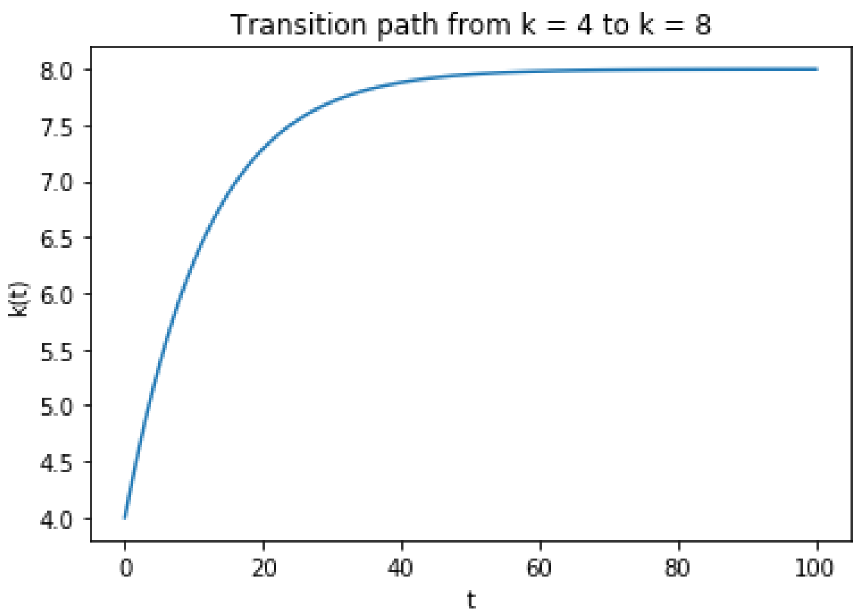
Back into our problem

$$u'(f(k) - k\_t+1) = \beta\text{^}t/(k\text{^}(1-\theta)*(z*h)\text{^}\theta + (1-\delta)k - k\_t+1)$$

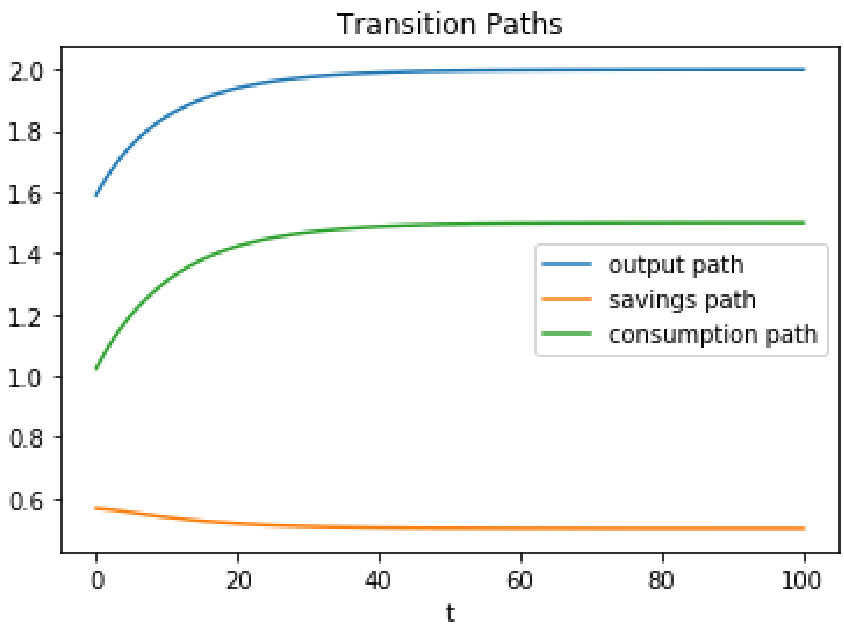
$$u'(f(k\_t+1) - k\_t+2) = \beta\text{ ^}(t+1)/(k\_t+1\text{^}(1-\theta)*(z*h)\text{^ }\theta + (1-\delta)k\_t+1 - k\_t+2)$$

$$f'(k\_t+1) = (1-\theta)*(z*h)\text{^ }\theta *k\_t+1\text{^}(-\theta) + 1 - \delta$$

Combining all of them in the above formula, we obtain an expression as a function of k, k_t+1 and k_t+2. Fixing a sufficient large number of periods, we can compute the sequence of k's by iterations. The obtained transition path is:



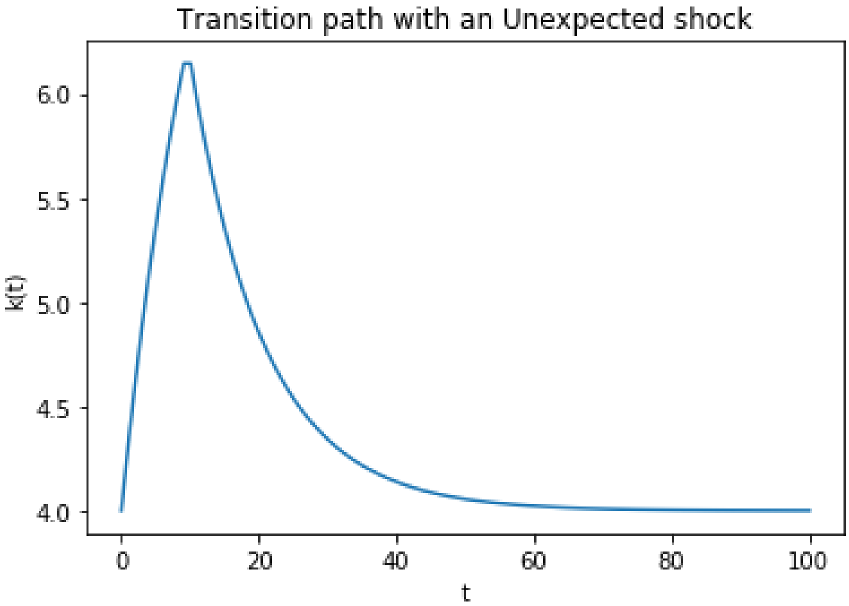
Computing now transition paths for consumption, savings and output is straight forward (labour is constant by assumption). We use the constrains and obtained:



d) Unexpected shocks. Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labour and output.

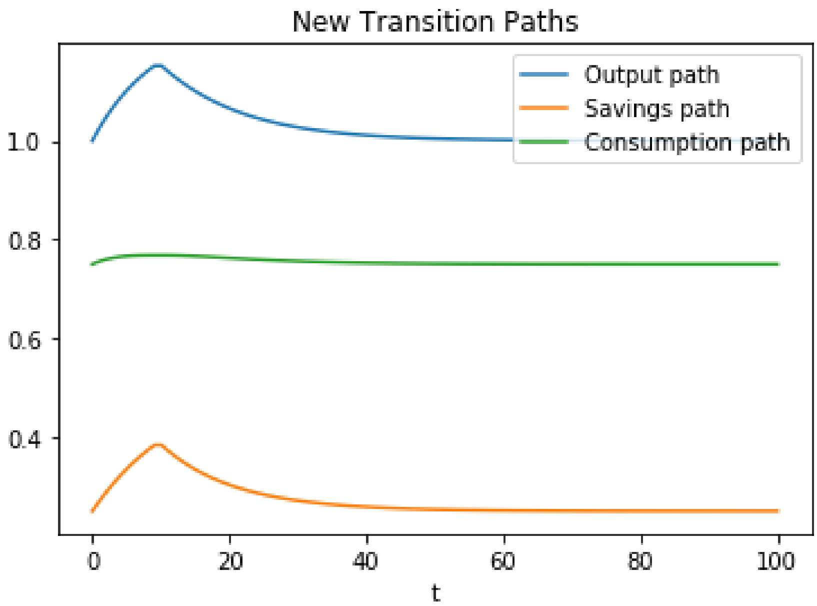
In this situation, the economy will behave as in the last exercise until period 10. After this period, the transition path will experiment a progressive decrease until the economy reach the initial steady state again. Therefore, we only need to compute its evolution from period 11 onwards, and then combining with the first part of the path (until period 10).

Evolution of the economy will be:



where the break point occurs at k = 6.145429367286361.

Computing transition paths of consumption, savings and output we obtain:



Question 2. Multi-country model with free mobility of capital (not labour) and progressive labour income tax

We are interested in studying the respective general equilibrium (GE) of closed economies of each of these countries and a the GE of a union with free mobility of capital. Labor and goods are not mobile.

Environment This is a multicountry static model $\ell = \{1, 2\}$. Each country is populated by a heterogenous households that differ in their permanent productivity η, supply labor, $h \in [0, 1]$ and capital $k_\ell \in [0, \bar{k}_\ell]$, where $\bar{k}_\ell$ is the country-specific capital endowment, and takes prices as given. Each country produces a single good with a representative CRS firm that operates in competitive markets.

Household Agents are heterogeneous in their permanent labor productivity η and face uncertainty on their wage. After they are born (and they realize their η) they receive an idiosyncratic shock ε_ℓ^i with probability π. Labor income taxation is according to Feldestein (1969) tax function that we saw in class.

An agent with productivity η in country ℓ solves:

$$\max_{\{c_\ell, k_\ell \in [0, \bar{k}_\ell], h_\ell \in [0,1]\}} \left(\frac{(c_\ell)^{1-\sigma}}{1-\sigma} - \kappa \frac{(h_\ell)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right)$$

subject to

$$c = \lambda \left(w_\ell \left(h_\ell \right) \eta_\ell \right)^{1-\phi_\ell} + r_\ell k_\ell^\eta + r_{-\ell} \left(\bar{k}_\ell - k_\ell \right)$$

where foreign investment is $\left(\bar{k}_\ell - k_\ell\right)$.

Firm problem Each country has a single representative firm that solves:

$$\max_{\{K_\ell^d, H_\ell^d\}} Z \left(K_\ell^d \right)^{1-\theta} \left(H_\ell^d \right)^\theta - w_\ell H_\ell^d - r_\ell K_\ell^d$$

2.1) Consider the case that each of these two countries are closed economies. Write (a) the equilibrium of a closed economy, (2) an algorithm to solve it and (3) solve the economy.

In order to compute equilibria:

- We set the maximization problem of the firm and solve it, obtaining prices as a function of total capital and labour

  r = (1-θ)*H^θ *K^(-θ)
  w = θ*H^(1-θ)*K^(1-θ)

- We set the maximization problem of the HH and solve it, obtaining the Euler equation

  dL/dc = c^(-σ) - μ = 0
  dL/dh = -κ*h^(1/ν) + μ *λ*(1- θ)*η*w*(η*h*w)^(- θ) = 0
  dL/dk = μ *(r1 - r2) = 0

Using these and the budget constrain, we can compute the equilibria in both countries. We only have to take into account that in this case there is no foreign investment, so the term

$$\cdot\, r_{-\ell}\left(\bar{k}_{\ell} - k_{\ell}\right)$$

goes away in the budget constrain. Computing equilibria for both countries we obtain:

AUTARKY EQUILIBRIA OF COUNTRY A:
w = 1.04920791035433
r = 0.172979458224847
Consumption of Low Type: 0.5763510963986865
Consumption of High Type: 1.3084763522273868
Labour Supply of Low Type: 0.2239354363915793
Labour Supply of High Type: 0.27066469915189667

AUTARKY EQUILIBRIA OF COUNTRY B:
w = 1.03932420714339
r = 0.175452799810180
Consumption of Low Type: 0.8823257167654504
Consumption of High Type: 0.9923996898803358
Labour Supply of Low Type: 0.25001386251964025
Labour Supply of High Type: 0.25642907012141924

2.2 (a) Write the equilibrium of the union economy, (b) the algorithm to solve it and (3) solve the economy for a given set of parameters.

Now we have to solve the same problem above, but instead to solve equilibrium separately, we need to solve it together at the same time. This is because there are movements of capital between countries, so decision of the agents is affecting not only their domestic economy, but also the economy of the foreign country. We compute the equilibria and obtain:

<u>**UNION ECONOMY EQUILIBRIA**</u>

EQUILIBRIA IN COUNTRY A:
w = 0.5527170907366663
r = 0.4524103440533619
Consumption of Low Type: 0.5727445908387657
Consumption of High Type: 1.2355841597545998
Labour Supply of Low Type: 0.12800028146280323
Labour Supply of High Type: 0.3791972311545574
Capital Supply of Low Type: 0.37739888879269184
Capital Supply of High Type: 0.6540358538104808

EQUILIBRIA IN COUNTRY B:
w = 0.6340494009801847
r = 0.36821563223234693
Consumption of Low Type: 0.7561818544017485
Consumption of High Type: 0.910500517444614
Labour Supply of Low Type: 0.3075794336250954
Labour Supply of High Type: 0.3401000427263777
Capital Supply of Low Type: 0.6227688246035569
Capital Supply of High Type: 0.6578773042080115

2.3) Discuss how you would choose the optimal progressive taxation of labour income for these economies? Write the planners problem and an algorithm to solve it.

Assuming a Fedelstein Tax Function form, labour income taxes will follow

$$T(y)=y(1-\lambda y-\varphi)$$

where the parameter φ determines the degree of progressivity. If we introduce taxes in this form, the budget constrain of agents will be

$$c_{li}= \lambda_l (1-T(y))(w_l h_{li}\eta_{li})_{1-\varphi l}+r_l \eta k_{l\eta li}+r_{-l}(k_{\bar{l}}-k_l)$$

With this, we can solve the maximization problem of the HH for every given degree of progressivity φ. We need to compute every possible equilibrium as a function of φ, and the solve a maximization problem with the social welfare function to obtain the optimal value of φ that deliver the maximum value of social welfare

$$SWF= \omega 1\ vAl(c,h)+\omega 2\ vAh(c,h)+\omega 3\ vBl(c,h)+\omega 4\ vBh(c,h)$$

Introducing a labor income taxation will change the agents budget constraint to the following:
$c_{li} = \lambda_l (1 - T(y))(w_l h_{li} \eta_{li})^{1-\varphi_l} + r_l \eta k_{l \eta_{li}} + r_{-l} (\bar{k}_l - k_l)$
Labor income is, in this case, y = $w_l h_{li} \eta_{li}$