1 Problem Statement

A chocolate bar has 6 rows and 8 columns (for a total of 48 small 1×1 squares). You break it into individual squares by repeatedly splitting one rectangular piece into two smaller rectangles. For example, in the first step you might split the 6×8 bar into a 6×3 piece and a 6×5 piece.

Question: What is the total number of breaks needed to end up with 48 individual squares?

2 Solution

We will prove a general formula for the number of breaks needed to separate any chocolate bar consisting of n small squares.

Definition

Let $f(n)$ denote the number of breaks required to split a chocolate bar containing n squares into n individual 1×1 pieces.

Small Cases

- $f(1) = 0$, since a single square needs no further splitting.
- $f(2) = 1$, since one split separates it into two single squares.
- $f(3)$: We can first split it into pieces of size 1 and 2 (or 2 and 1). Then we must further split the piece of size 2.

$$f(3) = 1 + f(1) + f(2) = 1 + 0 + 1 = 2.$$

- $f(4)$:

  1. Split into two pieces of size 2 and 2:

  $$f(4) = 1 + f(2) + f(2) = 1 + 1 + 1 = 3.$$

  2. Split into 1 and 3:

  $$f(4) = 1 + f(1) + f(3) = 1 + 0 + 2 = 3.$$

  In both cases, $f(4) = 3$, so the number of breaks is independent of how the bar is split.

Inductive Step

We will now prove by **strong induction** that:

$$f(n) = n - 1 \quad \text{for all } n \geq 1.$$

Base cases: From the calculations above, we have $f(1) = 0$, $f(2) = 1$, $f(3) = 2$, and $f(4) = 3$, all of which satisfy the formula $f(n) = n - 1$.

Inductive hypothesis: Assume that for all $m < n$, the formula holds, i.e.

$$f(m) = m - 1.$$

Inductive step: Consider a bar with n squares. Suppose we make the first split into two bars of sizes k and $n - k$, where $1 \leq k \leq n - 1$. This first split adds 1 to the total number of breaks, and the two smaller bars will require $f(k)$ and $f(n - k)$ additional breaks.

$$\begin{aligned} f(n) &= 1 + f(k) + f(n-k) \\ &= 1 + (k-1) + ((n-k) - 1) \quad \text{(by the inductive hypothesis)} \\ &= n - 1. \end{aligned}$$

Thus, the formula holds for n as well.

Conclusion: By the principle of strong induction,

$$\boxed{f(n) = n - 1 \text{ for all } n \geq 1.}$$

Answer for the Problem

For a 6×8 chocolate bar, $n = 48$, hence

$$f(48) = 48 - 1 = 47.$$

Therefore, **47 breaks are needed in total.**