Algorithm 6.1 (The eight-point algorithm for the fundamental matrix).

For a given set of image correspondences $(\boldsymbol{x}_1'^j, \boldsymbol{x}_2'^j)$, $j = 1, 2, \ldots, n$ $(n \geq 8)$, this algorithm finds the fundamental matrix F that minimizes in a least-squares sense the epipolar constraint

$$(\boldsymbol{x}_2'^j)^T F \boldsymbol{x}_1'^j = 0, \quad j = 1, 2, \ldots, n.$$

1. **Compute a first approximation of the fundamental matrix**
   Construct the $\chi \in \mathbb{R}^{n \times 9}$ from transformed correspondences $\boldsymbol{x}_1'^j$ and $\boldsymbol{x}_2'^j$ as in (6.76), namely,
   $$\chi^j = \boldsymbol{x}_1'^j \otimes \boldsymbol{x}_2'^j \quad \in \mathbb{R}^9.$$
   Find the vector $F^s \in \mathbb{R}^9$ of unit length such that $\|\chi F^s\|$ is minimized as follows: Compute the SVD of $\chi = U_\chi \Sigma_\chi V_\chi^T$ and define F^s to be the ninth column of V_χ. Unstack the nine elements of F^s into a square 3×3 matrix F. Note that this matrix will in general *not* be a fundamental matrix.

2. **Impose the rank constraint and recover the fundamental matrix**
   Compute the singular value decomposition of the matrix $F = U\text{diag}\{\sigma_1, \sigma_2, \sigma_3\}V^T$. Impose the rank-2 constraint by setting $\sigma_3 = 0$ and set the fundamental matrix to be
   $$F = U\text{diag}\{\sigma_1, \sigma_2, 0\}V^T.$$

Normalization of image coordinates

Since image coordinates $\boldsymbol{x}_1'$ and $\boldsymbol{x}_2'$ are measured in pixels, the individual entries of the matrix χ can vary by two orders of magnitude (e.g., between 0 and 512), which affects the conditioning of the matrix χ (see Appendix A). Errors in the values x' and y' will introduce uneven errors in the recovered entries of F^s and hence F. Since we know how to handle linear transformations in the epipolar framework, we can use that to our advantage and choose the transformation that "balances" the coordinates. This can be done, for instance, by transforming the points $\{\boldsymbol{x}_1'^j\}_{j=1}^n$ by an affine matrix $H_1 \in \mathbb{R}^{3 \times 3}$ so that the resulting points $\{H_1 \boldsymbol{x}_1'^j\}_{j=1}^n$ have zero mean and unit variance. This can be accomplished by transforming the pixel coordinates $\boldsymbol{x}_1'$ into the "normalized" coordinates $\tilde{\boldsymbol{x}}_i$ via

$$\tilde{\boldsymbol{x}}_i \doteq H_i \boldsymbol{x}_i' = \begin{bmatrix} 1/\sigma_{x_i} & 0 & -\mu_{x_i}/\sigma_{x_i} \\ 0 & 1/\sigma_{y_i} & -\mu_{y_i}/\sigma_{y_i} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix}, \tag{6.77}$$

where μ_{x_i} is the average (or mean) and σ_{x_i} is the standard deviation of the set of x-coordinates $\{(x_i')^j\}_{j=1}^n$ in the ith image $i = 1, 2$,

$$\mu_{x_i} \doteq \frac{1}{n}\sum_{j=1}^{n}(x_i')^j, \quad \sigma_{x_i} \doteq \sqrt{\frac{1}{n}\sum_{j=1}^{n}[(x_i')^j - \mu_{x_i}]^2}; \tag{6.78}$$