

# Supplementary methods

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22 October, 2020

## Description of *Milo*

### Building the KNN graph

Similarly to many other tasks in single-cell analysis, *Milo* uses a KNN graph computed based on similarities in gene expression space as a representation of the phenotypic manifold in which cells lie. While *Milo* can be used on graphs built with different similarity kernels, here we compute the graph as follows: for a gene expression matrix of  $N$  cells is projected onto the first  $d$  principal components (PCs) to obtain a  $N \times d$  matrix  $X_{PC}$ . Then, for each cell  $i$ , the euclidean distances to its  $k$  nearest neighbors in  $X_{PC}$  are computed and stored in a  $N \times N$  adjacency matrix. Then,  $D$  is symmetrized, such that cells  $i$  and  $j$  are nearest neighbors (i.e. connected by an edge) if either  $i$  is nearest neighbor of  $j$  or  $j$  is nearest neighbor of  $i$ . The KNN graph is encoded by the undirected symmetric version of  $\tilde{D}$  of  $D$ , where each cell has at least  $K$  nearest neighbors.

### Definition of cell neighbourhoods and index sampling algorithm

We define the neighbourhood  $n_i$  of cell  $i$  as the group of cells that are connected to  $i$  by an edge in the graph. Formally, a cell  $j$  belongs to neighbourhood  $n_i$  if  $\tilde{D}_{i,j} > 0$ . We refer to  $i$  as the index of the neighbourhood.

In order to define a representative subset of neighbourhoods that span the whole KNN graph, we implement a previously adopted algorithm to sample the index cells in a graph (Gut et al. 2015; Setty et al. 2016). Briefly, we start by randomly sampling  $p \cdot N$  cells from the dataset, where  $p \in [0, 1]$  (we use  $p = 0.1$  by default). Given the reduced dimension matrix used for graph construction  $X_{PC}$ , for each sampled cell we consider its  $k$  nearest neighbors  $j = 1, 2, \dots, k$  with PC profiles  $x_1, x_2, \dots, x_k$ . We measure the mean PC profile  $\bar{x}$  for the  $j$  cells and search for the cell  $i$  such that the euclidean distance between  $x_i$  and  $\bar{x}$  is minimized. This yields a set of  $M \leq p \cdot N$  index cells that are used to define neighbourhoods.

### Testing for differential abundance in neighbourhoods

*Milo* builds upon the framework for differential abundance testing implemented by *Cydar* (Lun, Richard, and Marioni 2017). In this section, we briefly describe the statistical model and adaptations to the KNN graph setting.

### Quasi-likelihood negative binomial generalized linear models

We consider a neighbourhood  $n$  with cell counts  $y_{ns}$  for each sample  $s$ . The counts are modelled by the negative binomial (NB) distribution, as it is supported over all non-negative integers and can accurately model both small and large cell counts. For such non-normally distributed data we use generalized-linear

models (GLMs) as an extension of classic linear models that can accomodate complex experimental designs. We therefore assume that

$$y_{ns} \sim NB(\mu_{ns}, \phi_n),$$

where  $\mu_{ns}$  is the mean and  $\phi_n$  is the NB dispersion parameter. The expected count value for neighbourhood  $n$  in sample  $s$   $\mu_{ns}$  is given by

$$\mu_{ns} = \lambda_{ns} N_s$$

where  $\lambda_{ns}$  is the proportion of cells belonging to sample  $s$  in  $n$  and  $N_s$  is the total number of cells of  $s$ . In practice,  $\lambda_{ns}$  represents the biological variability that can be affected by treatment condition, age or any biological covariate of interest. We use a log-linear model to represent the influence of the biological condition on the expected counts in neighbourhoods:

$$\log \mu_{ns} = \sum_{g=1}^G x_{sg} \beta_{ng} + \log N_s$$

where  $x_{sg}$  is the covariate vector indicating the condition applied to sample  $s$  and  $\beta_{ng}$  is the regression coefficient by which the covariate effects are mediated for neighbourhood  $n$ .

Estimation of  $\beta_{ng}$  for each  $n$  and  $g$  is performed by fitting the GLM to the count data for each neighbourhood, i.e. by estimating the dispersion  $\phi_n$  that models the variability of cell counts for replicate samples for each neighbourhood. Dispersion estimation is done using the quasi-likelihood method in `edgeR` (Robinson, McCarthy, and Smyth 2010), where the dispersion is modelled from the GLM deviance and stabilized with empirical Bayes shrinkage, to stabilize the estimates in the presence of limited replication.

## Adaptation of Spatial FDR to neighbourhoods

To control for multiple testing, we adapt the Spatial FDR method introduced by *Cydar* (Lun, Richard, and Marioni 2017). The Spatial FDR can be interpreted as the proportion of the union of neighbourhoods that is occupied by false-positive neighbourhoods. This accounts for the fact that some neighbourhoods are more densely connected than others. To control spatial FDR in the KNN graph, we apply a weighted version of the Benjamini-Hochberg (BH) method. Briefly, to control for FDR at some threshold  $\alpha$  we reject null hypothesis  $i$  where the associated p-value is less than the threshold

$$\max_i p_{(i)} : p_{(i)} \leq \alpha \frac{\sum_{l=1}^i w_{(l)}}{\sum_{l=1}^n w_{(l)}}$$

Where the weight  $w_{(i)}$  is the reciprocal of the neighbourhood connectivity  $c_n$ . As a measure of neighbourhood connectivity, we use the euclidean distance to the  $k$ th nearest neighbour of the index cell for each neighbourhood.

## References

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