Astrometry forecast

In progress...

November 13, 2024

1 Forecast

From the equation (2.2) at $t = t'$ we have

$$\langle \delta n^i(\mathbf{n}, t) \delta n^j(\mathbf{q}, t') \rangle = \frac{3H_0^2}{32\pi^3} \int df \, \frac{\Omega_{\rm GW}}{f^3} \left[H_{ij}^{(0)}(\mathbf{n}, \mathbf{q}) + \beta \, (4 - n_\Omega) \, H_{ij}^{(1)}(\mathbf{n}, \mathbf{q}, \mathbf{v}) \right] , \tag{1}$$

We identify the monopole and dipole coefficients:

$$M = \frac{3H_0^2}{32\pi^3} \int df \, \frac{\Omega_{\rm GW}}{f^3}, \tag{2}$$

$$D_\beta = \frac{3H_0^2}{32\pi^3} \int df \, \frac{\Omega_{\rm GW}}{f^3} \beta \, (4 - n_\Omega). \tag{3}$$

Besides this, we know

$$\Omega_{\rm GW}(f) = \frac{4\pi^2 f^3}{3H_0^2} I(f) = \frac{4\pi^2 f^3}{3H_0^2} I_0 \left(\frac{f}{f_\star} \right)^{n_I} , \tag{4}$$

Then

$$M = \frac{I_0}{8\pi} \int df \, \left(\frac{f}{f_\star} \right)^{n_I} , \tag{5}$$

$$D_\beta = \frac{I_0}{8\pi} \int df \, \left(\frac{f}{f_\star} \right)^{n_I} \beta \, (4 - n_\Omega). \tag{6}$$

On the other side, from equation (4.1)

$$\langle \delta n_a^i \, \delta n_b^j \rangle \equiv \boldsymbol{C} = \Sigma_n p_n H_{ab,n}^{ij} + N_{ab}^{ij} = \sum_n p_n \boldsymbol{H}_n + \boldsymbol{N} \, . \tag{7}$$

with $p_0 = M$ and $p_1 = D_\beta$. From the equations (5) and (6) we see

$$\Delta p_0 \simeq \frac{\Delta I_0}{8\pi} \int df \, \left(\frac{f}{f_\star} \right)^{n_I} , \tag{8}$$

$$\Delta p_1 \simeq \frac{\Delta I_0}{8\pi} \int df \, \left(\frac{f}{f_\star} \right)^{n_I} \beta \, (4 - n_\Omega) \tag{9}$$

There is a relation between I_0 and $\Omega_{\rm GW,0}$ given by

$$\Omega_{\rm GW,0} = \frac{2\pi^2}{3H_0^2} f_\star^2 A^2 = \frac{4\pi^2}{3H_0^2} f_\star^3 I_0 \tag{10}$$

Section 4.1 from the overleaf has a result for $\Delta p_0 = \sqrt{[\mathcal{F}^{-1}]_{00}}$ and $\Delta p_1 = \sqrt{[\mathcal{F}^{-1}]_{11}}$, where $\mathcal{F}$ is given byt the equation (4.13) in that section, times the total number of observations n_{obs}. With the relations above, the errors on the parameters p_n can be expressed as errors in $\Delta\Omega_{\rm GW,0}$ as follows,

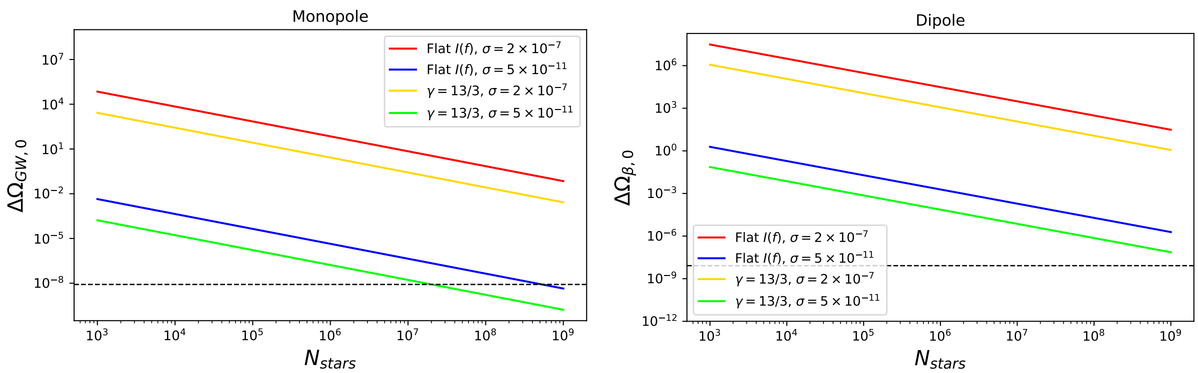

Figure 1: *Forecast for the estimators* p_0 *(***left panel***) and* p_1 *(***right panel***) in terms of the spectral energy density parameter* $\Omega_{\rm GW,0}$*. In this case* $\Omega_{\beta,0}$ *is just a way to make it evident that* $\Omega_{\rm GW,0}$ *is calculated this time with the dipole estimator.*

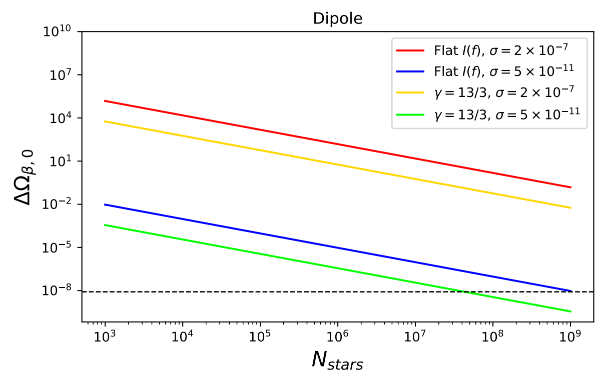

Figure 2: *Forecast for the estimator and* p_1 *in terms of the spectral energy density parameter* $\Omega_{\rm GW,0}$*. In this case* $\Omega_{\beta,0} = \beta(4 - n_\Omega)\Omega_{\rm GW,0}$.

$$\Delta\Omega_{\rm GW,0} \simeq \frac{4\pi^2}{3H_0^2} f_\star^3 \Delta I_0 = \frac{4\pi^2 f_\star^3}{3H_0^2}\left(\frac{8\pi\Delta p_0}{\int df \left(\frac{f}{f_\star}\right)^{n_I}}\right), \tag{11}$$

$$\Delta\Omega_{\rm GW,0} \simeq \frac{4\pi^2}{3H_0^2} f_\star^3 \Delta I_0 = \frac{4\pi^2 f_\star^3}{3H_0^2}\left(\frac{8\pi\Delta p_1}{\beta(4-n_\Omega)\int df \left(\frac{f}{f_\star}\right)^{n_I}}\right) \tag{12}$$

I can also plot equation (12) times the factor $\beta(4 - n_\Omega)$, the result for this case is in Figure 2. Nevertheless, in this way there is no much difference between the Dipole and the Monopole.

References