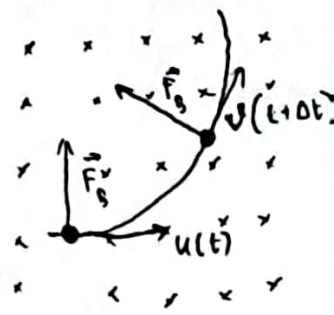
Diketahui

kecepatan partikel $\vec{v} = v_x \hat{i} + v_y \hat{j}$

Medan magnet $\vec{B} = -B_z \hat{k}$



(a) Hk 2 Newton

$$\sum F = ma$$

$$F_B = ma$$

$$q\vec{v} \times \vec{B} = m\frac{d\vec{v}}{dt}$$

$$q(v_x \hat{i} + v_y \hat{j}) \times (-B_z \hat{k}) = m\left(\frac{d}{dt}(v_x)\hat{i} + \frac{d}{dt}(v_y)\hat{j}\right)$$

(b) Persamaan terkopel kedua kecepatan

$$+\frac{qB_z}{m}(v_x \hat{j} - v_y \hat{i}) = \left(\frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}\right), \text{ misal } k = \frac{qB_z}{m}$$

Sumbu x $\quad -k v_y = \frac{dv_x}{dt} \quad \ldots (1)$

Sumbu y $\quad k v_x = \frac{dv_y}{dt} \quad \ldots (2)$

Substitusi (1) ke (2),

$$k v_x = \frac{d}{dt}\left(\frac{-1}{k}\frac{dv_x}{dt}\right) \Rightarrow -k^2 v_x = \frac{d^2 v_x}{dt^2} \Rightarrow \frac{d^2 v_x}{dt^2} + k^2 v_x = 0 \ldots (3)$$

(c) Persamaan $v_x(t)$, $v_y(t)$, $x(t)$, $y(t)$

Dari persamaan (3), substitusi $v_x = Ae^{mt}$, maka

$$\frac{d^2}{dt^2}(Ae^{mt}) + k^2(Ae^{mt}) = 0$$

$$Ae^{mt}(m^2 + k^2) = 0$$

persamaan auxiliary, $m^2 + k^2 = 0$, maka $\sqrt{m^2} = \sqrt{-k^2} \rightarrow m = \pm ik$

$$\left.\begin{aligned} m_1 &= 0 + ik \\ m_2 &= 0 - ik \end{aligned}\right\} \; m = \alpha + i\beta$$

Sehingga persamaan general,

$$v_x(t) = e^{\alpha t}[C_1 \cos \beta t + C_2 \sin \beta t]$$

$$\therefore v_x(t) = C_1 \cos(kt) + C_2 \sin(kt) \ldots (4)$$

Sedangkan kecepatan dlm arah sb-y sebagai berikut

$$v_y(t) = -\frac{1}{k}\frac{d}{dt}(C_1 \cos(kt) + C_2 \sin(kt))$$

$$\therefore v_y(t) = C_3 \sin(kt) - C_4 \cos(kt)$$

Persamaan posisi,

$$\frac{dx}{dt} = v_x$$

$$\int dx = \int v_x \, dt$$

$$\therefore x(t) = \frac{1}{k}\left(C_5 \sin(kt) - C_6 \cos(kt) \right) + C_7$$

$$\int dy = \int v_y \, dt$$

$$\therefore y(t) = -\frac{1}{k}\left(C_8 \cos(kt) + C_9 \sin(kt) \right) + C_{10}$$