Proof MLGS Exam

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Already given as answer in exam:

$$\begin{aligned} L(v)f = d(v)f(v) - \sum_{u \in V} w(u,v)f(u) = \\ \sum_{u \in V} w(u,v)f(v) - \sum_{u \in V} w(u,v)f(u) = \\ \sum_{u \in V} w(u,v)(f(v) - f(u)) = y(v) \end{aligned} \tag{1}$$

$$f^T L f = f^T y = \sum_{v \in V} f(v) \sum_{u \in N(v)} w(u,v)\left(f(v) - f(u)\right) \tag{2}$$

Missing part of the proof (just reformulation):

$$\begin{aligned} &= \sum_{v \in V} \sum_{u \in N(v)} w(u,v) f(v)\left(f(v) - f(u)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)\left(f(v) - f(u)\right) + f(u)\left(f(u) - f(v)\right)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)^2 - f(v)f(u) + f(u)^2 - f(u)f(v)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)^2 + f(u)^2 - 2f(v)f(u)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v) - f(u)\right)^2 \geq 0 \end{aligned} \tag{3}$$

The only part missing in the exam was the final reformulation due to time constraints. However, the core idea and the "hard" part of the proof were already completed correctly during the exam. I do not understand why points were deducted for not using the provided hint, as this enforces a particular solution approach, which should not be the objective of the assessment.