Proof MLGS Exam

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Already given as answer in exam:

$$\begin{aligned} L(v)f = d(v)f(v) - \sum_{u \in V} w(u,v)f(u) = \\ \sum_{u \in V} w(u,v)f(v) - \sum_{u \in V} w(u,v)f(u) = \\ \sum_{u \in V} w(u,v)(f(v) - f(u)) = y(v) \end{aligned} \tag{1}$$

$$f^T L f = f^T y = \sum_{v \in V} f(v) \sum_{u \in N(v)} w(u,v)\left(f(v) - f(u)\right) \tag{2}$$

Missing part of the proof (just reformulation):

$$\begin{aligned} &= \sum_{v \in V} \sum_{u \in N(v)} w(u,v) f(v)\left(f(v) - f(u)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)\left(f(v) - f(u)\right) + f(u)\left(f(u) - f(v)\right)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)^2 - f(v)f(u) + f(u)^2 - f(u)f(v)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v)^2 + f(u)^2 - 2f(v)f(u)\right) \\ &= \sum_{(u,v) \in E} w(u,v)\left(f(v) - f(u)\right)^2 \geq 0 \end{aligned} \tag{3}$$

The only part missing in my exam was the final reformulation, which I couldn't complete due to time constraints. However, the core idea and critical part of the proof were completed accurately during the exam.
I strongly disagree with the grading scheme. It appears that the

scheme expects students to follow the sample solution exactly, awarding points specifically for steps such as "using the hint," "applying linearity," "using definitions," and certain "simplifications," as mentioned in my grading. This effectively enforces a particular solution approach, which should not be the purpose of the assessment.
Points should not be subtracted for not using specific steps like "using the hint", "applying linearity", "using definitions" or for specific "simplifications" as they are not strictly necessary to prove the statement. My proof already addresses the essential parts of proofing the statement, with only the final calculations or reformulations missing — an omission worth only -0.5 points according to the grading scheme.
Given that my solution meets the proof's requirements aside from the "final calculations" (= missing reformulations above), I believe I should receive 2.5 points. Expecting a single, specific proof is unreasonable and overly restrictive.