Introduction to deep learning

Lecture 3

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Master 2 informatique
Université Paris Cité

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Today's lecture

1. Recap
2. Softmax regression and SGD: implementation
3. Vectorization
4. Going deep: Activation functions

Last week

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- Softmax regression

TP:

Reminder: Course webpage: https://samvangool.net/iap/

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TP:

- Implementing *softmax regression* in Numpy
- Over-fitting a single batch
- Training on a full dataset, achieving $> 80\%$ accuracy.

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- Read [Z, pp. 82-88, pp. 126-148], [P, Sections 5.1–5.2].
- Do [Z, Section 4.4.7], Exercise 1 and [P, Problem 5.2].
- If you need a reminder of Linear Algebra and Calculus: read [Z, Section 2.3 and 2.4], or [P, Appendix B].

Today's lecture

[Z, Section 4.4.7], Exercise 1

Numeric stability of `softmax` (see question 4(b) on TP2).

See notebook.

[P, Problem 5.2]

The loss function in the case of binary classification.

See notebook.

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- In today's TP, you will make last week's solution more efficient using vectorization.

A (slow) implementation of softmax regression

See notebook.

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Broadcasting

- Numpy arrays have a `shape` attribute.

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- Broadcasting happens when we perform an operation on two numpy arrays that do not have the same shape.

Broadcasting example

Example

(See notebook.)

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A = np.array([[1.],[2.],[3.],[4.]]) # shape: (4,1)
B = np.array([[5.,-5.,5.,-5.,5.]])  # shape: (1,5)
A + B
```

Images generated by Google ImageFX, input prompts: "An unhappy mathematician, putting his head against a blackboard" and "A happy computer, which can perform any computation you ask of it".

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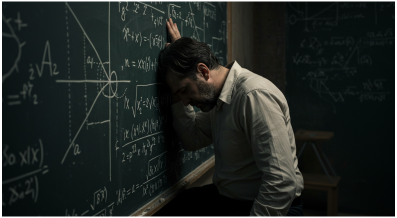

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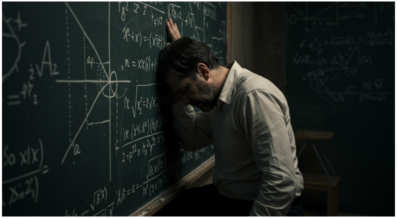

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Numpy: This is fine.

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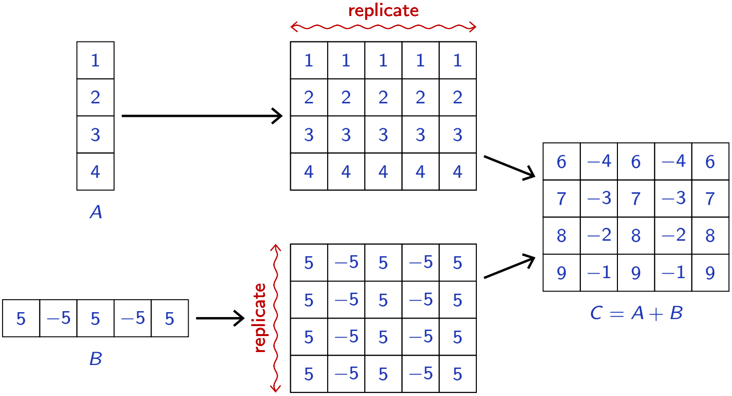
Broadcasting example visualized

```
A = torch.tensor([[1.], [2.], [3.], [4.]])
B = torch.tensor([[5., -5., 5., -5., 5.]])
C = A + B
```



Example & image credit: François Fleuret's Deep Learning Course.

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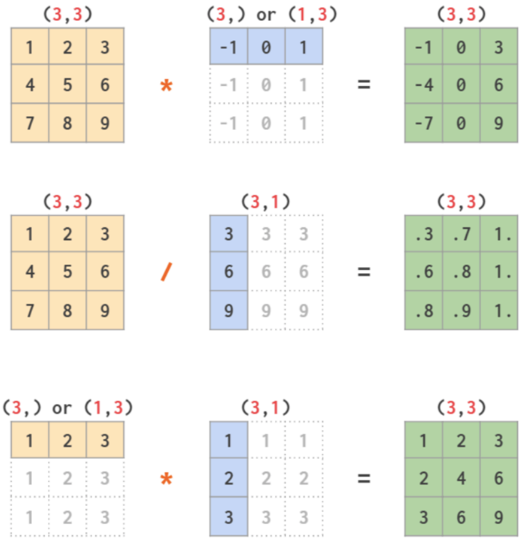

(From https://towardsdatascience.com/broadcasting-in-numpy-58856f926d73)

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- Numpy (and Pytorch) work *faster* than Python loops.
- We can **vectorize** the calculation `softmax(x @ W + b)`.

Batch matrix multiplication

- Instead of repeating `x @ W + b` for every row `x` separately, we can take advantage of broadcasting, and do it at once for the entire batch:

```
x.shape # (64, 784)
W.shape # (784, 10)
b.shape # (1, 10)
(x @ W).shape # (64, 10)
(x @ W + b).shape # (64, 10)
```

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Today's lecture

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- Its derivative is very easy to compute:

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- Multi-layer perceptron = stack of several linear layers, with ReLU in between each layer.

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Why does this give the same answer?

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Why is this the gradient?

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- Review the reflection questions on the last two slides above.

References

[P] Prince, S., **Understanding Deep Learning**, https://udlbook.com

[Z] Zhang, A. et al., **Dive into Deep Learning**, https://www.d2l.ai

Image sources:

- https://en.wikipedia.org/wiki/Omega
- https://en.wikipedia.org/wiki/Nevel_(instrument)