Aalto University, Institute of Industrial Engineering and Management
TU-E2210, Financial Risk Management I, Fall 2025
Kaila, Toepfer

Hedging call options
Assignment with real data

The goal of this assignment is to give you experience of working with real data, to learn hedging and to get an idea of how Financial Risk Management can be done with Python, R, Julia or Matlab. You can work either alone or in groups of 2-3 people. The report produced by a group should be more comprehensive than a report done alone, so that the workload is the same for everyone.

The assignment consists of three parts:

1. Searching real option price data,
2. Hedging call options; done in Python, R, Julia or Matlab,
3. Writing a report on the results.

The report should be returned no later than Sunday, December 8, 2025.

Task description: In this assignment, you should hedge call options using different hedging strategies (delta, delta-vega and delta-gamma hedging) and different re-hedging frequencies and compare the corresponding hedging results. A call option is hedged with a replicating portfolio consisting of a certain number of underlying assets and possibly replicating options. When the price of the underlying asset changes, the number of shares and options in the replicating portfolio must be readjusted. As a starting point, it is worth trying delta and delta-vega hedging. If you work in a group, you can also try delta-gamma hedging.

Data: You can select your own data. One easy choice is the SP500 index and option data. Alternatively, you can choose something more exotic. Instructions on how to obtain data from the financial databases provided by Aalto University are given in a separate paper.

Delta hedging: You can start with delta hedging. First, try to hedge an ATM (at-the-money) call C with a time to maturity of about $T = 45$ days. These options are the most stable and easiest to start with.

Assume that you have two portfolios. In the original portfolio OP, you have a long position in a call C. You want to replicate the changes in value of this portfolio. In the replicating portfolio RE, you have a short position: a number delta $\Delta = \partial C/\partial S$ of the underlying asset S. Initially, the value of OP is C_0 and the value of RE is $-\Delta_0 S_0$.

The value of the asset in the replicating portfolio changes as a function of time, and so does the value of the call. After some time, the hedge is not accurate anymore. Then, you can first

1. compute the difference A_i between the change in the value of OP, i.e, $C_{i+1} - C_i$, and the change in the value of the replicating portfolio RE, i.e., $\Delta_i(S_{i+1} - S_i)$. (Our aim was that $OP + RE$ would be immune to changes in the value of the underlying; we compute how well we did.);

2. adjust the replicating portfolio RE by holding a readjusted amount delta Δ_{i+1} of the underlying asset.

Here, $i = 0$. After some time, the hedge is not accurate anymore. Repeat the tasks 1. and 2. with $1 \leq i \leq n-1$ until you have reached the option maturity. Then, compute the total mean squared error,

$$E = \frac{1}{n-1} \sum_{i=1}^{n-1} A^2.$$

This total error E represents the average accuracy of the hedge. Why should you compute the sum of A^2 and not A?

You can test how the *rehedging frequency* affects the error E. You can try rehedging every day, every second day or once a week, for example. Then, you can try *hedging using different strikes* (OTM, ATM, ITM) and different *maturities*. Instead of a single option, you can hedge a portfolio of options or try to hedge a position, for example a butterfly.

You will need the implied volatility to compute the delta Δ. If you choose options that are deep ITM or deep OTM, you cannot compute any implied volatility. Instead, you will get NaN (Not-a-number). If you have even once NaN in your calculations, you will not get any result. For example, you cannot compute the total error E. Select only those options that allow you to calculate all implied volatilities.

Delta-vega hedging: When you feel that delta hedging is easy, you can try delta-vega hedging. Note that this time, in the replicating portfolio, you should hold a certain amount of both the underlying asset and the replicating option C^{rep}. This replicating option should have the same underlying and the same strike price but longer time to maturity T_2 than the original option C.

You can compare the accuracy of delta hedging and delta-vega hedging. In previous years, delta-vega hedging has proven to be much more difficult than delta hedging. You should get delta protection to work. If you don't get decent results from delta-vega-hedging, you can present the results you got and analyse how they differ from the theory (from the results you should have got).

Repeating the experiment for statistical results: To get at least some statistical insight into how different strategies and re-hedging frequencies affect hedge performance, you should repeat the hedge 10 times (same underlying, same strike price, same time to maturity BUT different expiration dates). Ten times is not enough in real life, but it is enough for us. From these repetitions, you can calculate averages and standard deviations of the accuracy of different hedging strategies.

Example on S&P100 options: The underlying of these options is the S&P100 index. Typically, these options open 6-3 months before the maturity. Each month, one option series matures, with 40-50 different strike prices. You could then use 10 sets of these options in your experiment, maturing between January and October 2017. In the experiment, you could use 3 different options: OTM, ATM, and ITM.

The above describes only one possible way of dealing with the hedging problem. However, you are very free to approach the problem in another way. There is no single right way to perform the task. The only thing that matters is that you understand the idea of hedging and how different parameters and hedging strategies affect the performance of the protection.

Hedging strategies - delta hedging and delta-vega hedging

The delta, denoted by $\varDelta$, is given by

$$\varDelta = \frac{\partial C^{\text{BS}}}{\partial S} = \mathcal{N}(d_1)$$

and the vega, denoted by κ, is given by

$$\kappa = \frac{\partial C^{\text{BS}}}{\partial \sigma} = \frac{S_t e^{-d_1^2/2}\sqrt{T-t}}{\sqrt{2\pi}} = S_t\sqrt{T-t}\,\mathcal{N}'(d_1).$$

Hedging strategies in incomplete markets depend on some dynamic risk-measure that has to be minimized. Common strategies include *delta hedging* and *delta-vega hedging*.

Consider two portfolios: OP consisting of a long call $C^{\text{BS}} = C^{\text{BS}}(t, S_t; E, T; \sigma)$ and the replicating portfolio RE consisting of a short amount of the underlying asset S_t. Changes in the value of the underlying affect the value of the option. If, for example, the asset price rises, the value of a call option increases. When the asset volatility is constant and trading is continuous, it is theoretically possible to keep $OP+RE$ protected against changes in the value of the underlying by holding an instantaneous delta number $\varDelta = \partial C^{\text{BS}}/\partial S$ of the underlying in replicating portfolio RE. Then, following this *delta strategy*, it is possible to eliminate all risk of loss if the option is executed. In practice, complete elimination is not possible as trading is done discretely.

When the asset price volatility is not constant, the perfect complete hedging is not possible even in theory. There are at least two sources of randomness: one from the asset price process, the *other one from the volatility process*. The volatility randomness is then hedged by holding in the replicating portfolio, in addition to the underlying assets, an instantaneous number of replicating options. These replicating options have the same underlying asset and roughly the same strike price as the original option being replicated. The expiry date is longer. We call this delta-vega hedging.

Let us have a practical look at the delta-vega hedging strategy. Now, the replicating portfolio RE consists of three main components: an instantaneous α amount of the underlying stock and an instantaneous amount η of a replicating option $C^{\text{Rep}} = C^{\text{Rep}}(t, S_t; E, T_2; \sigma)$ on the same underlying and with the same strike price than the original option, but a longer maturity, $T_2 > T$.

The amount α of assets to be held is

$$\alpha(\sigma) = \frac{\partial C^{\text{BS}}}{\partial S} - \frac{\partial C^{\text{BS}}/\partial\sigma}{\partial C^{\text{Rep}}/\partial\sigma}\frac{\partial C^{\text{Rep}}}{\partial S} \tag{1}$$

and the amount η of the replicating options to be held is

$$\eta(\sigma) = \frac{\partial C^{\text{BS}}/\partial\sigma}{\partial C^{\text{Rep}}/\partial\sigma}. \tag{2}$$

The hedging ratios α and η can be written in terms of the Greeks delta $\varDelta(\sigma)$ and vega $\kappa(\sigma)$ as

$$\alpha(\sigma) = \varDelta^{\text{BS}}(\sigma) - \frac{\kappa^{\text{BS}}(\sigma)}{\kappa^{\text{Rep}}(\sigma)}\varDelta^{\text{Rep}}(\sigma) \tag{3}$$

and

$$\eta(\sigma) = \frac{\kappa^{\text{BS}}(\sigma)}{\kappa^{\text{Rep}}(\sigma)}, \tag{4}$$

where Δ^{BS} and κ^{BS} refer to the delta and vega of the hedged option C^{BS}, and Δ^{Rep} and κ^{Rep} refer to those of the replicating option C^{Rep}.

Please check in your assignment that the signs of alpha and eta are correct (long or short position). In your report, explain why in delta-vega hedging, we hold an amount α and not Δ of the underlying.

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Also present your results as graphs and analyse the graphs in terms of theory.

To start with:

We choose an initial day, for example n=45 days to maturity. We denote this moment of time by t_0. We then choose a call option with price C_0 on that date. We compute its implied volatility I_0 needed to compute delta Δ_0. Then, we make two portfolios: we take a long position in portfolio OP containing one call and a short position in portfolio RE containing an amount delta of the underlying asset S. The initial values are $OP_0 = C_0$ and $RE_0 = \Delta_0 S_0$. Ideally, changes in the value of the option portfolio OP are neutralized by opposite changes in the replicating portfolio RE. Notice that the values of the portfolios are not the same; only changes in their values should be the same (for a short moment of time). We decide to rehedge every second day.

Then, the next day t_1, we compute how much the value of each portfolio has changed: $dOP_{01} = OP_1 - OP_0 = C_1 - C_0$ and $dRE_{01} = RE_1 - RE_0 = \Delta_0(S_1 - S_0)$. We denote $A_0 = dOP_{01} - dRE_{01}$. We don't want to rehedge yet.

The third day t_2, we compute how much the value of each portfolio has changed: $dOP_{12} = OP_2 - OP_1 = C_2 - C_1$ and $dRE_{21} = RE_2 - RE_1 = \Delta_0(S_2 - S_1)$. We denote $A_1 = dOP_{21} - dRE_{21}$. Then, we rehedge. We compute the implied volatility I_2 and the corresponding delta Δ_2. Then, we adjust the replicating portfolio so that it contains an amount Δ_2 of the underlying asset with value S_2. Now, $OP_2 = C_2$ and $RE_2 = \Delta_2 S_2$. (here, you don't need to think where the money comes from to buy the underlying assets. When later you feel comfortable with this assignment, you can make a variation were you keep track also of the money used, if you want to.)

The fourth day t_3, we compute how much the value of each portfolio has changed: $dOP_{32} = OP_3 - OP_2 = C_3 - C_2$ and $dRE_{32} = RE_3 - RE_2 = \Delta_2(S_3 - S_2)$. We denote $A_2 = dOP_{32} - dRE_{32}$. This time, we don't rehedge.

You can proceed by computing A_i and rehedging every second day until the maturity, day t_n. Then, you can compute the squared sum of A_i, $0 \le i \le n-1$. Alone, this squared sum does not tell you a lot. You can change the frequency of rehedging, the strike prices used, the maturities used, the expiration dates, and compare the squared sums $\sum_{i=0}^{n-1} A_i^2$ obtained.

Good luck with the assignment! Feel free to ask Ruth or Eljas if there are unclear issues.