Summary of informal proof rules

	Introduction Rule	**Elimination Rule**
$\rightarrow$	To prove $\Phi \rightarrow \Psi$, assume Φ and prove Ψ.	Given Φ and $\Phi \rightarrow \Psi$, conclude Ψ.
$\leftrightarrow$	To prove $\Phi \leftrightarrow \Psi$, prove both $\Phi \rightarrow \Psi$ and $\Psi \rightarrow \Phi$.	Given $\Phi \leftrightarrow \Psi$ and Φ, conclude Ψ. Given $\Phi \leftrightarrow \Psi$ and Ψ, conclude Φ.
$\neg$	To prove $\neg\Phi$, assume Φ and derive a contradiction.	Given $\neg\Phi$ and Φ, conclude Ψ.
$\wedge$	To prove $\Phi \wedge \Psi$, prove both Φ and Ψ.	Given $\Phi \wedge \Psi$, conclude Φ. Given $\Phi \wedge \Psi$, conclude Ψ.
$\vee$	To prove $\Phi \vee \Psi$, prove at least one of Φ or Ψ.	To use $\Phi \vee \Psi$ to prove Γ, first assume Φ and prove Γ. Then separately assume Ψ and again prove Γ.
$\forall$	To prove $\forall x \Phi(x)$, pick an arbitrary x_0 and prove $\Phi(x_0)$.	Given $\forall x \Phi(x)$, conclude $\Phi(x_0)$ for any x_0 of your choice.
$\exists$	To prove $\exists x \Phi(x)$, specify an x_0 satisfying $\Phi(x_0)$.	Given $\exists x \Phi(x)$, let x_0 be an element satisfying $\Phi(x_0)$.

Proof by contradiction

Use the ($\neg$)-introduction rule on $\neg\Phi$ to conclude $\neg(\neg\Phi) \equiv \Phi$.

Proof by cases

Add the tautology $\Phi \vee \neg\Phi$ to your givens and use the ($\vee$)-elimination rule to distinguish cases.

Modifying givens and proof goal

One can always replace the givens or the proof goal with logically equivalent statements. One may always add established true statements to the givens.