

$$(a) \quad \vdash (\neg(\varphi \wedge \neg\varphi))$$

$$\frac{\frac{\frac{\cancel{(\varphi \wedge \neg\varphi)}}{\varphi} \wedge E \quad \frac{\frac{\cancel{(\varphi \wedge \neg\varphi)}}{\neg\varphi} \wedge E}{\neg(\varphi \wedge \neg\varphi)} \neg I \quad (1)}{\perp} \neg E \quad (1)$$

$$(b) \quad \vdash ((\neg(\varphi \rightarrow \psi) \rightarrow \neg\psi)$$

$$\frac{\frac{\cancel{\neg(\varphi \rightarrow \psi)}}{\neg\psi} \rightarrow I \quad (1) \quad \neg(\varphi \rightarrow \psi)}{\neg\neg\psi} \neg I \quad (1)$$

$$\frac{\neg\neg\psi}{((\neg(\varphi \rightarrow \psi) \rightarrow \neg\psi))} \rightarrow I \quad (2)$$

(no discharging)

$$(c) \quad \{(\varphi \rightarrow \psi)\} \vdash ((\neg \psi) \rightarrow (\neg \varphi))$$

$$\frac{\frac{(\neg \psi)}{\text{②}} \quad \frac{(\varphi \rightarrow \psi) \quad \cancel{\varphi} \text{①}}{\psi} \rightarrow E}{\psi}$$

$$\frac{\frac{\perp}{(\neg \varphi)} (\neg I) \text{①}}{(\neg \psi) \rightarrow (\neg \varphi)} (\rightarrow I) \text{②}$$

$$(d) \quad \{(\varphi \rightarrow \psi)\} \vdash (\neg(\varphi \wedge (\neg \psi)))$$

$$\frac{\frac{\cancel{\varphi \wedge (\neg \psi)} \text{①}}{\neg \psi} \quad \frac{\frac{\cancel{\varphi \wedge (\neg \psi)} \text{①}}{\varphi} \quad (\varphi \rightarrow \psi)}{\varphi}}{\perp} (\neg E)$$

$$\frac{\perp}{\neg(\varphi \wedge (\neg \psi))} (\neg I) \text{①}$$