

Logic and Foundation with Haskell

Exercise sheet 10

Exercise 1. In the lecture, we defined ordered pairs as $\langle x, y \rangle := \{\{x\}, \{x, y\}\}$. Check (by informal proof) that this definition satisfies the characteristic properties of pairs:

$$\langle x, y \rangle = \langle x', y' \rangle \Leftrightarrow x = x' \wedge y = y'.$$

Exercise 2. Prove the following set theoretic identities (by informal proof):

$$\begin{aligned} \left(\bigcup_{i \in I} A_i \right) \cap B &= \bigcup_{i \in I} (A_i \cap B), \\ \left(\bigcap_{i \in I} A_i \right) \setminus B &= \bigcap_{i \in I} (A_i \setminus B), \\ (A \cap B) \times C &= (A \times C) \cap (B \times C) \\ (A \cup B) \times C &= (A \times C) \cup (B \times C) \\ (A_1 \times B_1) \cap (A_2 \times B_2) &= (A_1 \cap A_2) \times (B_1 \cap B_2) \end{aligned}$$