Tutoraggio…sviluppi di Taylor

lunedì 25 novembre 2024

20 Gennaio 2021

Calcolare il limite 0

$$\lim_{x\to+\infty}\frac{(e^x+x)\cdot\left[1-\frac{1}{2x^2}-\cos\frac{1}{x}\right]}{(e^{\frac{x}{2}}-x)^2\ln^4\left(1+\frac{1}{x}\right)} =$$

$(\infty-\infty)^2$

$\lim_{x\to+\infty} t = \lim_{x\to+\infty}\frac{1}{x}=0$

$\left[\ln(1+t)\right]^4 = \left[t-\frac{t^2}{2}+\frac{t^3}{3}-\frac{t^4}{4}+o(t^4)\right]^4 = t^4+o(t^4)$

$t=\frac{1}{x}$

$\lim_{t\to0}\frac{\left[-\frac{t^4}{24}+o(t^4)\right]}{(t^4+o(t^4))} = -\frac{1}{24}$

$1-\frac{t^2}{2}-\cos t = 1-\frac{t^2}{2}-\left[1-\frac{t^2}{2}+\frac{t^4}{24}+o(t^4)\right]=$

$=\cancel{1}-\cancel{\frac{t^2}{2}}-\cancel{1}+\cancel{\frac{t^2}{2}}-\frac{t^4}{24}+o(t^4)$

$\lim_{x\to+\infty}\frac{e^x+x}{(e^{x/2}-x)^2}\sim\frac{e^x\left(1+\frac{x}{e^x}\right)}{\left[e^{x/2}\left(1-\frac{x}{e^{x/2}}\right)\right]^2}=\frac{\cancel{e^x}\left(1+\frac{x}{e^x}\right)}{\cancel{e^x}\left(1-\frac{x}{e^{x/2}}\right)^2}$

$=\lim_{x\to+\infty}\frac{e^x\left(1+\frac{x}{e^x}\right)}{e^x\left(1-\frac{x}{e^{x/2}}\right)^2}\cdot\lim_{t\to0}\frac{\left[-\frac{t^4}{24}+o(t^4)\right]}{t^4+o(t^4)} = 1\cdot\left(-\frac{1}{24}\right)=-\frac{1}{24}$

$\lim_{x\to+\infty}\frac{\left[-\frac{1}{24x^4}+o\left(\frac{1}{x^4}\right)\right]}{\frac{1}{x^4}+o\left(\frac{1}{x^4}\right)}$

$$\lim_{x\to0^+}\frac{e^x-e^{\sin x}}{\sqrt{x}[x-\tan x]}\sin\left(\frac{x+3\sqrt{x}}{1-x}\right)$$

$\to 1$

$\frac{x+3\sqrt{x}}{1-x}$

$\frac{x+3\sqrt{x}}{1-x}=\frac{\sqrt{x}(\sqrt{x}+3)}{1-x}$

$\lim_{x\to0^+}\frac{e^x-e^{\sin x}}{x-\operatorname{tg}x}\cdot\frac{\cancel{\sqrt{x}}(\sqrt{x}+3)}{1-x}\cdot\frac{1}{\cancel{\sqrt{x}}}=$

$\operatorname{sen}x = x-\frac{x^3}{6}+o(x^3)$

$e^x=1+x+\frac{x^2}{2}+\frac{x^3}{6}+o(x^3)$

$e^{\operatorname{sen}x}=1+x-\frac{x^3}{6}+\frac{1}{2}\left(x-\frac{x^3}{6}\right)^2+\frac{1}{6}\left(x-\frac{x^3}{6}\right)^3+o(x^3)$

$1+x-\frac{x^3}{6}+\frac{x^2}{2}+\frac{x^3}{6}+o(x^3)$

$x-\operatorname{tg}x = x-\left(x+\frac{x^3}{3}+\cancel{\frac{2}{15}x^5+o(x^5)}\right)$ $o(x^3)$

$\lim_{x\to0^+}\frac{\cancel{1}+\cancel{x}+\cancel{\frac{x^2}{2}}+\frac{x^3}{6}+o(x^3)-\cancel{1}-\cancel{x}+\cancel{\frac{x^3}{6}}-\cancel{\frac{x^2}{2}}-\cancel{\frac{x^3}{6}}+o(x^3)}{x-\left(x+\frac{x^3}{3}+o(x^3)\right)}\cdot\frac{(\sqrt{x}+3)}{1-x}=\frac{\frac{1}{2\cancel{6}}}{-\frac{1}{3}}\cdot\frac{\cancel{3}}{1}=-\frac{3}{2}$

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$\lim_{n\to+\infty}\frac{\left(\sqrt{n^2+\ln(n^n)}-\sqrt{n^2+2}\right)\cancel{n!}}{\left(n^{2/n}-1\right)\cancel{(n+1)!}\left(1+\frac{n}{(n+1)!}\right)}\cdot\frac{\left(\sqrt{n^2+n\ln n}+\sqrt{n^2+2}\right)}{\left(\sqrt{n^2+n\ln n}+\sqrt{n^2+2}\right)}=$

$(n+1)$ $\to 0$

$\lim_{n\to+\infty}\frac{\cancel{n^2}+n\ln n-\cancel{n^2}-2}{\left(\sqrt[n]{n^2}-1\right)(n+1)\left(\sqrt{}+\sqrt{}\right)}=$

$n^{2/n}=e^{\ln n^{2/n}}$ $e^{\frac{2}{n}\ln n}$

$2\frac{\ln n}{n}\cdot\frac{\sqrt[n]{n^2}-1}{2\frac{\ln n}{n}}=\frac{e^{\ln n^{2/n}}-1}{}=\frac{e^{\frac{2\ln n}{n}}-1}{}$

$\frac{\cancel{n}\ln n-\cancel{2}}{(n+1)\left(\sqrt[n]{n^2}-1\right)\cancel{n}\left(\sqrt{1+\frac{\ln n}{n}}+\sqrt{1+\frac{2}{n^2}}\right)}=$

$\lim_{n\to+\infty}\frac{2\ln n}{n}=0$

$e^x=1+x$

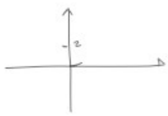
$=\frac{\ln n}{(n+1)\left(\cancel{1}+\frac{2}{n}\ln n\right)2}\sim\frac{n}{4(n+1)}=\frac{1}{4}$

$e^{\frac{2\ln n}{n}}=1+\frac{2\ln n}{n}+o\left(\frac{\ln n}{n}\right)$

22 giugno 2021

Esercizio 2 (6 punti). Determinare per quali valori di $\alpha\in\mathbb{R}$ la funzione $f:]-\infty,1]\to\mathbb{R}$ data da

$$f(x)=\begin{cases}\frac{1-\cos(2x)}{x^2+x^4} & \text{se } x<0\\ 0 & \text{se } x=0\\ \frac{\ln(1-(x-\sin x))}{x^\alpha} & \text{se } x>0\end{cases}$$



ammette un punto di salto in $x=0$ e calcolarne l'ampiezza.

$\lim_{x\to0^-}\frac{1-\cos 2x}{x^2+x^4}=\lim_{x\to0}\frac{1-\cos 2x}{4x^2(1+x^2)}4=\frac{1}{2}\cdot4=2$

$\alpha=3$ $\quad l^+=-\frac{1}{6}$ $\quad$ salto $=2-\left(-\frac{1}{6}\right)=\frac{13}{6}$

$\lim_{x\to0^+}\frac{\ln(1-(x-\operatorname{sen}x))}{x^\alpha}=\lim_{x\to0^+}\frac{-\frac{x^3}{6}+o(x^3)}{x^\alpha}=$

$\frac{x^3}{x}$

$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}$

$\alpha<3$ $\quad l^+=0$ $\quad$ salto $=2$

$\operatorname{sen}x=x-\frac{x^3}{6}+o(x^3)$

$\ln(1-(x-\operatorname{sen}x))=-\frac{x^3}{6}+o(x^3)$

$-(x-\operatorname{sen}x)=-\frac{x^3}{6}+o(x^3)$

31 Agosto 2021

Calcolare al variare di $\alpha>0$ il limite

$$\lim_{x\to0^+}\frac{\sin(\log(1+3x))-e^{3x}+\cos x}{(\sin x)^{5\alpha}}$$

Esercizio 2 (6 punti). Determinare per quale valore di $\alpha > 0$ il limite

$$\lim_{x\to 0^+} \frac{\ln(1+x^2) - \sin x^2}{1 - \sqrt{x^\alpha + 1}}$$

esiste finito e non nullo e calcolarlo.

Calcolare al variare di $\alpha \in \mathbb{R}$ il limite

$$\lim_{x\to 0^+} \frac{e^x - \frac{1}{2}\ln(1+2x) - 1}{x^\alpha \sqrt{1+x^3} - \cos x}$$