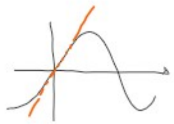
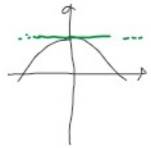
Derivate

lunedì 4 novembre 2024

$D\,\mathrm{tg}\,x = 1 + \mathrm{tg}^2 x$





$$D\,\mathrm{tg}\,x = D\frac{\mathrm{sen}\,x}{\cos x} = \frac{\cos x \cdot \cos x - \mathrm{sen}\,x \cdot (-\mathrm{sen}\,x)}{\cos^2 x} = \frac{\overbrace{\cos^2 x + \mathrm{sen}^2 x}^{1}}{\cos^2 x} = 1 + \frac{\mathrm{sen}^2 x}{\cos^2 x} = 1 + \mathrm{tg}^2 x$$

Derivate

$$D\,x^{\alpha} = \alpha\, x^{\alpha - 1}$$

$f(x) = x \log x - x$ $\qquad f'(x) = \log x + \cancel{x} \cdot \frac{1}{\cancel{x}} - 1 = \log x$

$f(x) = x\,\mathrm{sen}\,x + \cos x$ $\qquad f'(x) = 1 \cdot \cancel{\mathrm{sen}\,x} + x \cdot \cos x - \cancel{\mathrm{sen}\,x} = x \cos x$

$f(x) = \mathrm{tg}\,x + \frac{1}{\cos x}$ $\qquad f'(x) = \frac{1}{\cos^2 x} + \frac{\mathrm{sen}\,x}{\cos^2 x} = \frac{1 + \mathrm{sen}\,x}{\cos^2 x}$

$$\frac{1}{\cos x} = (\cos x)^{-1} = -1(\cos x)^{-2} \cdot (-\mathrm{sen}\,x)$$

$f(x) = x^2 \cdot 2^x$ $\qquad f'(x) = (D x^2)(2^x) + x^2 (D\, 2^x) = 2x \cdot 2^x + x^2\, 2^x \ln 2 = x 2^x (2 + x\ln 2)$

$$\frac{1}{\cos x} = \frac{0 - 1 \cdot (-\mathrm{sen}\,x)}{\cos^2 x} = \frac{\mathrm{sen}\,x}{\cos^2 x}$$

$f(x) = 2^x \log_2 x$ $\qquad f'(x) = 2^x \ln 2 \log_2 x + 2^x \cdot \frac{1}{x \ln 2} = 2^x \cancel{\ln 2} \frac{\ln x}{\cancel{\ln 2}} + 2^x \frac{1}{x \ln 2} = 2^x \left[\ln x + \frac{1}{x \ln 2}\right]$

$$D\,a^x = \log a \cdot a^x$$

$$a^x = e^{\log a^{x}} = e^{x \cdot \log a}$$

$$D\,a^x = \log a \;\; a^x$$

$$D\,e^x = e^x$$

$$D \log_a x = \frac{1}{x \ln a}$$

$$D \log_a x = D\frac{\ln x}{\ln a} = \frac{1}{\ln a} D \ln x = \frac{1}{\ln a\; x}$$

$x > 0$ $\qquad f(x) = \frac{x + \sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x} \cdot \cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} + \frac{\cancel{\sqrt{x}}^{\,1}}{\cancel{\sqrt{x}}_{\,1}} = (x)^{1/2} + 1$

$$f'(x) = \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}}$$

$f(x) = \frac{1 + x^{\frac{3}{4}}}{1 - x^{3/4}}$

$$f'(x) = \frac{\frac{3}{4}x^{-1/4}(1 - x^{3/4}) - (1 + x^{3/4}) \cdot \left(-\frac{3}{4}x^{-1/4}\right)}{(1 - x^{3/4})^2} = \frac{\frac{3}{4}\frac{1}{\sqrt[4]{x}} - \cancel{\frac{3}{4}\sqrt{x}} + \frac{3}{4}\frac{1}{\sqrt[4]{x}} + \cancel{\frac{3}{4}\sqrt{x}}}{(1 - x^{3/4})^2} = \frac{3}{\cancel{4}_{\,2}} \cdot \frac{\cancel{2}}{\sqrt[4]{x}} \frac{1}{(1 - \sqrt[4]{x^3})^2} = \frac{3}{2} \frac{1}{\sqrt[4]{x}(1 - \sqrt[4]{x^3})^2}$$

$f(x) = \cos x \cdot (\mathrm{tg}\,x - 1)$

$\cos x\,\mathrm{tg}\,x - \cos x$

$$f'(x) = D\cos x\,(\mathrm{tg}\,x - 1) + \cos x\, D(\mathrm{tg}\,x - 1) =$$

$$= -\mathrm{sen}\,x(\mathrm{tg}\,x - 1) + \cancel{\cos x} \cdot \frac{1}{\cos^2 x} =$$

$$-\frac{\mathrm{sen}^2 x}{\cos x} + \mathrm{sen}\,x + \frac{1}{\cos x} = \frac{1 - \mathrm{sen}^2 x}{\cos x} + \mathrm{sen}\,x =$$

$$= \frac{\cos^2 x}{\cancel{\cos x}} + \mathrm{sen}\,x = \cos x + \mathrm{sen}\,x$$

$f(x) = \frac{\log x}{x^n}$

$f(x) = x^n \log x$

$f(x) = \frac{e^x + e^{-x}}{2}$ $\qquad f'(x) = \frac{1}{2}(e^x - e^{-x}) = \frac{e^x - e^{-x}}{2}$

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$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$

$$f'(x) = \frac{2e^{2x}(e^{2x} + 1) - (e^{2x} - 1)\,2e^{2x}}{(e^{2x} + 1)^2} = 2e^{2x}\left[\frac{1}{e^{2x} + 1} - \frac{e^{2x} - 1}{(e^{2x} + 1)^2}\right] = \frac{2e^{2x}}{e^{2x} + 1} \cdot \left[1 - \frac{e^{2x} - 1}{e^{2x} + 1}\right]$$

$f(x) = \frac{e^x - 1}{e^x + 1}$

$$f'(x) = \frac{e^x(e^x + 1) - (e^x - 1) \cdot e^x}{(e^x + 1)^2} = \frac{\cancel{e^{2x}} + e^x - \cancel{e^{2x}} + e^x}{(e^x + 1)^2} = \frac{2e^x}{(e^x + 1)^2}$$

$$f(x) = \frac{e^x}{\operatorname{sen} x}$$

$$f(x) = \frac{1 + \cos x}{\cos x}$$

$$f(x) = \frac{x - \operatorname{sen} x \cos x}{2} \qquad \operatorname{sen}^2 x$$

$$f(x) = \log \log x \qquad f'(x) = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

$$f(x) = \log |x| = \begin{cases} \log x & \text{se } x > 0 \\ \log(-x) & \text{se } x < 0 \end{cases} \qquad f'(x) = \begin{cases} \frac{1}{x} & x > 0 \\ \frac{1}{-x}(-1) & x < 0 \end{cases} \qquad \frac{1}{x}$$

$$f'(x) = \frac{1}{x}$$

$$f(x) = 3^{\operatorname{sen} x} \qquad f'(x) = 3^{\operatorname{sen} x} \cdot \ln 3 \cdot \cos x = \ln 3 \cos x \cdot 3^{\operatorname{sen} x}$$

$$f(x) = 9^{\operatorname{arctg} x}$$

$$f(x) = (\operatorname{sen} x)^3 \qquad f'(x) = 3 (\operatorname{sen} x)^2 \cdot (\cos x) = 3(1 - \cos^2 x) \cos x =$$

$$f'(x) = 9^{\operatorname{arctg} x} \cdot \ln 9 \cdot \frac{1}{1 + x^2}$$

$$f(x) = x^x = e^{\ln x^x} = e^{x \ln x}$$

$$D\, x^x = D\, e^{x \ln x} = e^{x \ln x} \cdot \left(1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}}\right) =$$

$$x^x (\ln x + 1)$$

$$f(x)^{g(x)} = e^{g(x) \ln f(x)}$$

$$f(x) = (3x^4+x)^{(\cos x + \operatorname{sen} x)} = e^{\ln (3x^4+x)^{(\cos x + \operatorname{sen} x)}} = e^{(\cos x + \operatorname{sen} x)\ln[3x^4+x]}$$

$$f'(x) = (3x^4+x)^{(\cos x + \operatorname{sen} x)}\left((-\operatorname{sen} x + \cos x)\cdot \ln[3x^4+x] + (\cos x + \operatorname{sen} x)\,\frac{12x^3+1}{3x^4+x}\right)$$