QUESTIONS FOR THE FUTURE

MARIUS NIELSEN

ABSTRACT. This is an extended list of questions to think about in the future.

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Filtered stacks:

1. Is decalagé an endofunctor on filtered stacks?
    1. If it is, is there then a relation like the following: If $R \to S$ is (phantom) descendable of index n. Is it then true that
    $$\operatorname{spec}^f S \to \operatorname{spec}^f R$$
    becomes a cover after applying decalage n times. What if we assume $R \to S$ to be flat aswell?
    2. What about if we consider index 1. Recall that this gives you the (phantom) right anchored topology and the phantom right anchored topology is the universal grothendieck topology for which
    $$\operatorname{Sp} \xrightarrow{\tau_{\geq \bullet}} \operatorname{FilSp}$$
    is a sheaf. So we can put the subcanonical topology on the right hand side.
2. What are Frobenius maps in filtered spectral algebraic geometry?
    1. Is it possible to define a tate valued Frobenius map in FSAG?
    2. If it is, is it possible to define height? (Say for p-complete things)
    3. What are the geometric properties of $(-)^{tC_p}$. Does it preserve covers? What about flat covers of filtered things?
3. Does Roberts trick about etale extensions work for filtered stacks in general?
    1. Can we classify Galois extension of TMF using these tricks? What about $\mathrm{KO} \to \mathrm{KU}$?
    2. Is it possible to compute the Balmer spectrum of $\mathrm{QCoh}(X)$ for a filtered stack X using Roberts tricks and the ideas from Ishan, Piotr and Roberts computation of $\mathrm{Spc}(\mathrm{Syn}_E)$?
    3. What if we input the even filtration on $\mathrm{TC}(-)$ or $\mathrm{THH}(-)$?
        - I would be very interested in computing all Galois extension of $\mathrm{TC}(\mathbb{Z}_p^\wedge)$ from say the Galois covers of $\Delta_{\mathbb{Z}_p^\wedge}$. What about the story for the Adams summand $\mathrm{TC}(\ell)$?
4. Can cohomological dimension be made into a reasonable notion of filtered stacks?
    1. Particular interest is the even-stack of a ring spectrum R.

IMF, Norwegian University for Science and Technology, Trondheim, Norway

Email address: marius.v.b.nielsen@ntnu.no

URL: mariusnielsen.github.io