IDEAS FOR MASTER'S THESIS PROJECTS

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(1) Exact categories:
   (a) Study the relationship of exact categories and synthetic categories. Exact categories with suspensions automatically have an induced local grading $[1]\colon D(\mathcal{E}) \xrightarrow{\sim} D(\mathcal{E})$ and a natural transformation: $\tau\colon \Sigma(-)[-1] \to \mathrm{id}$. Goal prove that this gives rise to a $\mathcal{S}p^{\mathrm{fil}}$-module structure in Pr^L. One can also study how much the results from Piotr's paper on Synthetic and his paper with Irakli can be generalized to this setting.
   (b) Goodwillie calculus for exact categories. Give a definition of n-excisive functors from and exact category into a (maybe even exact) stable category. Prove polynomial functors extend uniquely to polynomial functors from the stable envelope. More precisely that restriction along the inclusion induces an equivalence:
$$\mathrm{Fun}^{\mathrm{e-poly}}(D(\mathcal{E}), \mathcal{C}) \xrightarrow{\sim} \mathrm{Fun}^{\mathrm{e-poly}}(\mathcal{E}, \mathcal{D}).$$
   It seems likely to be able to deduce that K-theory is functorial in polynomial functors on exact categories using these methods and the results of Barwick–Glasman–Mathew

(2) π_*-flat SAG
   (a) Is there a Lazard's theorem in this setting? Proof idea would be to use Jacob's proof for the t-structure flat topology on $\mathcal{S}p^{\mathrm{fil}}$ and perhaps the version in Dirac Geometry.
   (b) Study the filtered stack $\operatorname{Spec} KU^{\tau}/C_2$ and show that it may be presented as the Adams stack of KO.

(3) TT-geometry
   (a) Compute the Balmer spectrum of the periodic derived category of a ring. With its structure sheaf.
   (b) Study the decategorification stack of a tt-category. That is the functor from $\operatorname{Spec}\mathcal{T}\colon\ \mathrm{CAlg}_{\geqslant 0} \to \mathcal{S}$ given by $R \mapsto \mathrm{map}_{\mathrm{CAlg}(\mathrm{Cat}^{perf})}(\mathcal{T}, \mathrm{Perf}(R))$. It would be very interesting to know for which topologies this is a sheaf. That should tell us about the existence of categoriefied versions of this topology. It would also be interesting to come up with good covers of this stack by affines (or relatively affine things related to $\mathcal{T}$) and understand $\mathrm{QCoh}(\operatorname{Spec}\mathcal{T})$.
   (c) Is there a good notion of the Picard graded *thick subcategory*. For a symmetric monoidal stable category $\mathcal{C}$, subcategory $\mathcal{D}$ is thick if it is closed under finite limits and colimits, closed under tensor products

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with Picard elements and closed under retracts. The definition is made this so that $\langle \Sigma^{0,1} X \rangle = \langle X \rangle$ in $\mathcal{S}p^{\mathrm{fil}}$.

(4) More to come.