Work Sheet 3

Multidimensional Distributions and Importance Sampling

Exercise 1

Normally distributed multivariate random numbers

a) Implement a way to generate normally distributed random vectors of dimensions 2, 5 and 10 with mean vector $\boldsymbol{\mu} = \mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma} = \boldsymbol{I}$.

b) Use the random vector generator from 1 a) to create a sample set of random vectors. Estimate the parameters of the underlying normal distribution from this sample set. What do you observe? How accurate is your parameter estimate? What happens if the size of the sample set changes?

c) Implement a way to generate normally distributed random vectors of different dimensions with mean vectors $\boldsymbol{\mu} \neq \mathbf{0}$ and covariance matrices $\boldsymbol{\Sigma} \neq \boldsymbol{I}$ of your choice.

d) Use the random vector generator from 1 c) to create a sample set of random vectors. Estimate the parameters of the underlying normal distribution from this sample set. What do you observe? How accurate is your parameter estimate? What happens if the size of the sample set changes? What if the dimensionality changes?

Exercise 2

Basic Importance Sampling example

a) Given a set of N random numbers x that follow a standard normal distribution $x \sim \mathcal{N}(0.0, 1.0)$.

Estimate the probability that these random numbers satisfy the condition $x < \theta$ for any θ with and without Importance Sampling. Think about suitable Importance Sampling proposal functions.

b) Plot the estimation result using the two techniques (with and without Importance Sampling) as a function of the parameter θ in a meaningful way.

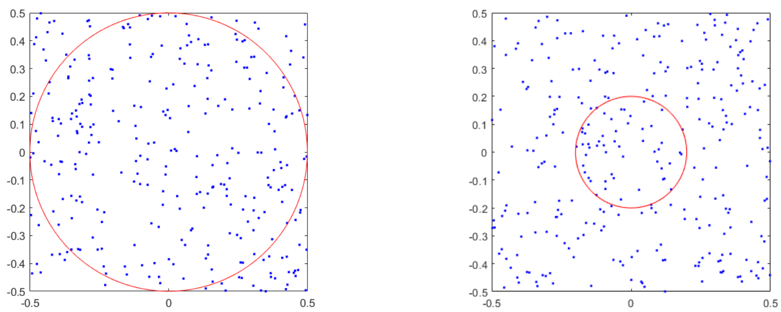

Fig. 1: Square with side length 1, circles with radius $r = 0.5$ resp. $r = 0.2$ and uniformly distributed samples within the square.

Exercise 3

Circle area estimation using Importance Sampling

a) Given a square with side length 1. Place a circle of varying radius $r \leq 0.5$ in the center of this square. Generate uniformly distributed random samples $\boldsymbol{x}$ in the area of the interior of the square. See Fig. 1 for an illustration how the scenario could look like.

b) Estimate the area covered by the circle by counting the percentage of samples that fall into the interior of the circle. I.e. all samples that have a distance smaller than r from the center of the circle. How accurate is your estimate compared to the real area $r^2\pi$ of the circle?

c) What happens to your estimation when r gets very small (e.g. $r = 10^{-10}$)? Find a solution for the problem using Importance Sampling. Use and compare two different proposal distributions.

d) Think about the proper number of samples required for a stable estimate of the area. Find a quality measure for the number of samples and plot your quality as a function of the number of samples.