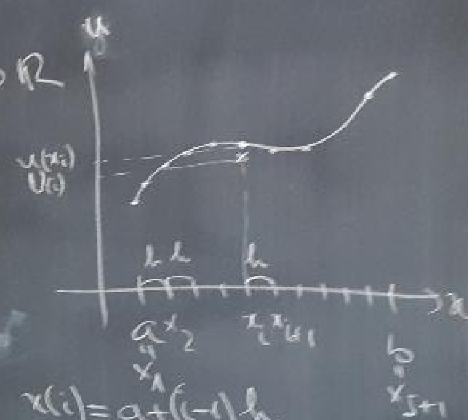
Ec. integrală Fredholm de sp. II. Met numerică

(Ei) $u(x) = \lambda \int_a^b k(x,y)\,u(y)\,dy + f(x), \quad x \in [a,b] \implies u : [a,b] \to \mathbb{R}$

Date pt $\underbrace{a, b, \lambda}_{\text{nr}}$, $\underbrace{k(\cdot,\cdot)}_{\text{funcție}}$, $\underbrace{f(\cdot)}_{\text{funcție}}$

Date utilizator I $(2, 4$



$[a,b] \to \underset{a}{x_{(1)}} < x_{(2)} < \dots < x_{(i)} < x_{(i+1)} \dots < x_{(I)} < \underset{b}{x_{(I+1)}} \implies h = \frac{b-a}{I}$ pt $i = \overline{1, I+1}$ $x(i) = a + (i-1)h$

(Ei) $\xrightarrow{x = x(i)}$ pt $i = \overline{1, I+1}$ $u(x(i)) = \int_a^b \underbrace{\lambda k(x(i), y)\, u(y)}_{g(y)}\, dy + f(x(i))$

$\xrightarrow{\text{f. trapez}}$ pt $i = \overline{1, I+1}$ $u(x(i)) \simeq \frac{h}{2} \lambda k(x(i), \underset{x(1)}{y(1)})\, u(\underset{x(1)}{y(1)}) + \sum_{j=2}^{I} h \lambda k(x(i), \underset{x(j)}{y(j)})\, u(\underset{x(j)}{y(j)}) + \frac{h}{2} \lambda k(x(i), \underset{x(I+1)}{y(I+1)})\, u(\underset{x(I+1)}{y(I+1)}) + f(x(i))$

Caut $U(i) \simeq u(x(i))$ a.î.

$\forall\, i=\overline{1,I+1}$

$$U(i)=\underbrace{\frac{h}{2}\lambda k(x(i),x(1))}_{A(i,1)}\cdot U(1)+\sum_{j=2}^{I}\underbrace{h\cdot\lambda\cdot k(x(i),x(j))}_{A(i,j)}\cdot U(j)+$$

$$+\overbrace{\frac{h}{2}\lambda\cdot k(x(i),x(I+1))}^{A(i,I+1)}\cdot U(I+1)+f(x_i)$$

Notez $A(i,j)=\begin{cases}\frac{h}{2}\lambda k(x(i),x(1)), & \text{dc } j=1\\ h\lambda k(x(i),x(j)), & \text{dc } j=\overline{2,I}\\ \frac{h}{2}\lambda k(x(i),x(I+1)), & \text{dc } j=I+1\end{cases}$ $\quad i=\overline{1,I+1}$

A matrice $(I+1)\times(I+1)$

U matrice coloană $(I+1)\times 1$, $U=\begin{pmatrix}U(1)\\U(2)\\ \vdots\\ U(I+1)\end{pmatrix}$; F matrice coloană $(I+1)\times 1$ $F=\begin{pmatrix}f(x(1))\\f(x(2))\\ \vdots\\ f(x(I+1))\end{pmatrix}$

$(*)$ $i=\overline{1,I+1}$ $\quad U(i)=\sum_{j=1}^{I+1}A(i,j)U(j)+F(i)$

$$\Rightarrow \underset{I_{I+1}U}{\underset{\parallel}{U}}=A\cdot U+F \Leftrightarrow (I_{I+1}-A)U=F \Rightarrow U=(I_{I+1}-A)^{-1}\cdot F$$

reprezint grafic (X, U)

$$\int_a^b g(y)\,dy \simeq \sum_{j=1}^{I+1}\omega_j\, g(y_j) \qquad (*)$$

$(y(i))$ diviziune uniformă

a h h … h … b

$y(1)$ $y(2)$ … $y(j)$ $y(j+1)$ … $y(I+1)$

x_1 x_2 … x_j x_{j+1} … x_{I+1}

f. trapezelor

$$\int_a^b g(y)\,dy \simeq \frac{h}{2}g(y(1))+\sum_{j=2}^{I}h\,g(y(j))+\frac{h}{2}g(y(I+1))$$

$a=0,\ b=\pi,\ \lambda=3,\ f(x)=\cos x$

$$k(x,t)=\begin{cases}\sin x\cos t, & 0\le x\le t\\ \sin t\cos x, & t<x\le\pi\end{cases}$$