HAND IN MODULE 3

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1. Task 1:

a)

S = standard basis $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix} for \mathbb{R}^2$

B = the two vectors $\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$

Gaussian Elimination: $\begin{pmatrix}1 & 1 & 0\\-1 & 0 & 0\end{pmatrix} -> \begin{pmatrix}1 & 1 & 0\\0 & 1 & 0\end{pmatrix} -> \underline{\underline{\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\end{pmatrix}}}$

From the Gaussian Elimination we can see that it is linear independent and has unique solution.

b)

$\begin{pmatrix}1 & 1 & 1\\-1 & 0 & 2\end{pmatrix} -> \begin{pmatrix}1 & 1 & 1\\0 & 1 & 3\end{pmatrix} -> \underline{\begin{pmatrix}1 & 0 & -2\\0 & 1 & 3\end{pmatrix}}$

$\begin{pmatrix}1\\2\end{pmatrix} -> \underline{\underline{(-2) \cdot \begin{pmatrix}1\\-1\end{pmatrix} + 3 \cdot \begin{pmatrix}1\\0\end{pmatrix}}}$

c)

Scalar product: $\frac{1}{\sqrt{2}} \cdot (1 \cdot 1) + \frac{1}{\sqrt{2}} \cdot (-1 \cdot 1) = 0$

Length of vector u: $\frac{1}{\sqrt{2}} \cdot \sqrt{1^2 + 1^2} = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$

Length of vector v: $\frac{1}{\sqrt{2}} \cdot \sqrt{(-1)^2 + 1^2} = \frac{1}{\sqrt{2}} \cdot \sqrt{2} = 1$

<u>So the vectors u and v are orthogonal and has the length 1.</u>

d)

$Projection_{\binom{1}{-1}} \begin{pmatrix}1\\4\end{pmatrix} = \begin{pmatrix}1\\4\end{pmatrix} \cdot \frac{1}{\sqrt{1^2 \cdot (-1)^2}} \cdot \begin{pmatrix}1\\-1\end{pmatrix} \cdot w$

$= \frac{1}{\sqrt{2}} \cdot w \cdot (1 \cdot 1 + 4 \cdot (-1))$

$= -\frac{3}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$= -\frac{3}{2} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -\frac{3}{2} \cdot 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{3}{2} \cdot (-1) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \text{STANDARD BASIS}$

$-\frac{3}{2} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -\frac{3}{2} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}_C = \text{BASIS C}$

e)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 4 \end{pmatrix}$$

By performing gaussian elimination on the matrix we get:

$$\begin{pmatrix} 1 & 0 & (\sqrt{2} - \frac{4\cdot\sqrt{2}+\sqrt{2}}{2}) \\ 0 & 1 & (\frac{4\cdot\sqrt{2}+\sqrt{2}}{2}) \end{pmatrix}$$

Which translates to:

$$\underline{\underline{\begin{pmatrix} 1 \\ 4 \end{pmatrix} \text{ on basis C} = \begin{pmatrix} \sqrt{2} - \frac{4\cdot\sqrt{2}+\sqrt{2}}{2} \\ \frac{4\cdot\sqrt{2}+\sqrt{2}}{2} \end{pmatrix}_C}}$$

2. Task 2:

a)

b)

c)

3. Task 3:

a)

$u = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Orthogonal?

$u \cdot v = 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 = 1 \neq 0$

which means <u>u and v are NOT orthogonal.</u>

b)

Let's try to find a vector y to make the orthonormal basis u, y for plane H.

span{u,v} = span{u,y}