# Empirical Methods in Finance

# Project 1

Groupe 11

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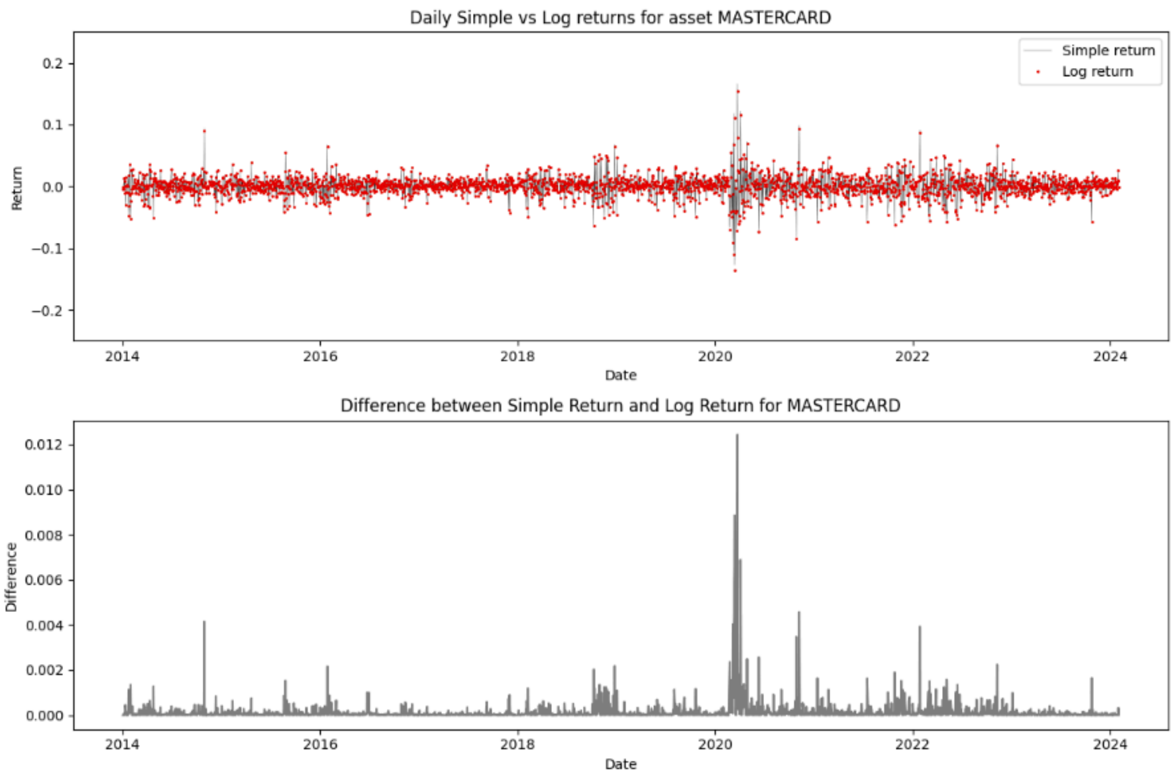
# 1 Descriptive Statistics

## Q.1.1

In daily analysis, simple and log returns exhibit notable similarities for small magnitudes, yet log returns gain preference for their ease in mathematical manipulations, like calculating compounded growth rates and facilitating statistical modeling. These log returns standardize distributions and mitigate the impact of outliers.

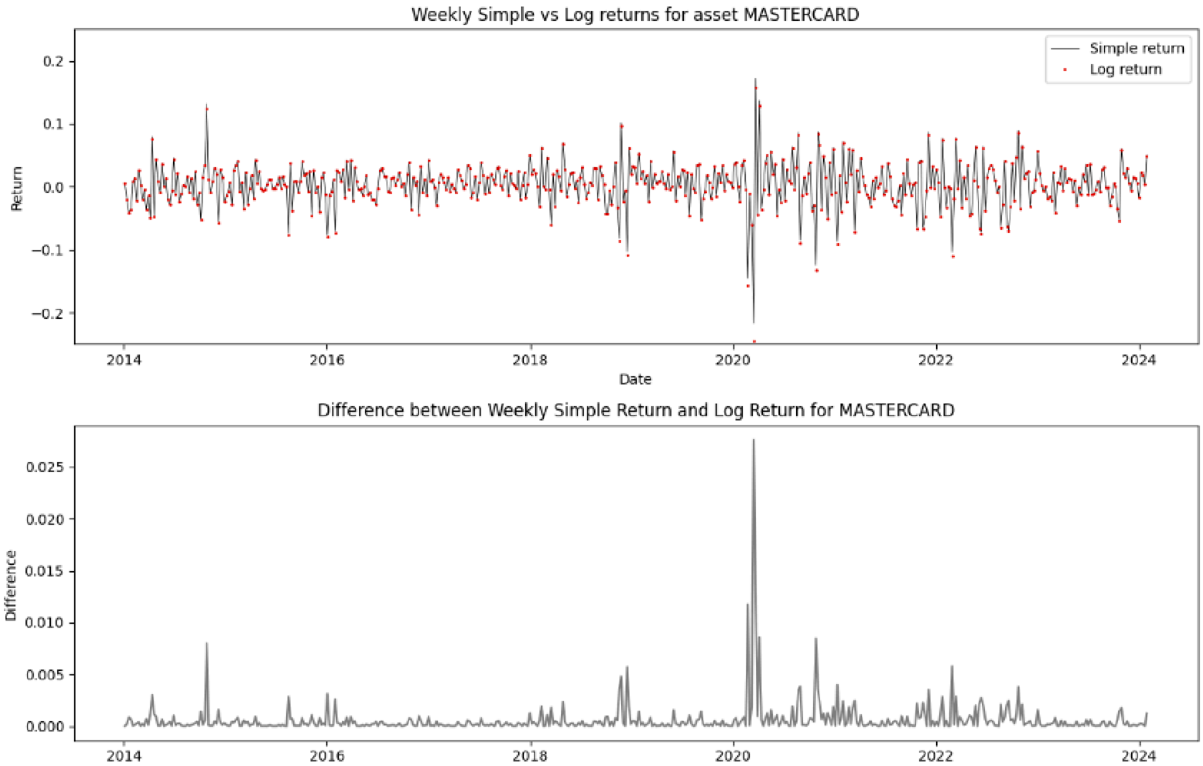
While the mean of simple returns is higher than the log, in figure 1, the variance between simple and log returns remains similar based on the results from our code . The log returns, however, tend to show reduced skewness without significantly altering kurtosis, demonstrating their effectiveness in handling extremes.

Volatility accentuates the differences between the two return types. In this context, simple returns outsize log returns for positive shifts, whereas log returns, marked by red dots, dives deeper into the negative territory for downturns. This behavior reflects their respective approaches to encoding growth and loss: simple returns escalate positive developments, while log returns deepen the perception of losses. Such contrasts grow more visible with escalating volatility, as depicted in the graph. Here, the red dots underscore log returns' tendency towards more negative values in downturns, emphasizing the gap between simple and log returns under high volatility conditions.



### Q.1.2

In the weekly time frame the findings are similar. We can see that the mean for the weekly returns tends to be higher than for weekly log returns. The variances stay similar. The skewness becomes more negative and the kurtosis and the extremes stay similar.
In the difference graph we can see that the spread becomes bigger in high volatility time. For the big volatility period in 2020 we notice a big spike stating a difference between simple and log return of 0.025, and in the daily frequency that was only 0.012 of difference.




### Q.1.3

In the descriptive tables we can see that for the daily log the mean is higher, the variance is much lower and skewness becomes more negative.
When doing weekly returns, the mean return is much higher, almost by half. The variance is smaller but almost the same. There are higher extremes when doing weekly returns (smaller min and bigger max), especially smaller min. And the skewness tends to be more negative meaning that the curve goes to the right and that the values are more concentrated at the right. And the kurtosis tends to be smaller as well.

## 2 Stationarity

### Q.2.1

To continue our study we can look at the stationarity of our times series. To do this, we can create this AR(1) process for each series $p_t = log(Pt)$: $p_t = \mu + \phi p_{t-1} + \varepsilon_t$ (1) and test the stationarity with a Dickey-Fuller Test. The null hypothesis of the Dickey-Fuller test posits the existence of a unit root within the time series, indicating nonstationarity. Formally, it can be expressed as $H_0$: $\phi$=1. In contrast, the alternative hypothesis suggests a bounded parameter value, typically $H_1$: -1<$\phi$<1.

Considering the implications of a non-zero mean on the asymptotic distribution of the t-test for $H_0$, it is assumed that a unit root exists conditional to μ = 0. To explain the intuition we need to understand how is made an $AR_1$ model :

$$p_t = \mu + \phi p_{t-1} + \varepsilon_t$$

$$p_t = \mu + \phi(\mu + \phi p_{t-2} + \varepsilon_{t-1}) + \varepsilon_t$$

$$p_t = \mu + \phi\mu + \phi^2 p_{t-2} + \phi\varepsilon_{t-1} + \varepsilon_t$$

The Dickey-Fuller test assesses how past values influence the present state of a time series. If -1<$\phi$<1, the influence of past values diminishes as we move further back in time. As we backtrack, the variable is multiplied by $\phi$, gradually approaching 0, signifying stationarity. Conversely, when $\phi \geq 1$, the impact persists and even amplifies over time. In such cases, the time series remains non-stationary due to the continuous multiplication with $\phi$, reinforcing past influences indefinitely.

## Q.2.2

Now, we can compute the test-statistics by using the estimated coefficients from our OLS regression from (1). The t-stat is :

$$t = \frac{\widehat{\phi} - \phi_0}{SE(\widehat{\phi})} \quad (3)$$

with $\widehat{\phi}$ = estimated coefficient, $\phi_0$= 1 the null hypothesis

To decide how you should reject or accept the null hypothesis, we need to compare the t-stat obtained and the critical value related to the test. If the t-statistic is below the critical value associated with a specified confidence level (e.g. 5%), then the null hypothesis is not rejected. On the other hand, if the t-statistic is above the critical value, we reject the null hypothesis. This implies that the coefficient $\phi$ is significantly different from 1.

## Q.2.3

However, it is also possible to run a new regression to test the stationarity. The new regression is $r_t = \mu + \phi p_{t-1} + \varepsilon_t$. In fact we could see the link with the previous regression, by subtracting the old regression by $p_{t-1}$

$$p_t - p_{t-1} = \mu + \phi p_{t-1} + \varepsilon_t - p_{t-1} \Leftrightarrow p_t - p_{t-1} = \mu + (\phi - 1)p_{t-1} + \varepsilon_t$$

The new regression leads to another coefficient, define $\phi_2$ and offering a relation between $\phi_2$ and $\phi$, $\phi_2$ = ($\phi$ - 1). We can apply the same steps as before to test the stationarity but with $\phi_2$, making our hypotheses more intuitive to write ($H_0$ implies 0).

$H_0$ : $\phi_2$ = ($\phi$ - 1) and $\phi$ = 1 so $\phi_2$ = 0

$H_a$ : $\phi_2 = (\phi - 1)$ and $-1 < \phi < 1$ so $-2 < \phi_2 < 0$

## 2.1 Critical Values

### Q.2.4

By simulating with random walks on log-transformed stock prices, we can capture realistic representation of stock prices movements. We aim to capture the inherent volatility and persistence commonly observed in financial time series. The idea is that prices follow the previous price plus a random shock, a part of the price is available and another part depends on future movements, seen as random.

Indeed, under random walks, we can build an empirical distribution and determine critical values that serve as thresholds for rejecting the null hypothesis. This distribution provides insights into the behavior of the test statistic under the null hypothesis of stationarity.

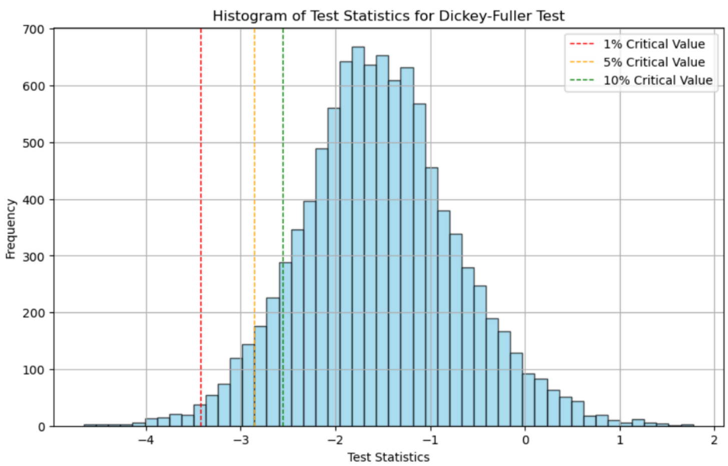
### Q.2.5

The distribution obtained looks like a normal distribution centered approximately on -1,8. After a deeper analysis we can see that's not the case. The Q-Q plot shows that the right tail seems to move away from a normal distribution. Moreover, we have a skewness of 0.23 which is not equal to 0 as the normal. Finally, we performed 2 tests, the shapiro-wilk test and the Anderson-Darling test. Both tests reject the normal distribution with a high significance (Note that the shapiro-wilk's p-value may not be accurate with N > 5000).

| *Shapiro-Wilk Test* | |
|---|---|
| *test* | ***0.9952*** |
| *P-value* | ***0.0000*** |

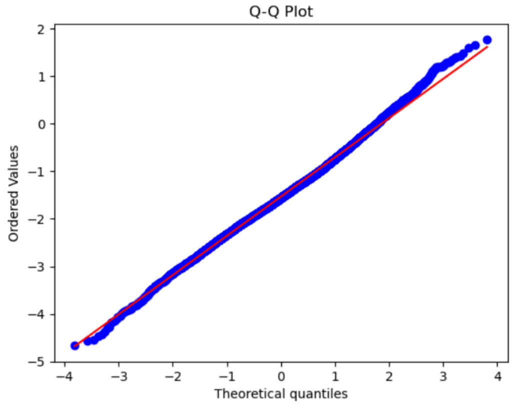
| *Anderson-Darling Test* | |
|---|---|
| *AD Statistic* | ***11.3800*** |
| *Critical Values* | ***0.576/0.656/0.787/0.918/1.092*** |
| *Significance Levels %* | ***15. / 10. / 5. / 2.5 /1.*** |

The distribution of the Dickey-Fuller test statistic is asymmetrical because the null hypothesis being tested is typically that the time series has a unit root, i.e., it is non-stationary. This implies that the distribution of the test statistic under the null hypothesis does not follow a symmetric distribution. Indeed, a unit root suggests that a time series




variable is non-stationary, meaning it exhibits a systematic pattern over time and its statistical properties change over time.

The distribution reflects the potential values for the test-statistics utilized in our analysis. Specifically, we conducted a t-test to assess the stationarity of a random walk, a process known not to exhibit stationarity. Our null hypothesis posited non-stationarity within the time series. Consequently, obtaining results close to zero indicates a failure to reject the null hypothesis, as anticipated.



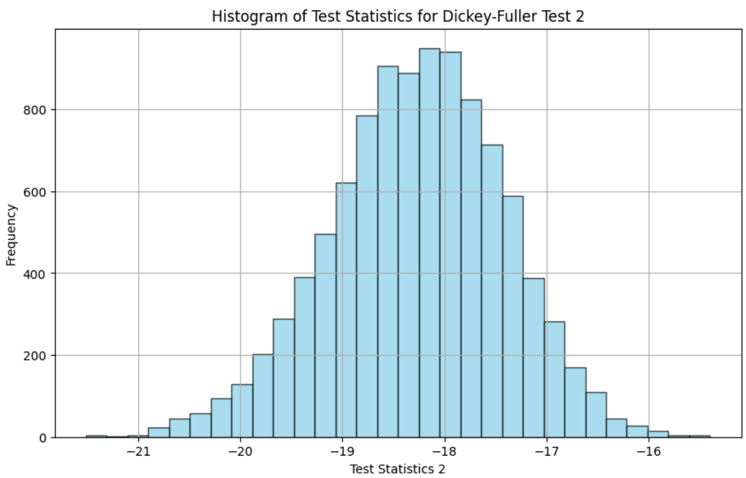

## Q.2.6

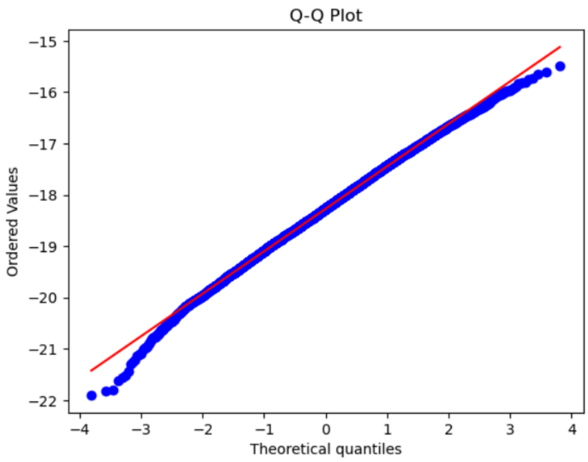
These simulations enabled us to find our critical values linked to our dickey fuller test for confidence intervals of 10%, 5% and 1%. Moreover, we can compare these values with the critical values provided by fuller, and logically they coincide. However, they are more negative than the critical values usually used for a t-student distribution and implies a bigger chance to not reject the null hypothesis.

| | ***Critical Value*** |
|---|---|
| *1%* | -3.442 |
| *5%* | -2.872 |
| *10%* | -2.586 |

## Q.2.7

If we assume that the AR(1) process is $p_t^{(i)} = 0.2\, p_{t-1}^{(i)} + \varepsilon_t^{(i)}$. In this scenario, we're dealing with an autoregressive (AR) model that exhibits stationarity due to the condition $|\phi| < 1$, ensuring that the model neither explodes nor vanishes over time. We observe that it looks like a normal distribution but it isn't. The distribution looks like our first monte-carlo simulation distribution, except this one is centered on -18. The t-stats are notably distant from previously established critical values, a logical outcome given our rejection of stationarity.




Upon conducting tests for normality, utilizing both the Shapiro-Wilk and Anderson-Darling tests, we find consistent evidence against normality, particularly evident when T=500. This is supported by the slightly negative skewness (conversely to the positive skewness in our first Monte-Carlo simulation) of the distribution meaning that the tail is longer on the left side. However, our results are due to the small size of our sample. As we increase the sample size (T), an interesting trend emerges: the distribution gradually converges towards a normal distribution. This is consistent since a stationary process follows a normal distribution.

| *Shapiro-Wilk Test* | |
|---|---|
| *test* | ***0.9991*** |
| *P-value* | ***0.0000*** |

| *Anderson-Darling Test* | |
|---|---|
| *AD Statistic* | ***1.4458*** |
| *Critical Values* | ***0.576/0.656/0.787/0.918/1.092*** |
| *Significance Levels %* | ***15. / 10. / 5. / 2.5 /1.*** |

# 2.2 Testing Non-stationarity

## Q.2.8

We can test the null hypothesis: log-price has a unit root. We obtain t-stats between -0.28 and -1.24 which is far from the critical values we found earlier with our Monte-Carlo simulation. Hence, we cannot reject the null hypothesis that our data follow a random walk for all significance levels ($\alpha$ = 1%,5%,10%). Since a random walk is non-stationary, the results tell us that all our log stock prices are non-stationary. It makes sense with our previous results. Indeed, the lagged coefficient in our AR models was estimated at a number really close to 1 for each stock. The plot of our regressions clearly showed a positive trend for each stock.

***Non-stationarity test***

| y \ X | t-stat | Critical Value α = 1% | P-value | Reject $H_0$ |
|---|---|---|---|---|
| **MA** | ***-0.28*** | ***-3.40*** | ***0.78*** | ***No*** |
| **GS** | ***-1.24*** | ***-3.40*** | ***0.22*** | ***No*** |
| **AXP** | ***-0.83*** | ***-3.40*** | ***0.40*** | ***No*** |
| **MS** | ***-1.15*** | ***-3.40*** | ***0.25*** | ***No*** |
| **V** | ***-0.99*** | ***-3.40*** | ***0.32*** | ***No*** |

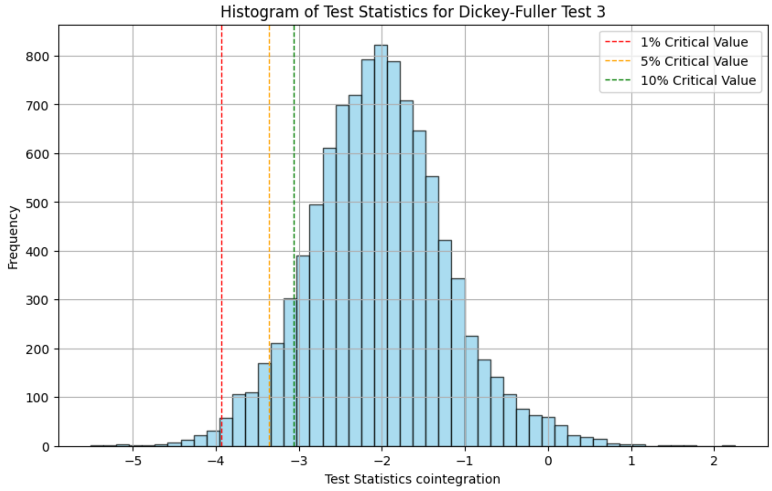
Q.2.9

Cointegration is a concept that arises when dealing with multiple non-stationary time series. Two or more non-stationary time series are said to be cointegrated if a linear combination of them is stationary, even though individually they are not. Since all our stocks are non stationary we could test the cointegration between them and find a long-term relationship among these variables, despite the fact that individually they may not be stationary.

# 3 Cointegration

Q.3.1

The histogram shows the distribution of the test statistic of $\square^{(i)}$ and not $\square^{(i)}$-1 as before since we are performing now an augmented Dickey-Fuller test. We are testing the stationarity of the residuals of the regression between our 2 random walks, meaning that we are looking at the possible value we can have in the case of no cointegration (null hypothesis).
As described in Q2.5 the distribution looks normal but is not. The positive skewness shows that the distribution is asymmetrical ( longer tail on the right side), due to non-stationarity.




The critical values for T = 2632 and T = 500 are reported below. If we compare them to the critical values provided by Phillips and Ouliaris, we see that our critical values are really close for the different significance levels.

| ALPHA | T = 500 | T = 2632 |
|---|---|---|
| **1%** | -3.9353 | -3.8749 |
| **5%** | -3.3711 | -3.3360 |
| **10%** | -3.0696 | -3.0606 |

## Q.3.2

The idea of testing cointegration is by posing the null hypothesis that the residual $z_t$ is non-stationary. If $z_t$ is non-stationary, this implies that we suppose no cointegration. Hence if t-stat < critical values , we reject the null hypothesis and assume cointegration.

The following Tables shows the cointegration test results and p-value:

***Test statistics for cointegration***

| y \ X | MA | GS | AXP | MS | V |
|---|---|---|---|---|---|
| **MA** | | *-1.6757* | *-2.3853* | *-1.6198* | *-4.2763* |
| **GS** | *-1.9558* | | *-2.4822* | *-3.4025* | *-1.9494* |
| **AXP** | *-2.3220* | *-2.2317* | | *-2.5700* | *-2.2234* |
| **MS** | *-1.8725* | *-3.3743* | *-2.7579* | | *-1.8329* |
| **V** | *-4.3440* | *-1.8369* | *-2.4732* | *-1.7353* | |

*P-values associated*

| y \ X | MA | GS | AXP | MS | V |
|---|---|---|---|---|---|
| MA | | 0.0939 | 0.0171 | 0.1054 | 0.0000 |
| GS | 0.0506 | 0.0171 | 0.0131 | 0.0007 | 0.0514 |
| AXP | 0.0203 | 0.0257 | | 0.0102 | 0.0263 |
| MS | 0.0612 | 0.0008 | 0.0059 | | 0.0669 |
| V | 0.0000 | 0.0663 | 0.0135 | 0.0828 | |

*Rejection of the null hypothesis*

| y \ X | MA | GS | AXP | MS | V |
|---|---|---|---|---|---|
| MA | | No | No | No | Yes |
| GS | No | No | No | Yes | No |
| AXP | No | No | | No | No |
| MS | No | Yes | No | | No |
| V | Yes | No | No | No | |

By comparing the test results to our critical values, 4 pairs of assets stand out from the crowd :

- MA on V
- V on MA
- GS on MS
- MS on GS

Note that the pair MA → V and V→MA has a significance of 1% compared to the 2 others that have a significance level of 5%.

Q.3.3

The following tables shows the estimated α and β:

*Alpha for cointegration*

| y \ X | MA | GS | AXP | MS | V |
|---|---|---|---|---|---|
| MA | | -3.5683 | -2.2732 | 0.2944 | 0.4690 |
| GS | 3.5346 | | 1.8569 | 2.7741 | 3.3656 |
| AXP | 2.2205 | 0.7000 | | 1.8003 | 2.0773 |
| MS | 0.8677 | -3.6671 | -1.5157 | | 0.6207 |
| V | 0.4405 | -2.5580 | -1.2971 | 0.7220 | |

Alpha shows the fundamental level or equilibrium point of the relationship. It may represent the long-term average value that the variables converge towards when they are in equilibrium. In our economic context, alpha could represent the fundamental value of stocks in the absence of divergence due to idiosyncratic shocks.

***Beta for cointegration***

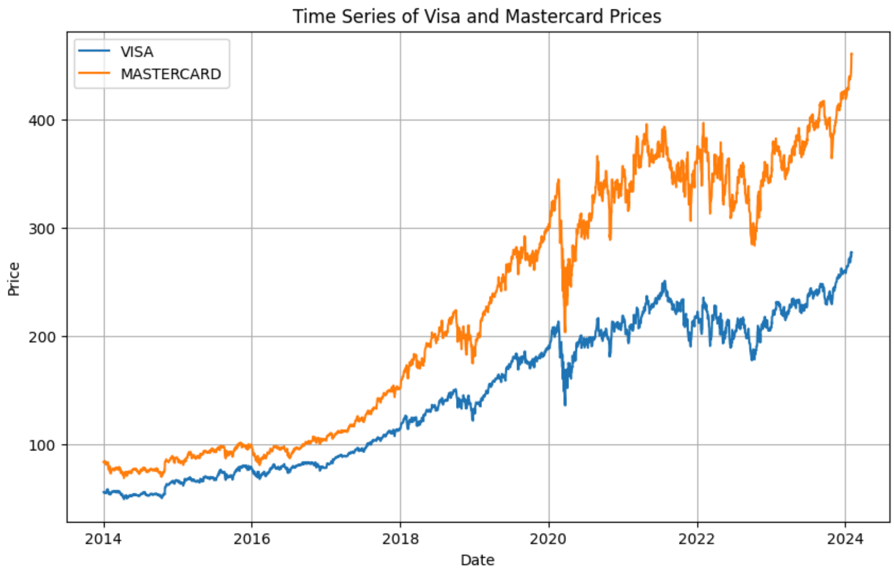
| y \ X | MA | GS | AXP | MS | V |
|---|---|---|---|---|---|
| MA | | 1.6147 | 1.6112 | 1.2614 | 1.1746 |
| GS | 0.3672 | | 0.7721 | 0.6841 | -0.4306 |
| AXP | 0.4666 | 0.9832 | | 0.7303 | 0.5325 |
| MS | 0.5831 | 1.3908 | 1.1660 | | 0.6796 |
| V | 0.8435 | 1.3599 | 1.3205 | 1.0555 | |

The β represents the effect of a one-unit change in the independent variable on the dependent variable. In the context of cointegration testing, a beta coefficient of 1 implies that the two time series are perfectly cointegrated, while a coefficient less than 1 suggests that they are cointegrated but there might be some adjustment process involved. A beta coefficient of zero indicates that the two time series are not cointegrated.

## Q.3.4

The most cointegrated pair is Visa /Mastercard, It may simply be due to the fact that these two institutions operate in the same sector, the payment transactions. When one suffers a shock of demand/supply, it's highly likely that the other will do the same. As also for their market perspectives.

## Q.3.5



# 4 Pair Trading

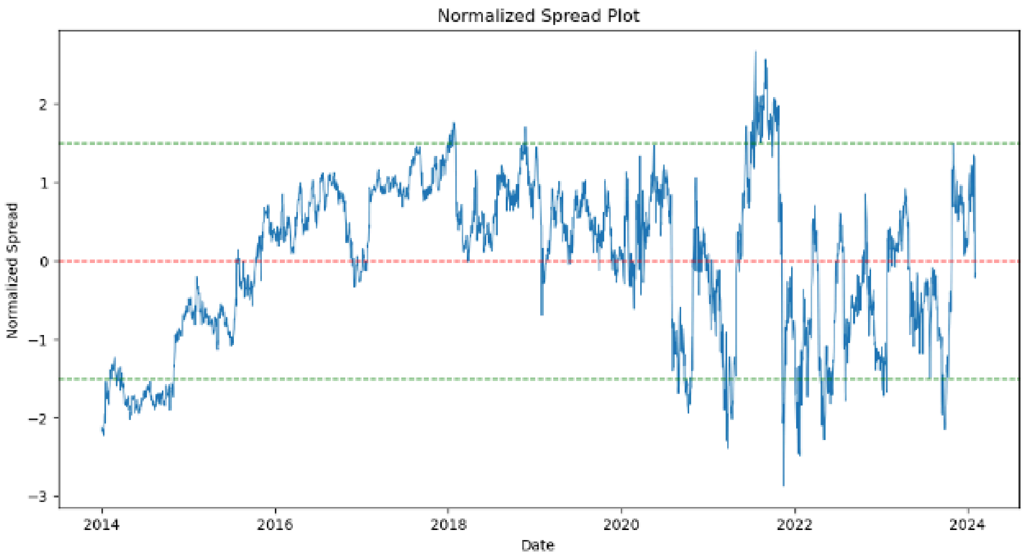
From the cointegration test we selected the pair assets for our pair-trading strategy. To make money, we aim at exploiting statistical arbitrage underlying their cointegration relationship with a pair-trading strategy. The first step is to define the spread between the prices that we will use as a signal for trading.

Q.4.1

If the pair (Visa,Mastercard) is cointegrated, which can be defined as a linear combination of the 2 stocks exhibits mean-reversion behavior. This means that deviations from the long-term relationship between A and B, as represented by zt, tend to be temporary, and the series has a tendency to return to its long-term equilibrium. If zt >> 0 it suggests that Visa is overvalued relative to Mastercard, based on their historical relationship.

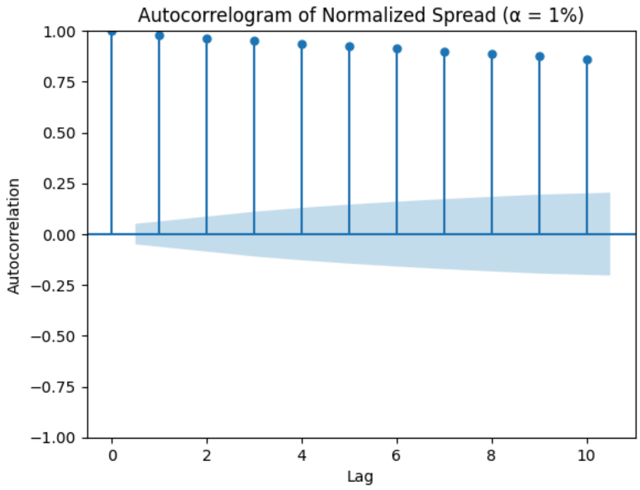
The arbitrage strategy would be based on the exploitation of temporary deviations from the long-term equilibrium relationship between the 2 cointegrated assets. The idea would be that if zt >> 0 we sell Visa stocks and buy Mastercard stocks and wait until zt goes back to its long-term equilibrium to make profit. Conversely, if zt << 0 Visa is undervalued relative to Mastercard, so we buy Visa stocks and sell Mastercard stocks.

Q.4.2




Q.4.3

The Ljung-Box test is designed to determine whether there is any autocorrelation in a time series that deviates from zero. Specifically, we aim to ascertain whether historical data can generally explain the current price at time t, considering up to 10 lags for our example. Our strategy revolves around the spread, meaning it's inadequate to solely examine the correlation. Should there be a declining trend within the sector, the likelihood of observing mean reversion becomes minimal. The figure below illustrates that, with a 10-day lag, even the most dated information has a significant impact on the spread observed today.




High positive autocorrelation with all the lags, with very high significance.

**Ljung- box**

| | *T-statistic* | *P-value* |
|---|---|---|
| *1* | ***2531.2134*** | ***0.0000*** |
| *2* | ***4976.4618*** | ***0.0000*** |
| *3* | ***7355.9297*** | ***0.0000*** |
| *4* | ***9677.3004*** | ***0.0000*** |
| *5* | ***11941.3503*** | ***0.0000*** |
| *6* | ***14141.4985*** | ***0.0000*** |
| *7* | ***16280.6828*** | ***0.0000*** |
| *8* | ***18863.5914*** | ***0.0000*** |
| *9* | ***20384.9768*** | ***0.0000*** |
| *10* | ***22336.9648*** | ***0.0000*** |

The p value is 0, so we reject the null hypothesis (white noise) with very high significance.

The high autocorrelation in $z_t$ indicates that there is a serial dependence or predictability in the deviations from the long-term relationship between the assets. Indeed, the positive autocorrelation of $z_t$ means that a positive deviation from the equilibrium level in one period is more likely to be followed by another positive deviation in the subsequent period.

Hence, the predictability of the deviation makes our pair-trading strategy effective.

### Q.4.4

A self-financing strategy ensures that the proceeds from closing one position are sufficient to finance the opening of another position, maintaining a constant capital base without requiring additional capital injection or external financing. In the provided strategy, the quantities of assets held in each position do not directly offset each other (if 1≠Beta), leading to imbalanced cash flows and requiring additional capital for position sizing.

Therefore, the provided algorithmic trading strategy is not self-financing.

## Q.4.5

Let define :

$P_i^A$ the price of asset A when we initiate the trade

$P_i^B$ the price of asset B when we initiate the trade

$P_c^A$ the price of asset A when we close the trade

$P_c^B$ the price of asset B when we close the trade

Initial amount invested : $P_i^A + \beta P_i^B$

Profit on A : $P_i^A - P_c^A$

Profit on B : $\beta(P_c^B - P_i^B)$

We know that $z_t = P_t^A - \alpha - \beta P_t^B$ and $\tilde{z}_t = \frac{z_t}{\sigma_{zt}}$

So : $\tilde{z}_{in} = \frac{P_i^A - \alpha - \beta P_i^B}{\sigma_{zt}} \Leftrightarrow P_i^A = \alpha + \beta P_i^B + \tilde{z}_{in}\sigma_{zt}$ and $\tilde{z}_0 = 0 = \frac{P_c^A - \alpha - \beta P_c^B}{\sigma_{zt}} \Leftrightarrow P_c^A = \alpha + \beta P_c^B$

**Hence, the profit on A:**

$P_i^A - P_c^A = \tilde{z}_{in}\sigma_{zt}$

For $P^B$ we know that:

$P_i^B = \frac{P_i^A - \alpha - \tilde{z}_{in}\sigma_{zt}}{\beta}$ and $P_c^B = \frac{P_c^A - \alpha}{\beta}$

**Hence, the profit on B:** $\beta(P_c^B - P_i^B) = \beta\left(\frac{P_c^A - \alpha}{\beta} - \frac{P_i^A - \alpha - \tilde{z}_{in}\sigma_{zt}}{\beta}\right) = -(P_i^A - P_c^A) + \tilde{z}_{in}\sigma_{zt} = 0$

Finally the total profit = Profit on A + Profit on B = $\tilde{z}_{in}\sigma_{zt}$

## Q.4.6

A lower value of $\tilde{z}_{in}$ makes the trading strategy more sensitive to deviations in the spread between the two assets. It means the strategy will initiate trades more frequently, capturing smaller deviations from the mean. This increased sensitivity can potentially lead to more trading opportunities and higher potential profits, especially if the spread between the assets tends to mean-revert quickly.

Conversely, a higher value of $\tilde{z}_{in}$ makes the trading strategy less sensitive to small deviations in the spread. It requires larger deviations from the mean before initiating trades. While a higher threshold may reduce the frequency of trades, it can also reduce the likelihood of false signals and overtrading. This can lead to a more robust trading strategy that is less affected by noise or temporary fluctuations in the spread.

## 4.2 Pair-trading Strategy

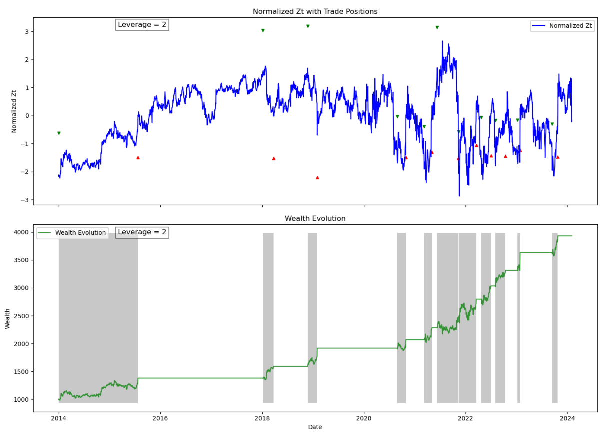
### Q.4.7

The simulation ended with 11 trades in total. Most of the time, when we closed with a losing position in one of the stocks, the other one managed to close with a winning position bigger than the losing one. This managed to make our wealth grow from 1000$ to a final wealth of 3932$ (with the lowest wealth of 980$ and highest of 3932$) . This represents 29.32% of average annual returns for our trading strategy which isn't beating a traditional buy&hold strategy in this case.
We can conclude that we got good signals because our lowest wealth is only 20$ under our initial wealth whereas the highest went close to 1932$ up.

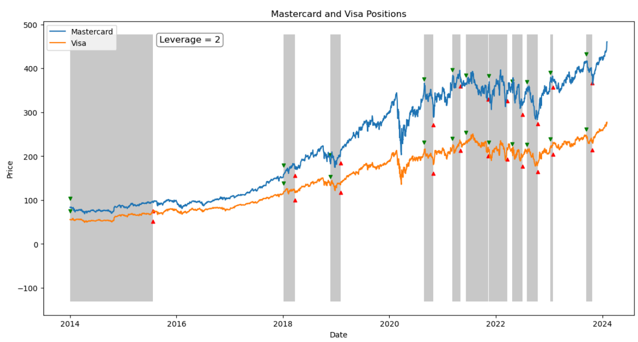
***Results***

| | |
|---|---|
| **Final wealth** | *3'932.92$* |
| **Profit** | *2'932.92$* |
| **Largest wealth** | *3'932.92$* |
| **Lowest wealth** | *980.03$* |
| **Total return** | *293.29%* |
| **Buy and hold strategy – V** | *397.90%* |
| **Buy and hold strategy– MA** | *451.29%* |
| ***Number of trades*** | *11* |

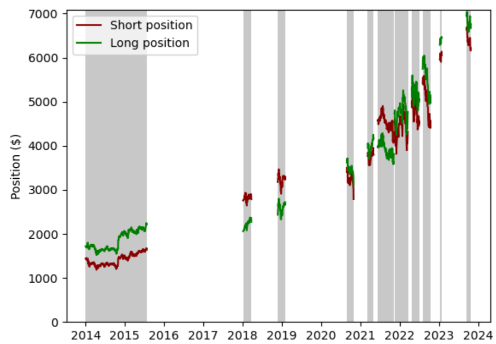
***Normalized spreads & Wealth***



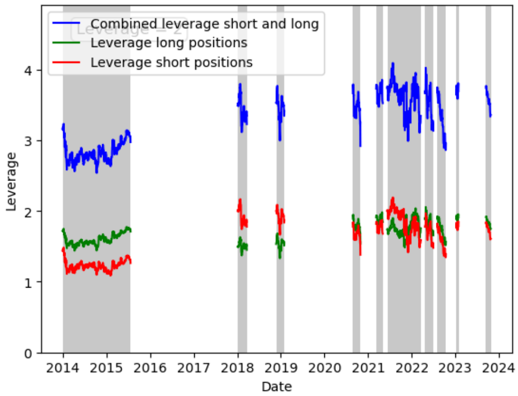

***Positions on price***




***long/short positions***



***Leverage***



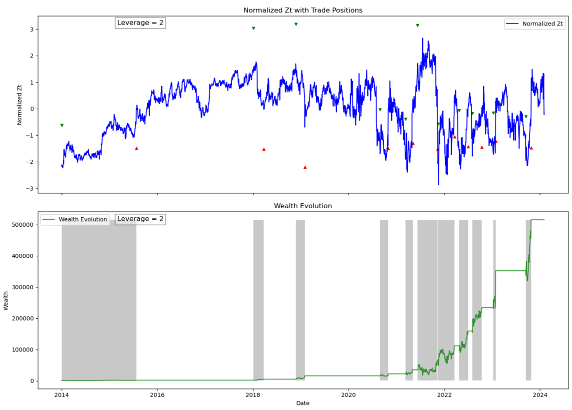

Q.4.8

With a leverage = 20 we have a lowest wealth of 943$ and a largest wealth of  515 '636$. . In this case we have higher gain , but higher exposure. We can grow our portfolio account much faster with winning trades but also lose it much quicker with losing one. In this simulation, the winning trades were succeeding which made our portfolio grow exponentially from 2021 to 2024.
We can also notice that while our positions were open, the leverage initially set to a maximum of 20 at the entry peaked almost at 80 around the end of 2021. If we look at the red line, we see that the short position contributed a leverage of around 42 despite the entry limit.

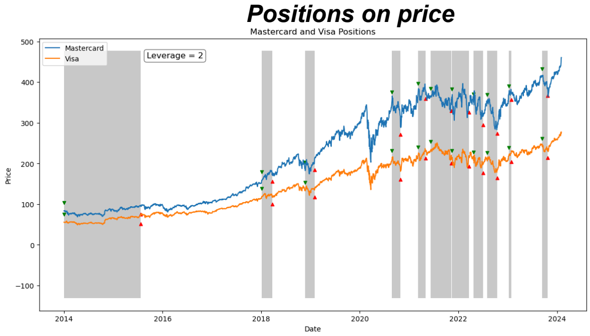
***Results***

| | |
|---|---|
| **Final wealth** | ***515'636.77$*** |
| **Profit** | ***514'636.77$*** |
| **Largest wealth** | ***515'636.77$*** |
| **Lowest wealth** | ***943.32$*** |
| **Total return** | ***51463.68%*** |
| **Buy and hold strategy – V** | ***397.90%*** |
| **Buy and hold strategy– MA** | ***451.29%*** |
| ***Number of trades*** | ***11*** |

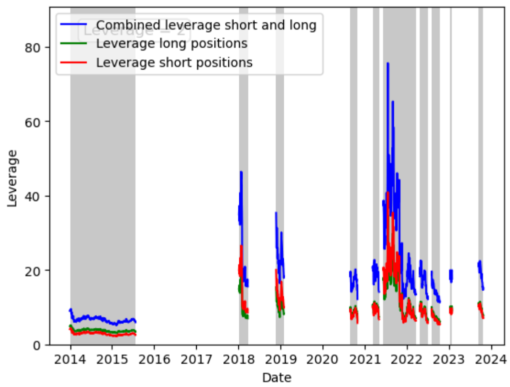
***Normalized spreads & Wealth***



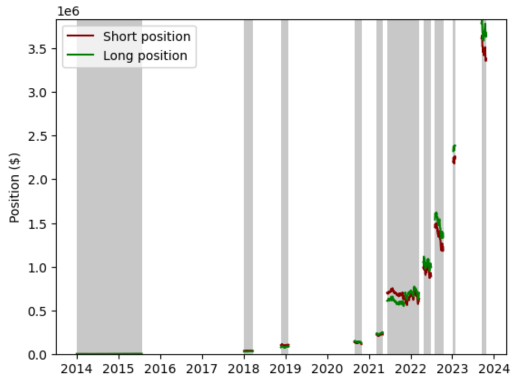
***Positions on price***




***Leverage***




***Long/short positions***




## Q.4.9

The stop loss rule is there to prevent our portfolio from experiencing big losses. If we decide to enter in a signal that could be too early in reality, the stop loss can save us from this mistake. But it is also something that can prevent us from hitting our profit signal if we set it too close to our entering position. So one must find an optimum place to set its stop loss, not too far to prevent big losses, and not too close to prevent from closing potential winning positions.

In our case, setting the stop loss to 1.75 with an entry signal at 1.5 gives us a probability of closing the position at the next observation of 3.69%.

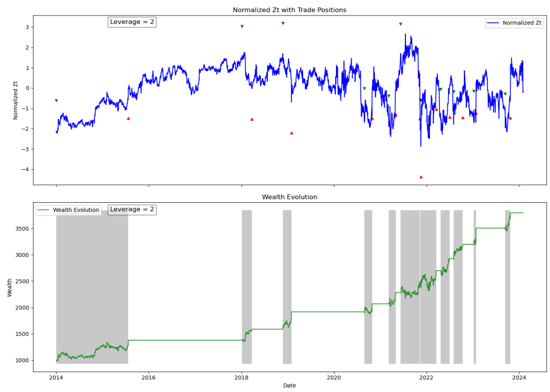
## Q.4.10

In our case, the wealth evolution with and without stop loss isn't significant. Meaning that the stop loss wasn't hit a lot of times and that our initial signals for exiting were most of the time good ones. This added only one trade more than without stop loss.
In conclusion, stop loss can be more useful in positions with bigger leverages, because we can win exponential money but also lose exponentially. Setting a stop loss in this setting seems more valuable in order to preserve our account from big losses.

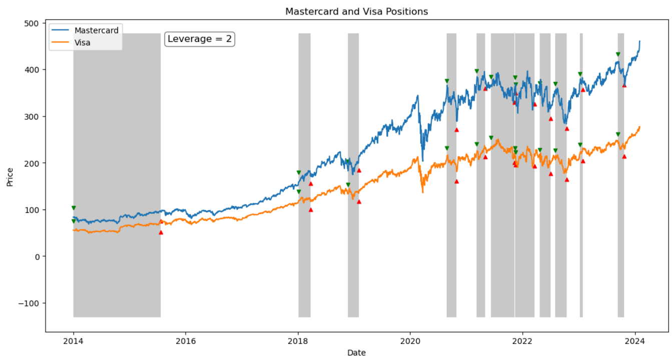
## *Results*

| | |
|---|---|
| Final wealth | 3'794.96 $ |
| Profit | 2'794.94 $ |
| Largest wealth | 3'794.94 $ |
| Lowest weath | 980.03 $ |
| Total Return | 279.50% |
| Buy and Hold Strategy Comparison - Visa | 397.90% |
| Buy and Hold Strategy Comparison - Mastercard | 451.29% |
| Number of trades | 12 |

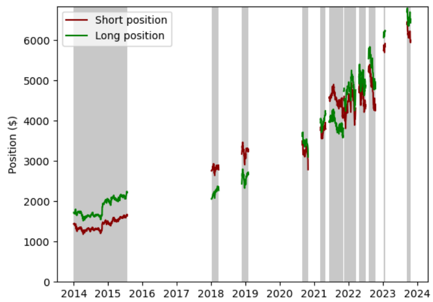
## *Normalized spreads & Wealth*



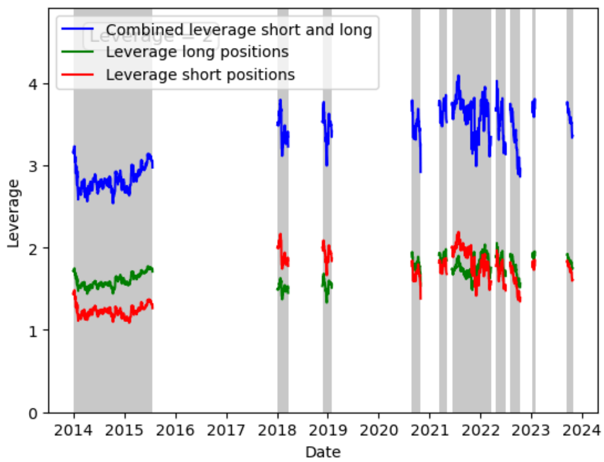

## *Positions on price*



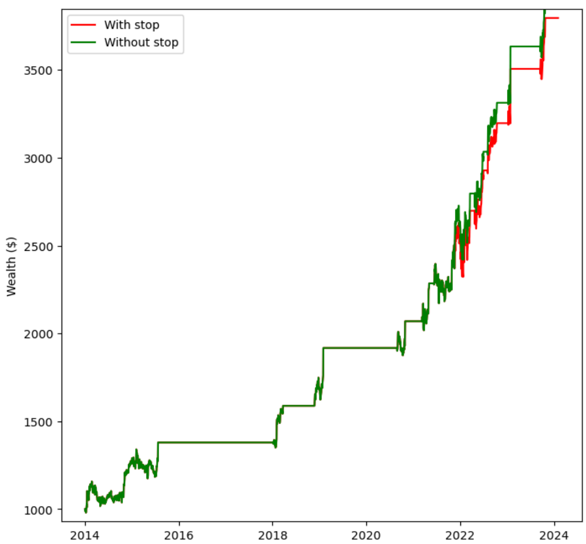

## *long/short positions*




## *Leverage*




## *Wealth comparison*



Q.4.11

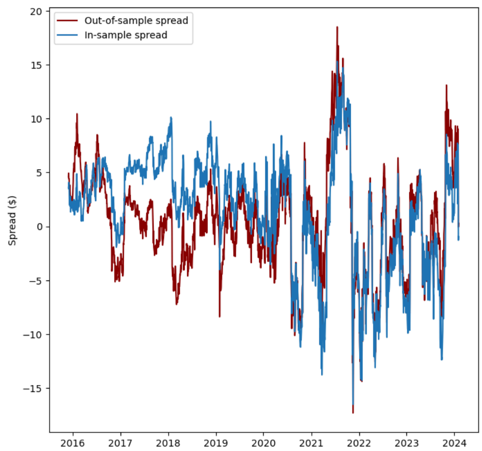
***Rolling correlation***


The rolling correlation between the prices is overall bigger than the rolling correlation between the returns. The correlation between prices is more volatile with the biggest drawdowns, using returns is more precise. They both increase over our sample.

Q.4.12


The rolling estimates started far from the full sample estimates but then as the rolling window advanced it got closer to the full sample estimates. This shows that since 2018, the time series didn't change a lot in its parameters properties compared to 2014-2016 where the parameters can be assumed to be different.

***In and out-of-sample spread***




We can see what we described about the parameters rolling estimates, the rolling estimates are starting to get closer to the full sample estimates by 2019, which is mirroring the parameters because the normalized spread computation depends on the rolling parameters.

Q.4.13

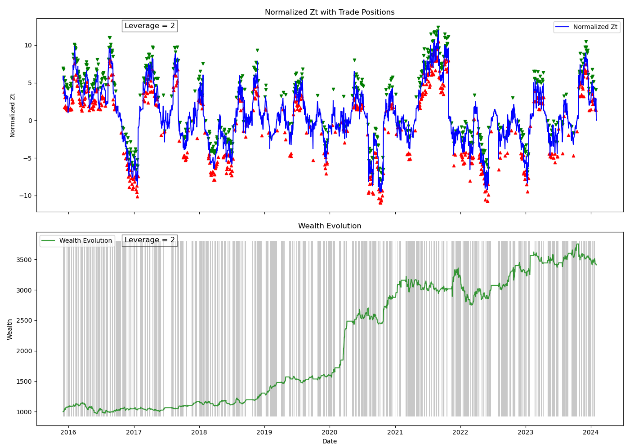
Using a rolling window for the estimates gives us a little bit less of final wealth with a lot more trades. Because of recomputing the parameters every 20 observations we get a lot more signals to get in and out of positions. In practice we have to do the rolling estimates, it is better than a full sample estimate on historical data because we update our parameters very often. This way, we aren't “cheating”.
Because in Q4.7, we estimated the full sample parameters and then made a simulation on it, it is like getting the full sample parameters from the “future” in a real life situation.

With this number of trades, we can also have more confidence in the robustness of this method. Having more than 500 trades and still being over 3400 dollars of final wealth is a good indicator of consistency for getting more winning trades overall than losing ones.

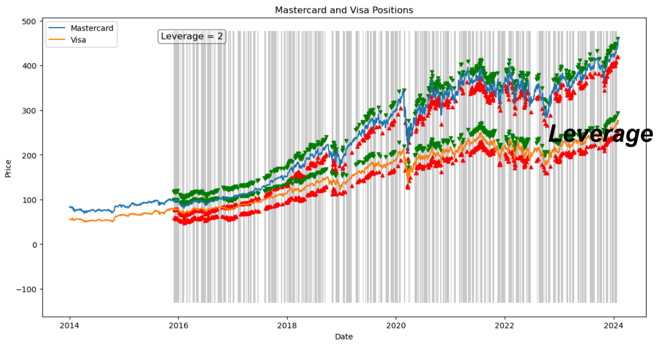
***Results***

| | |
|---|---|
| Final wealth | 3'411.27 $ |
| Profit | 2'411.27 $ |
| Largest wealth | 3'754.9 $ |
| Lowest weath | 970.24 $ |
| Total Return | 241.13% |
| Buy and Hold Strategy Comparison - Visa | 397.90% |
| Buy and Hold Strategy Comparison - Mastercard | 451.29% |
| Number of trades | 546 |

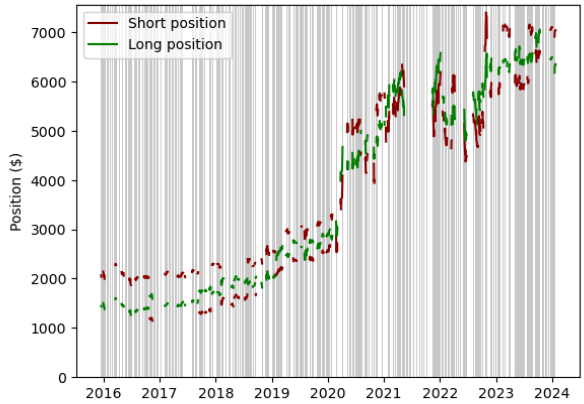
***Normalized spreads & Wealth***



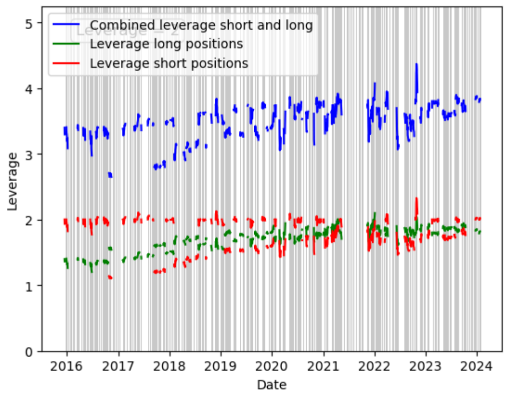
**Positions on price**




**long/short positions**

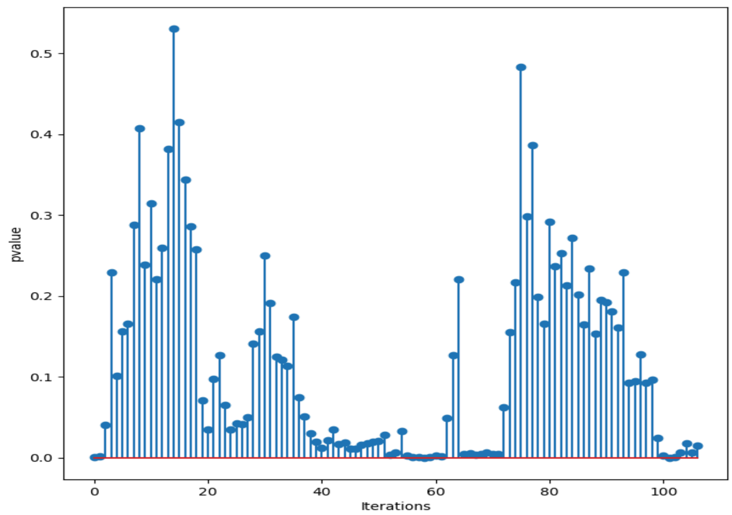



**Leverage**




Q.4.14

**P-value of cointegration**




We can see that we have cointegration especially in the middle and the very end of our graph for the subsamples. This implies that opening or even letting already opened positions with parameters that are not validated by the cointegration test is not statistically adequate. We have to trade only during the periods where cointegration is valid.

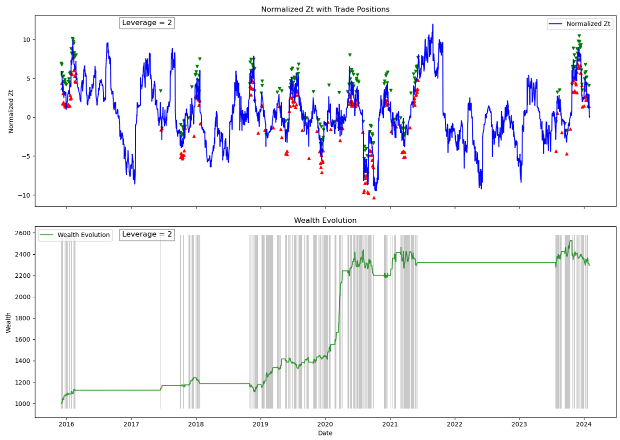
## Q.4.15

Now that we implemented the cointegration condition in our strategy, we can see on the graph that all the trades that happened did it during the cointegration validation if we compare it with the p-values graph. This of course diminished our number of trades and thus, this led us to lower final wealth of 2296 dollars. But this isn't necessarily bad, because this diminution of trades also prevented us from losing important ones, our lowest wealth never went under our initial wealth. This shows that doing it this way is more robust and safe for our wealth.

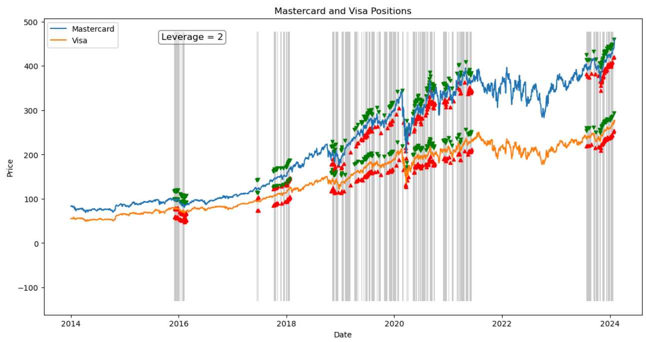
***Results***

| | |
|---|---|
| Final wealth | 2'295.52 $ |
| Profit | 1'295.52 $ |
| Largest wealth | 2'526.75 $ |
| Lowest weath | 1000.0 $ |
| Total Return | 129.55% |
| Buy and Hold Strategy Comparison - Visa | 397.90% |
| Buy and Hold Strategy Comparison - Mastercard | 451.29% |
| Number of trades | 203 |

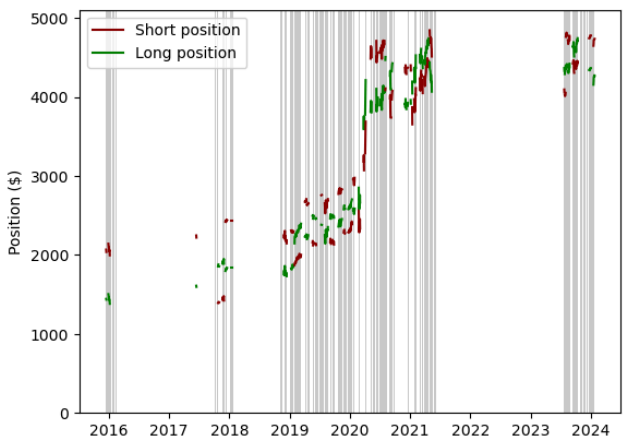
***Normalized spreads & Wealth***



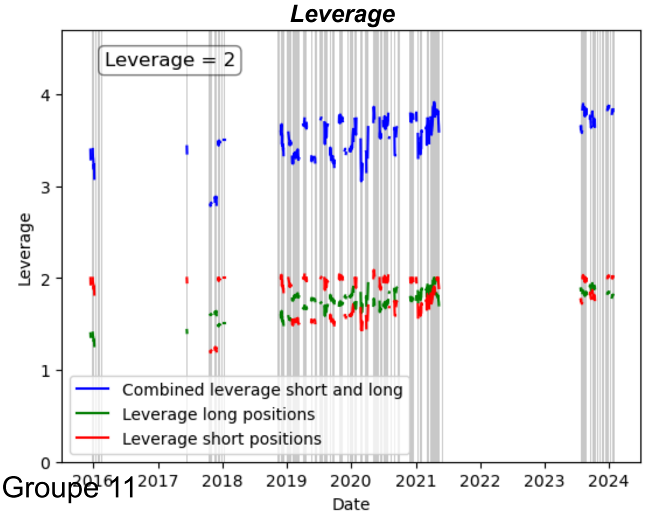

***Positions on price***




***long/short positions***




***Leverage***



Q.4.16

In conclusion, we augmented our simulation strategy little by little and not necessarily saw a jump in returns, but more importantly a jump in robustness and confidence in our strategy. We diminished the frequency of bad signals to keep a more qualitative wealth growth.
This strategy is considered market-neutral as it doesn't rely on the direction of the market but rather on the relationship between the pair.
Before opening our hedge fund we have :

- to get very fast internet latency and computer infrastructure, because in live trading we also have to consider this variable that adds more uncertainty.
- pay for private information about the market that isn't publicly available.
- to get better access to the quality of our data, for example directly working with tick data for intraday strategies or even daily to be able to simulate live trading.
- implementing a transaction cost structure that replicates the broker fees.
- considering liquidity and margin risks while having our position open.
- having a more complete risk management that relies on our percentage of our wealth that we are willing to risk per trade, instead of risking our whole account at every trade.
- considering other more advanced empirical methods that makes use of machine learning for potential returns prediction to complete our strategy