Available online at www.sciencedirect.com
SCIENCE DIRECT®
Optik 117 (2006) 49–57
Optik Optics
www.elsevier.de/ijleo

A simple method to simulate diffraction and speckle patterns with a PC

F. Gascón[a], F. Salazar[b,*]

[a]ETS Arquitectura (US). Avda. Reina Mercedes 2, 41012 Sevilla, Spain
[b]ETSI Minas (UPM). C/Ríos Rosas 21, 28003 Madrid, Spain

Received 20 January 2005; received in revised form 29 June 2005; accepted 3 July 2005

Abstract

A simple, intuitive and pedagogical method is proposed in order to simulate the phenomenon of light diffraction in simple cases. A diffracting plane obstacle is simulated by means of a model with a variable transmittance from one point to another. A numerical matrix is built with transmittance values at those points taken as samples. The matrix is handled with the Matlab program, and the Fraunhofer approximation is used. The method is applied to variable module and constant phase transmittance obstacles in single slit, double slit, square window and round window cases. The method is also applied to an obstacle with a constant reflectance module and random variable phase with Gaussian statistics, which gives rise to speckle phenomenon.

PACS: 01.50.Lc; 42.25.Fx; 42.30.Ms

Keywords: Simulation; Matlab; Diffraction; Speckle

1. Introduction

In the last two decades, simulation has played an increasingly important role in science and technology. Both, saving time and resources, are attained since it allows the study of problems under different conditions without having to carry out complex experiments to reach an approximate solution. The use of algorithms in Optics has been a particularly powerful tool in digital speckle interferometric systems for automatic measurement, as well as in speckle pattern simulations [1]. However, the programming of these algorithms is usually difficult. In these fields of Optics, the most employed computing languages are C and C++, which require a great quantity of steps and at the same time they have complex architecture (essentially, the difference between C++ and C consist in the bookkeeping with regard to valid and invalid access to certain data structures).

In the present work, we are interested in the simulation of diffraction patterns in two different situations: the generation of the intensity diagram of any geometry and the speckle pattern produced by a rough medium when it is illuminated by a laser beam. Very easy algorithms based on Matlab language are employed. In the procedure proposed in this paper, these diffraction patterns can be obtained with a program of about 30 lines.

2. Diffraction

Our interest lies in the diffraction of light. A parallel monochromatic light beam, with wavelength λ, falls

*Corresponding author. Tel.: +34 913364179; fax: +34 913366952.
E-mail address: fsalazar@dfarn.upm.es (F. Salazar).

0030-4026/$ - see front matter © 2005 Elsevier GmbH. All rights reserved.
doi:10.1016/j.ijleo.2005.07.001

perpendicularly onto a diffracting plane obstacle located on the OXY co-ordinate plane from $z<0$. The plane has a complex transmittance $t(x,y)$, which is variable from one point to another in a certain domain, while outside this domain there is no transmittance as a result of an opaque mask. The mask has a maximum linear size D; e.g., D is the diameter if we are dealing with a circular mask. An observation screen is located in front of and parallel to the diffracting plane at a distance z from that diffracting plane. The points on the observation plane are specified by means of their x', y' co-ordinates, with respect to the $O'X'Y'$ co-ordinate axis system on the same plane.

The diffraction phenomenon can be analysed by applying the Kirchhoff–Huygens principle for the case where there is only the mask, as long as the linear dimension of the mask is much longer than the length of the wave, $D \gg \lambda$, which is the usual situation in the field of Optics [2]. In this case, the diffraction can be dealt with in accordance with the Fresnel [3] approach if the distance from any obstacle point to another on the screen is much longer than λ. The more limiting Fraunhofer, or paraxial, approximation is to be to applied, which is appropriate if $z \gg \pi(x^2+y^2)/\lambda$. This inequality is verified if the co-ordinates of all of the aforementioned plane points are much smaller than the distance between the diffracting plane and the observation plane. A similar criterion [4] is that the square of the mask diameter be much less than λz. In short, the mask must be bigger in comparison with λ and the greater the size of the obstacle, the greater the distance to the observation plane.

The transmittance $t(x, y)$ is a function whose value remains approximately constant over a distance d. It seems logical to apply the same reasoning as applied to the size limits of the mask to size d. Hence, the Kirchhoff–Huygens principle and everything that follows only becomes meaningful when the distance d between two points, between which there is an important variation of the $t(x, y)$, is longer than the wavelength λ of the light used. On the contrary, if $t(x, y)$ varies considerably in distance d less than λ, the aforementioned principle cannot be applied.

In short, the principle requires

$$D>\lambda, \qquad d>\lambda.$$

It goes without saying that the observation must be carried out at a distance z much greater than D which is even greater than d.

According to Fraunhofer's diffraction theory, amplitude of the diffracted wave is proportional to the square modulus of the Fourier transform (FT) of the transmittance. The transmittance $t(x, y)$ is a function of co-ordinates of the points on the diffracting plane. Let us suppose that the light falls in the direction of the OZ-axis. The amplitude of the wave is a function of the spatial frequencies $\alpha = \cos\theta_x/\lambda$ and $\beta = \cos\theta_y/\lambda$.

3. The sampling

This paper deals with the numerical calculus of diffracted light intensity, and not its analytical solution. For this reason, the transmittance of the diffracting plane is specified by means of sampling. Let us take the samples at the cross points of the lines of a grid supported on the OXY co-ordinate plane. The vertical lines of the grid, perpendicular to the OX-axis, are the OY-axis and parallel lines at the respective distances u, $2u$, $3u$,… The distance u is called the horizontal sampling period and its value is known, or taken as understood, before starting the measurements. The sampling frequency is $f_s = l/u$, which is measured in m^{-1}. Analogously, the horizontal lines of the grid are separated by a distance v.

Let us first take N samples on one of the horizontal grid lines. The resulting sampling values form a string of N numbers are the transmittance measurements over the said line of the diffracting plane. The set of these transmittance values is grouped in the form of a matrix T of dimension $N \times 1$. The field variation of the matrix index i is $i = 1, 2, 3, \ldots, N$. This index stands for the respective co-ordinate points $x = 0, u, 2u, 3u, \ldots, (i-1)u, \ldots, (N-1)u$ (Fig. 1(a)). The correspondence between the i index and x is the u factor. Two neighbouring elements of the T matrix contain the transmittance values of two points on the plane separated by distance u in metres. Indeed, any two points whose indices are i_1 and i_2 are separated by a distance $x_1 - x_2 = (i_2 - i_1)u$.

Sampling always introduces a loss of information, given that no information exists between two neighbouring data. We lose, approximately, the frequency harmonic components that are greater than the sampling frequency.

When calculating the discrete Fourier transform (DFT) of the T sequence, a string of N elements is obtained, with which the A matrix of $N \times 1$ dimension is formed (Fig. 1(b)). The r row index varies from 1 to N and represents the harmonic components, whose frequencies are either $\alpha = 0$, $1/(Nu)$, $2/(Nu)$, …, $(r-1)/(Nu)$, …, $(N-1)/(Nu)$, or equally $\alpha = 0, f_s/N, 2f_s/N, \ldots,$

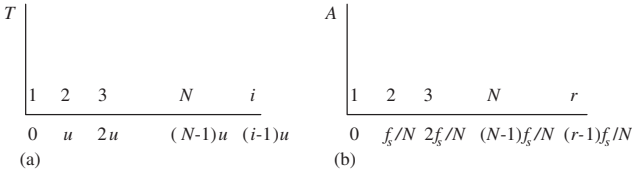

Fig. 1. (a) Matrix T of $N \times 1$ dimension and (b) discrete Fourier transform of string T.

$(r-1)f_s/N$, …, $(N-1)f_s/\mathrm{N}$. There is, therefore, a correspondence for A between index r, which is a pure number, and spatial frequency α, which is measured in m^{-1}, by means of f_s/N. Two frequencies, respectively, characterized by their values r_1 and r_2, have a spatial frequency difference of $\alpha_2-\alpha_1=(r_2-r_1)/(Nu)$. The direction $\theta_x=\cos^{-1}(\lambda f_s/N)$ corresponds to the frequency f_s/N, and the direction $\theta_x=\cos^{-1}[(N-1)\lambda f_s/N]$ refers to the maximum frequency $\alpha=(N-1)f_s/N$.

A co-ordinate point $x=(i-l)u$ on the sampled line corresponds to the ith element of matrix T. Given that i is an integer, the x co-ordinate of a point at which the sample is taken is an integer multiplied by the sampling period. It seems logical to introduce the adimensional variable $(x/u)=(i-1)$ (Fig. 2(a)). The new variable represents the distance from the origin of co-ordinates (measured in units of u) to the $(i-1)$th element of matrix T. In accordance with this adimensional interpretation, the neighbouring elements of matrix T represent the transmittances of two points 1 unit apart (that is to say, one u).

If r is the index of an element of matrix A, then this element represents the harmonic frequency $\alpha=(r-1)f_s/N=(r-1)/(Nu)$. By introducing adimensional numbers in matrix A, the adimensional number $\alpha u=r-1$ represents the frequency measured in units of $(1/Nu)$ of the $(r-1)$th element of the A matrix.

The aforementioned procedure can be easily performed as a laboratory measurement. Nevertheless, we are not interested at present in measurement, but in simulation. The elements of the T string are complex numbers, to which we are going to apply the DFT, and more specifically the fast Fourier transform (FFT). The resulting A string is easily obtained with a computer. The fact that the distance u between two close points, at which the measurements have supposedly been made, does not come up in the calculation process, must be borne in mind and can be taken as a parameter. The resulting A is "universal" with respect to u, in the sense that it is independent of the same. The DFT can be taken as pure mathematics, which gives the string of numbers A based on a T sequence. The units used in the measurement are unimportant, as are the resulting magnitudes, given that the DFT only handles numbers. Nevertheless, having started from supposed physical measurements T with evident physical meaning, it can be said that the element i of T is the measurement of the point transmittance located at position $x=(i-1)u$.

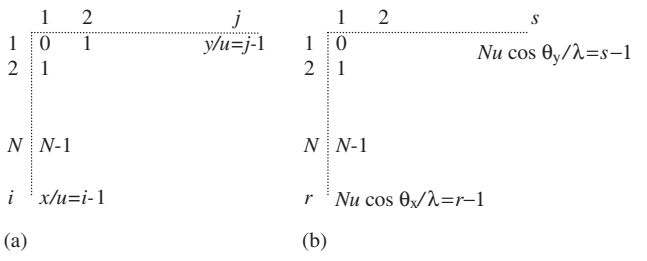

Fig. 2. (a) Non-dimensional co-ordinates. Two neighbouring elements of matrix T represent the transmittances of two points located a distance 1 unit apart. (b) Frequencies in units of u of the A matrix. The numbers denoted by i, j, r and s do not have units.

Matrix A can also be interpreted by its relation with α. When certain details interest us, we can assign any value to u, expressed in metres, and the resulting A gives us the harmonic amplitude, the frequency of which will automatically be expressed in m^{-1}. For example, assigning an arbitrary value of $10^{-7}\,\mathrm{m}$ to u and proposing that $N=128$, the first non-zero frequency component is $\alpha=1/(Nu)\equiv 78,125\,\mathrm{m}^{-1}$. Thus, supposing that N and u are data, α is defined. With α known and λ specified, θ_x is determined.

Moreover, it is quite true that two close points of an FFT row (or column) represent the complex amplitude of two harmonics, the frequency difference of which is $1/(Nu)$, and it is also true that the A matrix represents, in its row number r, the amplitude of the harmonic corresponding to the direction $\cos\theta_x=(r-1)\lambda/(Nu)$. Likewise, the non-dimensional co-ordinate axes $Nu\cos\theta_x/\lambda$ can be used (Fig. 2(b)). Moreover, the sampling theorem implies that the sample must be carried out over a period u, which must be fewer than half of the characteristic distances D and d. Consequently, Fraunhofer's approach will only be applicable to our study if

$$D>u, \qquad d>u$$

or if

$$f_s>1/(Nu)\gg 1/(Nd).$$

The FFT of the transmittance lacks negative frequencies. The zero spatial frequency occurs at $r=1$. Multiplication, element by element, of $T(j)$ by $(-1)^j$ gives a product matrix. The A matrix is then the FFT of this product matrix. Matrix A has the zero frequency centred and is made up of one harmonic of null frequency and $N/2$ harmonics with positive frequency, while the rest of the frequencies are negative. It is clear that when N is large, it can be said that the number of positive and negative frequencies is approximately $N/2$. In the FFT, the independent variable is the spatial frequency, and in the representation with positive and negative frequencies its maximum value of the frequency is $f_s/2$.

At a distance z from the scattering plane, a screen is placed whereupon the diffraction pattern appears (Fig. 3). For small observation angles θ the following can be written:

$$\alpha=\cos\theta_x/\lambda=x'/(z\lambda)\Rightarrow x'/z=\alpha\lambda,$$

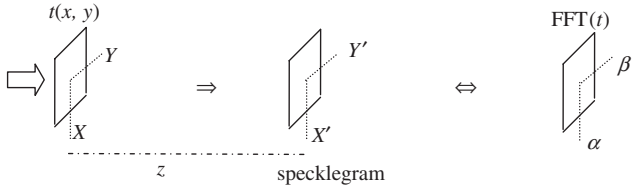

Fig. 3. This figure depicts a typical experimental set-up to obtain the diffraction pattern of an obstacle with the Fraunhofer approximation.

where the x' co-ordinate is proportional to the spatial frequency. On the other hand, the expression $\alpha = (r-1)/(Nu)$, which relates r with α, enables us to write

$$\cos\theta_x = x'/z = (r-1)\lambda/(Nu).$$

Therefore, the x' co-ordinate is a linear function of r. The maximum value of x' is $zf_s(\lambda/2)$.

This set out is easily applied to a two-dimensional simulation. The x and y co-ordinates of the point where the transmittance is supposed known must be taken into account. The transmittance values are placed in a two-dimensional matrix T. We are only going to study phenomena with a sampling period u along axis OX, which is identical to v for the OY-axis and with the same N number of samples in both directions. The matrix T of the supposed transmittance will therefore be of $N \times N$ dimensions.

Two diffracting plane points of co-ordinates (x_1, y_1) and (x_2, y_2) are separated by the distance $((x_2-x_1)^2+(y_2-y_1)^2)^{1/2}$. In the T matrix, two elements, (i_1, j_1) and (i_2, j_2), represent points separated by the distance $((i_2-i_1)^2+(j_2-j_1)^2)^{1/2}u$. Matrix A represents the amplitude for each spatial frequency. Two elements (r_1, s_1) and (r_2, s_2) of A represent points separated by the "distance" $((r_2-r_1)^2+(s_2-s_1)^2)^{1/2}/Nu$. The y' co-ordinate is a linear function of s in the same way as x' is of r.

Given that the frequent use of matrices is foreseen in this work, it seems logical to use the programming language known as Matlab throughout the rest of this paper.

4. The program

Two scattering plane models are to be proposed. The first has a transmittance whose module is variable with respect to the position and whose phase is constant. The second has a transmittance whose module is constant at all its points and whose phase is variable from one point to another. Let us now have a closer look at the first model.

The T transmittance matrix of $N \times N$ elements is going to be built. Each element of T contains the complex transmittance of a surface point separated from its neighbouring row and column by a distance equal to the sampling period u. The FFT of the T matrix provides the A matrix. The intensity of the diffracted light due to the diffracting plane is proportional to the square modulus of the FFT of T. The diffraction pattern is obtained by calculating the square of each element of A and drawing a figure whose points have a clarity proportional to $A.*A$.

The resulting two-dimensional FT lacks negative frequencies. When a two-dimensional FT is computed, the zero peak of the spatial frequency occurs not at the centre of the array but in the upper leftmost corner. To force the frequency origin to approach the centre of the array and the diffraction figures to be centred, the matrix $C(i,j) = (-1)^{i+j}$ is built and the data of matrix T are multiplied, element by element, by $C(i,j)$. The two-dimensional FFT of this product matrix gives matrix A, which has positive and negative frequencies and supplies the resulting centre pattern.

As has already been foreseen, the Matlab programming language is used. The program always begins with the allocation of the number of samples N along each axis. Let N be a power of 2 in order to simplify the calculation. The matrices used are of the form $M(i,j)$, the first element of which is $M(1,1)$ and the last is $M(N,N)$.

The matrix intensity is calculated as the square of the A matrix, the square being understood as the product of each element of the matrix by itself. The resulting I matrix is the true intensity, except for a numerical factor.

The elements $I(N/2+1,:)$, which are all in the almost central I row, can be plotted with the aim of visualizing the intensity profile along the most centred straight line possible.

With the command *imagesc*(T), the positive photo of the diffracting plane is drawn; with *imagesc*(I), the proportional diffraction figure is obtained, and with *plot*($I(N/2+1,:)$), the intensity is drawn as a function of the spatial frequency r along the horizontal axis. The aforementioned method is applied to those simple cases that appear in Optics manuals.

5. Results

5.1. Single slit

Take a plane that is opaque to light except for a transparent slit of width b equal to W times u. The slit is perpendicular to the OY-axis. A monochromatic light beam is perpendicularly incident on the plane. The

intensity of the resulting light measured from a long distance and its dependence on the angle θ is sought.

The width W of the slit is specified in units of u, that is to say, W is the j_2-j_1 interval of j at which the object is transparent. The transmittance matrix T is introduced in such a way that, for $j > N/2 - W/2$ and $j \leqslant N/2 + W/2$, their elements are set at value 1 and for the rest of elements they are zero. The program is very easy and takes up less than 30 lines (see Appendix A). Computation time and plot by a Pentium II are about 3 s. The results for $N = 128$ and $W = 8$ are given in Figs. 4–6.

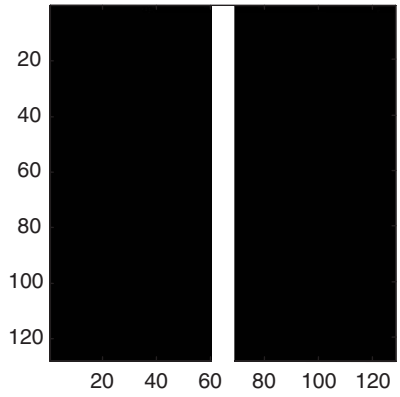

Fig. 4. A single slit of width 8.

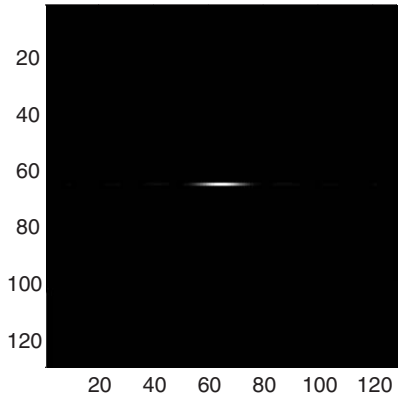

Fig. 5. Diffraction pattern of a single slit.

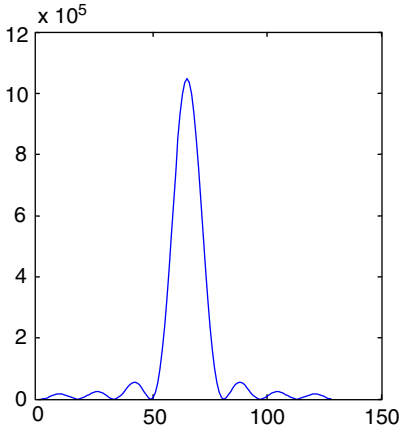

Fig. 6. Graphic of the light intensity diffracted by a single slit. The first lateral minimum and the central maximum are 16 units apart.

In accordance with the analytical theory of the diffraction of a slit, the first lateral minimum appears when $\cos\theta_y = \lambda/b = \lambda/(Wu) \Rightarrow \beta = 1/(Wu) = 1/(8u)$. On the other hand, from Fig. 6 a difference of $s_2 - s_1 \approx 16$ from the central maximum to the first minimum is deduced, hence $\beta_2 - \beta_1 = (s_2 - s_1)/(Nu) = 1/(8u)$, in complete agreement with the theory.

5.2. Double slit

Let us consider a plane that is opaque to light except for a transparent slit of width W, and another identical slit parallel to the first and $d \times u$ metres away (Fig. 7). Both slits are perpendicular to the OY-axis and the width W of the slits and distance d are specified.

The double slit transmittance is simulated by making the elements equal to one for

$$j > N/2 - W/2, \quad j \leqslant N/2 + W/2$$

and

$$j > N/2 - W/2 + D, \quad j \leqslant N/2 + W/2 + D$$

and zero for the rest of the elements. The results for $N = 128$, $W = 8$ and $d = 16$ are given in Figs. 8 and 9.

In case of the double slit, it is well known that first minimum is given by $\cos\theta_y = \lambda/(2du) = \lambda/(32u)$. If we examine the intensity pattern in Fig. 9, it shows a difference $s_2 - s_1 \approx 4$ between the central maximum and the first lateral minimum. Therefore, $\beta_2 - \beta_1 = (s_2 - s_1)/(Nu) = 4/(128u) = 1/(32u)$, which agree with the theoretical value.

In order to show the general validity of the result obtained, it is possible to study the diffraction maxima, in a similar way as carried out for minima. So, the equation for the maxima is $\cos\theta_y = ((n-1)\lambda)/(du)$, where n denotes an integer concerning the order of maximum ($n = 1$ corresponds to the direction of central

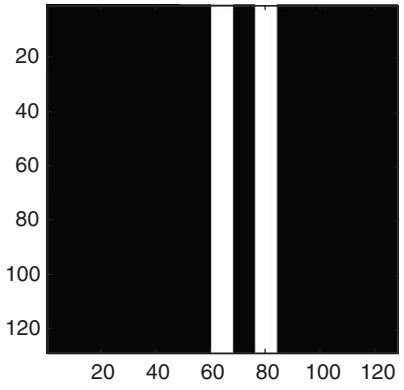

Fig. 7. A double slit a distance $d = 16$.

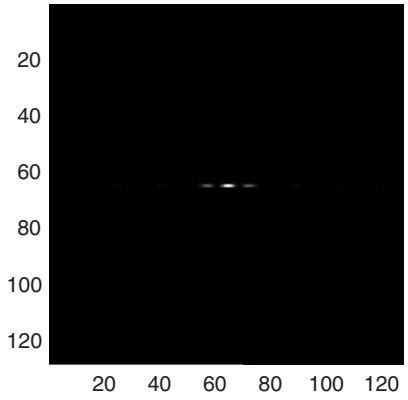

Fig. 8. Diffraction pattern of two parallel slits.

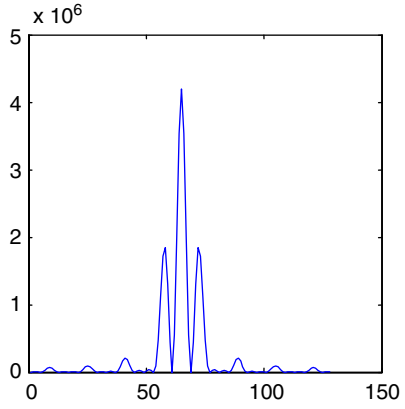

Fig. 9. Intensity distribution corresponding to a double slit.

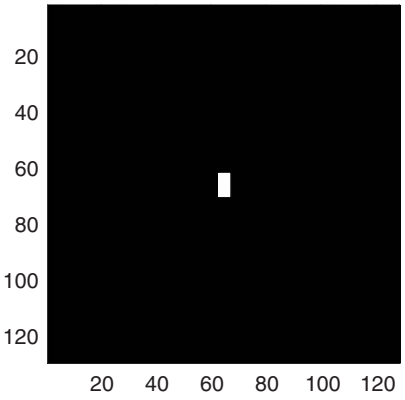

Fig. 10. A rectangular window 4×8.

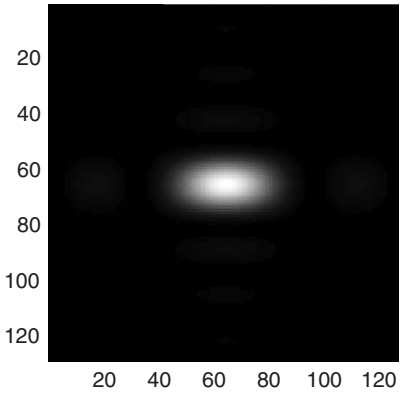

Fig. 11. Diffraction by a rectangular aperture.

maximum intensity). For $n = 2$ (first lateral maximum), and for the aforementioned value of d, that equality becomes $\cos\theta_y = \lambda/(16u)$. On the other hand, taking Fig. 9 into account, the difference from the first to the central maximum is $s_2 - s_1 \approx 8$, hence $\beta_2 - \beta_1 = \beta_2 = (s_2 - s_1)/(Nu) = 8/(128u) = 1/(16u)$. This result is again in concordance with another one.

5.3. Rectangular window

A diffracting plane is opaque to light except for a rectangular region which has width W and height H, both measured in term unit u.

The values of parameters N, W and H are introduced into the program. The transmittance is simulated by making transmittance matrix elements equal to one for $i > N/2 - H/2$, $i \leqslant N/2 + H/2$, $j > N/2 - W/2$ and $j \leqslant N/2 + W/2$, and by introducing the rest equal to zero. The values for $N = 128$, $W = 4$ and $H = 8$ are plotted in Figs. 10 and 11.

For that rectangular hole, a good concordance between numerical simulation and theory is obtained. To show it, we consider the solution for the diffraction pattern of that geometry, in which the maxima are easy identifiable. Valid analytical expressions for lateral maxima on both perpendicular frequency axes are

$$\beta_n = \left(\frac{(n-1)}{Wu} + \frac{1}{2Wu}\right)$$

and

$$\alpha_m = \left(\frac{(m-1)}{Hu} + \frac{1}{2Hu}\right) \qquad (n, m = 2, 3, \ldots),$$

respectively (as we seen in Section 5.2, these equations work from $n, m = 2$). Substituting first the value of W into β_2, one sees that $\beta_2 = 3/(2Wu) = 3/(8u) \approx 0.37$. If we now calculate the location of that maximum by means of Fig. 11, it follows that if $s_2 - s_1 \approx 46$, then $\beta_2 - \beta_1 = \beta_2 = (s_2 - s_1)/(Nu) = 46/(128u) = 23/(64u) \approx 0.36$, result similar to the analytical one. For other maxima on the perpendicular axis the procedure is the same.

5.4. Circle

Let us study a plane that is opaque, except for a circle with a diameter equal to D times u (Fig. 12). The data N and D are entered into the program. The transmittance is made equal to one in the matrix elements which verify

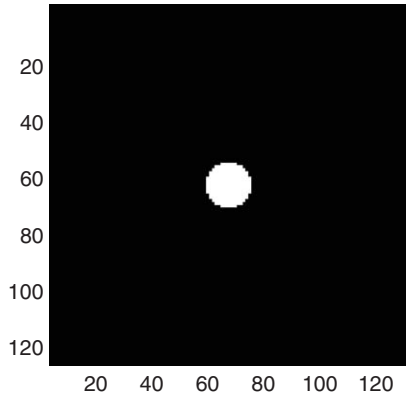

Fig. 12. Circular aperture of diameter 16.

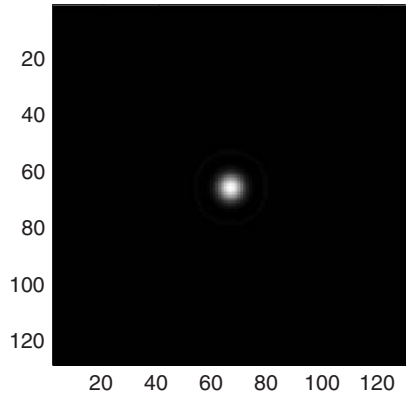

Fig. 13. Diffraction of a circular aperture.

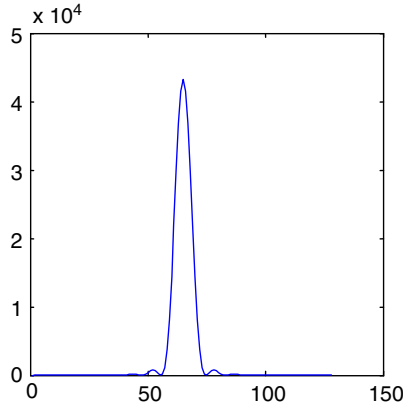

Fig. 14. Intensity of the light diffracted by a circle. The Airy disc has diameter of 20 units.

$(i-0.5-N/2)^2+(j-0.5-N/2)^2<(D/2)^2$ and zero for the rest.

The results for $N=128$ and $D=16$ are given in Figs. 13 and 14. The diffraction pattern calculated shows the classical Airy disc corresponding to diffraction through a hole circular in shape. The diameter of the disc can be estimated by Fig. 14 in 20 units of $1/(Nu)$, approximately. The physical diameter is $\Phi=Du$. Therefore, the angle θ, complementary to that under which the radius of the Airy disc is shown, is related with the spatial frequency q, given by $q=\sqrt{\alpha^2+\beta^2}$, through the following expression:

$$\cos\theta=\lambda q=\frac{1}{2}\,20\,\frac{1}{Nu}\,\lambda=10\,\frac{1}{(N\Phi/D)}$$
$$=10\,\frac{1}{(128\Phi/16)}=1.25\,\frac{\lambda}{\Phi},$$

which is completely in agreement with the expected value.

5.5. Speckle pattern

It is well known that when a beam of coherent light illuminates a rough surface the so-called speckle phenomenon arises. This phenomenon is attributed to the fact that the electromagnetic wave falling on the surface has a different optical path to the point of observation, depending on its point of incidence. When the surface is uneven, the complex reflectance is a random function and there will be a certain probability that at some point in space and in its surroundings, there is a maximum intensity. These brilliant spots have irregular shape and irregular distribution and they provide speckle on an observation screen.

A reflecting surface, e.g., a flat sheet of metal which is not well polished, is illuminated perpendicularly; whereby the diffracted wave travels a path δ which is different at each obstacle point. A model of diffusing and non-absorbent surface is therefore proposed. Points of the model are at different heights above a reference plane and heights follow random Gaussian distribution. It is supposed that the aforementioned surface is illuminated by a parallel coherent light beam and the diffraction pattern is calculated by means of the FFT, with Fraunhofer's approximation. The result is a typical speckle pattern.

The optical path and wavelength are handled as a single adimensional variable, δ/λ, with the aim of the results being applicable to any wavelength. This single variable is made equal to a constant multiplied by a random number. The aforementioned constant is denoted as *RF* and it represents a roughness modulating factor. Random numbers with Gaussian distribution are generated by the command *randn*. The final expression for reflectance is *exp(i2πRFrandn)*. Thus, an element of *RFrandn*(*N*) is a number equal to an optical path measured in wavelengths. For example, an $RF=1$ and a $randn=2$ give rise $RFrandn=2$, which indicates a path difference of 2λ, that is to say, a scratch on the reflecting surface with depth equal to λ. However with the same *randn*(*N*), but with the modulating factor equal to 0.1, the roughness would be a tenth part. The height of the

sample points is, therefore, $\lambda RF.\ Randn(N)/2$. Hence, the RF factor represents the roughness measured in wavelengths.

The study of the reflecting surface is restricted by means of a round absorbent mask of diameter D. In theory, the latter can be any size, as long as it is longer than the wavelength and the sampling period. Nevertheless, if one wishes the result to be representative of the Gaussian distribution, the number of points sampled inside the mask area must be large enough.

A reflectance mask of $N \times N$ elements is constructed. Each element of the said matrix contains the complex reflectance of a surface point separated from its row and column neighbours by a distance equal to the sample period u. The area corresponding to the mask is represented by points outside a circle with null reflectance. The FFT of the reflectance gives rise to matrix A. The intensity of the diffracted light corresponding to the rough surface is proportional to the square of each one of the reflectance matrix FFT elements, which are exactly the diffraction pattern of the rough surface.

The program includes:

(1) Building the matrix $RAN = randn(N)$.
(2) Specifying the diameter D of the light beam or mask, and building the BS matrix representing the mask centred on the $N \times N$ matrix.
(3) Giving the roughness modulating coefficient RF a value.
(4) Obtaining the optical path expressed in wavelengths, $OP = 2*RF*RAN$.
(5) Calculating $RANR = RF*RAN$.
(6) Calculating the product matrix (element by element) $RANR.*BS$.
(7) Building the reflectance matrix $T = BS.*\exp(i2\pi OP)$.
(8) Making the matrix $C(i,j)$.
(9) Preparing the reflectance matrix to centre the pattern, by means of $C.*T$.
(10) Calculating matrix A, which is the two-dimensional FFT matrix.

The program takes up fewer than three dozen lines.

Figs. 13–18 show the numerical results carried out with a PC. The starting data were $N = 128$ and $D = 16$. Figs. 13 and 14 correspond to $RF = 0$, and therefore, the surface under study is a perfect round mirror of diameter $16u$, surrounded by a black mask. The diffraction pattern corresponds to diffraction through a hole (again the same as Section 5.4). Figs. 15 and 16 refer to a coefficient $RF = 0.4$. The profile shows peaks and troughs. The result is a diffraction pattern similar to Fig. 13 in the centre (compare Figs. 14 and 16), surrounded by diffuse spots. Figs. 17 and 18 correspond to $RF = 1$ and have high peaks. The effect of the mask

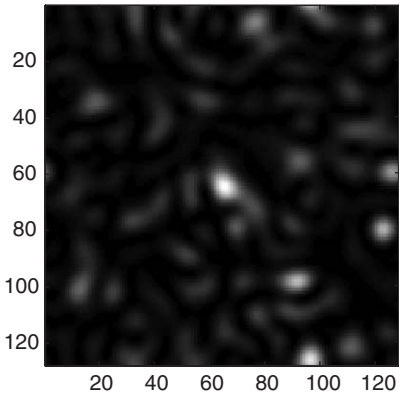

Fig. 15. Diffraction pattern of a circle with roughness factor $RF = 0.4$. It shows a central maximum surrounded by a blurred pattern.

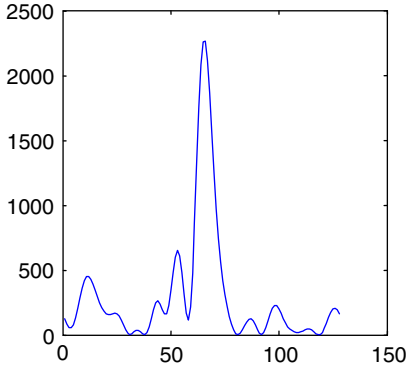

Fig. 16. Intensity of the light diffracted by a surface with $RF = 0.4$.

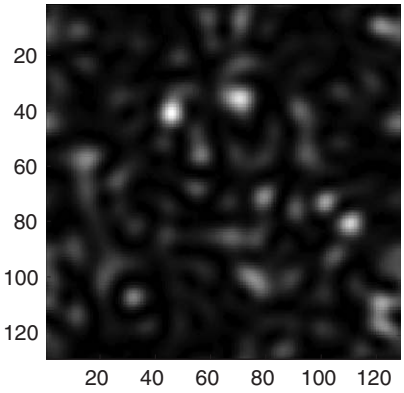

Fig. 17. Typical speckle pattern ($RF = 1$).

circle has disappeared. Only the spots remain, both irregularly shaped and distributed. This phenomenon is called speckle, well known in the field of applied Optics.

With the purpose of corroborate the proposed speckle model, let us consider the mean width of the intensity peaks shown in Fig. 18. So, we assume as a possible measure of the width, the length of a rectangle whose

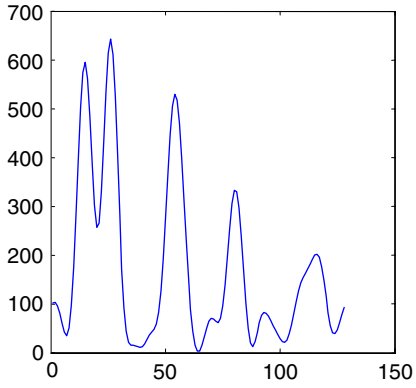

Fig. 18. Plot of the speckle intensity pattern on a direction.

height is equal to the half of the intensity maximum (this is more or less the same as the distance, on the non-dimensional axis, from the speckle centre to the first minimum). In our case, a rough estimation of it leads to a difference of $(s_2 - s_1)$ in the interval which runs from 7 to 12, approximately (it depends on the spot selected). Thus, the special frequencies for such a speckle are of the order of $(s_2 - s_1)/(Nu) \approx 9.5/(128u) = 0.07/u$. On the other hand, if we assume the average speckle diameter σ_s at about $(1.2\lambda)/D$, the following expression applies:

$$\frac{\sigma_s}{\lambda z} \approx \frac{\cos\theta}{\lambda} = \beta = \frac{1.22}{D} = \frac{1.22}{16u} = \frac{0.08}{u},$$

whose value is analogous to simulation.

Appendix A

```
%Diffraction of a single slit:
clear
N = 128% row number N and column number N
for i = 1:N
   for j = 1:N
      C(i,j) = (-1)^(i+j);% C is to centre the pattern
   end
end
C;
W = 8%W is the slit width
for i = 1:N
   for j = 1:N
      if j>N/2-W/2&j<=N/2+W/2
         tr(i,j) = 1;% actual transmittance
      else
         tr(i,j) = 0;
      end
   end
end
t = C.*tr;
fft = fft2(t);%two-dimensional fast Fourier transform
I = abs(fft).*abs(fft);%intensity
colormap(gray)
subplot(2,2,1)
imagesc(tr)
subplot(2,2,2)
imagesc(I)
subplot(2,2,3)
plot(I(N/2+1,:))
return
```

References

[1] J.M. Huntley, Speckle photography fringe analysis: assessment of current algorithms, Appl. Opt. 28 (1989) 4316–4322 (see references therein).
[2] J.A. Stratton, Théorie de l'électromagnétisme, Dunod, Paris, 1961 (p. 531).
[3] T. Kreis, Holographic Interferometry, Akademie Verlag, Berlin, 1996 (p. 28).
[4] S.G. Lipson, H. Lipson, Tannhauser: Optical Physics, Cambridge University Press, Cambridge, 1995 (p. 162).