Modelling of interference pattern produced by Michelson interferometer

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Modeling of interference pattern produced by Michelson interferometer

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ABSTRACT

Using of Michelson interferometer is shown in the field of measurement of periodical displacements of the controlled object. The foundations of optical interferometry are presented. The features of Michelson interferometer are described. The mathematical model of interference pattern produced by Michelson interferometer is created. It takes in consideration such parameters as the angles at which the mirrors are located and the lengths of two optical paths.

Keywords: Interferometer, Michelson interferometer, mathematical model, vibrations, MATLAB

1. INTRODUCTION

Modern technologies require continuous monitoring of the parameters of manufacturing process and control of equipment. Parameters of mechanical motion are among of the most important parameters, in particular parameters of periodical displacements of the controlled object (vibrations).

Measuring of vibration displacement is necessary in different spheres:[1,2] semiconductor electronics (control of vibrations of crystal growth apparatus), microelectronics[3] (vibrations of photomask processor), machine building (vibration of machines and jumping of details), automobile industry (control of vibrations of cars elements), in railway transport (sensors for detection of approaching train), aircraft building (control of turbines jumping), power engineering (control of vibrations of turbine blades).

2. MAIN METHODS OF VIBRATIONS DETECTION

There are two big groups of vibration detection methods. The first group includes contact methods. These methods arent sensitive to high-frequency vibrations and vibrations with low amplitudes. Also, if the controlled object has low weight, such contact detectors could influence the character of vibration.

Mechanical connection between the object and detector is not always acceptable and recent years the focus in on the development of non-contact methods of vibrations control. They have low persistence and do not influence of the controlled object.

Due to the short wavelength of visible light interferometry allows determining small change in optical path length. Principle of operation of an interferometer is to divide the beam of electromagnetic radiation into two or more coherent beams, after which they pass through different optical paths and directed to the screen, where the phase difference between two beams (one is reflected from the surface of the object and the other is the reference beam, reflected from the mirror) can be measured from the interference pattern.

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3. MAIN FOUNDATIONS OF INTERFEROMETRY

Interference of light is the phenomenon in which two or more light waves superpose to form a resultant wave of greater, lower or the same amplitude in certain spatial region. The resultant intensity is expressed by the equation 1:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi \tag{1}$$

where I_1, I_2 are intensities of initial waves, $\Delta\phi$ is the phase difference between waves in the point of observation. Interference is possible only when using coherent sources. Coherent waves have a constant phase difference and the same frequency.

An interference intensity distribution presents a system of alternating minimums and maximums of intensity, that is, bright and dark stripes.

From formula 1 maximum of intensity is observed when the phase difference $\cos\Delta\phi = 1$, i.e. when the phase difference between the waves in the point of observation equal to an even number of π:

$$\Delta\phi = 2\pi k, (k = 0, 1, 2). \tag{2}$$

Minimum of intensity is observed when $\cos\Delta\phi = -1$, i.e. when the phase difference between the waves in the point of observations equal to odd number of π:

$$\Delta\phi = (2k + 1)\pi, (k = 0, 1, 2). \tag{3}$$

When the intensities of two superposing waves equal to each other ($I_1 = I_2 = I_0$) the resultant intensity is expressed by formula:

$$I = 2I_0(1 + cos\Delta\phi) \tag{4}$$

Thereby intensity in maximums is four times greater than the intensities of each wave $I = 4I_0$.

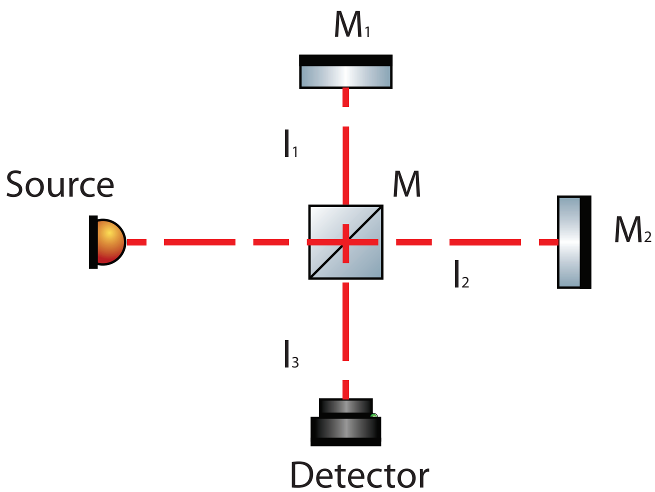
4. THE MICHELSON INTERFEROMETER

The Michelson interferometer is the device for observation of interference patterns by superposing of two waves derived from one initial light wave by amplitude division. The scheme of the Michelson interferometer is on the Fig. 1a. Interferometer consists of two mirrors (M_1 and M_2) and beam splitter (M), which represents a plane-parallel glass plate with a semireflecting coating. The resultant interference pattern can be observed on the screen or can be examined using light radiation receiver.

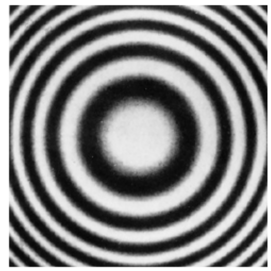
The light beam from source falls on the beam splitter where its divided into two beams: the first one after passing beam splitter goes towards mirror M_1 and the second is directed towards mirror M_2 after reflection off the beam splitter. After reflection off the mirrors the beams go back to the beam splitter where the first beam reflects and the second passes the beam splitter once more. Then the beams are directed to the detector and overlap.

Distances between the beam splitter and the mirrors are called arms. The Michelson interferometer is well-suited for such application as measuring vibrations[4,5] parameters because two optical paths are well separated. The most interferometers for measurement of linear displacements are based on the Michelson interferometer scheme. Also it's easy to set up and align.

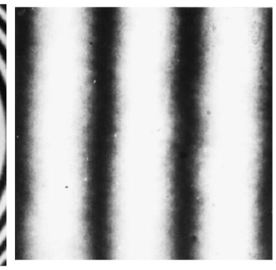
One of the mirrors (M_1) can be rotated about vertical and horizontal axes and the other can be mover along the axis of the system. It allows both to change the path difference between beams and the direction of their propagation. Depending on the mirrors position different interference patterns could be observed. The interference pattern changes its appearance depending on the angle between two reflecting surfaces. If two surfaces are strictly perpendicular to each other, the interference pattern will be fringes of equal inclination



(a) Optical scheme of the Michelson interferometer.



(b) Fringes of equal inclination



(c) Fringes of equal thickness

Figure 1: Optical scheme and interference picture in different conditions

(fig. 1b), localized at infinity. If the mirrors have deviation of perpendicularity pattern takes the form of fringes of equal thickness (fig. 1c).

There are three conditions of producing fringes of fringes of equal inclination (the same intensity value corresponds to the same inclination): mirrors should be exactly perpendicular to each other; the difference between lengths of two arms should be nonzero; the source point should be used. If lengths difference is zero the uniform bright background will be observed.

The Michelson interferometer allows producing the other type of interference pattern, in particular fringes of equal thickness (they look like parallel stripes). For observation of this type of pattern one of the mirrors should be rotated for a small angle from perpendicularity and the wave from the source should be plane.

5. MATHEMATICAL MODEL OF INTERFERENCE PATTERNS PRODUCED BY THE MICHELSON INTERFEROMETER

During the work the mathematical model of interference patterns were created in MATLAB environment. The mathematical model takes in consideration such parameters as the angles at which the mirrors are located and the lengths of two optical paths (fig. 2).

At the beginning values of lengths of the interferometers arms l_1,l_2,l_3, the thickness of the beam splitter d and its material refractive index n, the mirror tilt angle θ, the distance between two neighboring rays Δ, the operating wavelength λ, the intensity of the initial ray I_0 and the number of accountable rays are entered from keyboard.

Then the program calculates optical paths of transmitted and reflected rays.

Optical path for the reflected ray:

$$S_1 = 2l_1 + \frac{d}{\cos\sin^{-1}\frac{\sin\alpha}{n}} + l_3 \tag{5}$$

Where $\alpha = \pi/4$ is the angle of the beam splitter arrangement. If the second ray goes i. e. 1 mm higher ($\lambda = 1$ mm), the optical path is 1 mm less. So, in general

$$S_{1,g} = 2 \cdot (l_1 - \frac{i}{\Delta}) + \frac{d}{\cos\sin^{-1}\frac{\sin\alpha}{n}} + l_3 \tag{6}$$

Figure 2: The modeled interference pattern.

Where i is the number of the cycle iteration (or the number of the ray).

Optical path for transmitted ray consists of three summands. The optical path in the beam splitter:

$$S_{2,glass} = \frac{d}{\cos(\sin^{-1}\frac{\sin\alpha}{n})} + \frac{2d}{\cos(\sin^{-1}\frac{\sin[\alpha+2\theta]}{n})} \tag{7}$$

The optical path between the beam splitter and the mirror M2:

$$S_{2M2} = l_2 - i - \frac{i}{\tan(2\alpha-\theta)} + \frac{(i_2 - i - \frac{i}{\tan(\pi/2-\theta)}) \cdot \sin 3\alpha}{\sin\alpha + 2\theta} \tag{8}$$

The optical path between the beam splitter and the detector:

$$S_{2,d} = \frac{l_3 + i - [2d \cdot \tan\sin^{-1}\frac{\sin\alpha+2\theta}{n}]}{\sin(2\alpha-2\theta)} \tag{9}$$

The total optical path of the transmitted ray is:

$$S_{2,\sum} = S2, glass + S2M2 + S_{2,d} \tag{10}$$

After calculating the optical paths the program computes the phase difference:

$$\Delta\phi = \frac{2\pi(S_1 - S_{2\sum})}{\lambda} \tag{11}$$

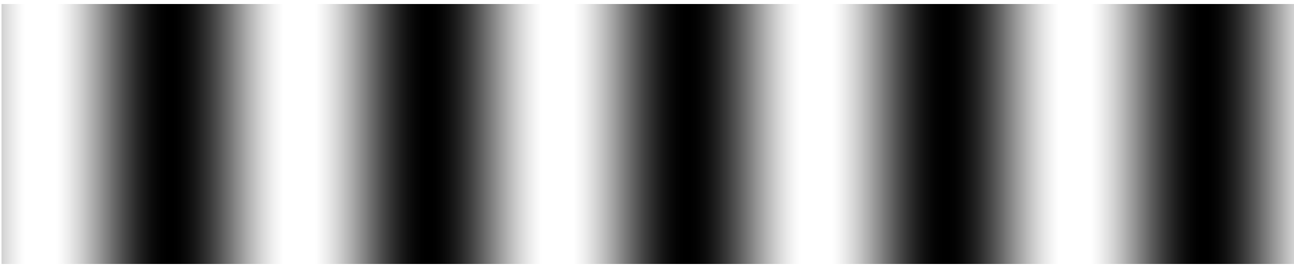
After that the intensity of the interference pattern point is calculated using formula 4.

These operations are repeated as many times as many accountable rays are specified. Consequently the program generates an image as shown on the fig. 2.

The program allows modeling the movement of the mirrors. Moving the mirror $M1$ changes the fringes positions and it should be noticed that each time the pattern returns to an identical position, the mirror has moved half-wavelength.

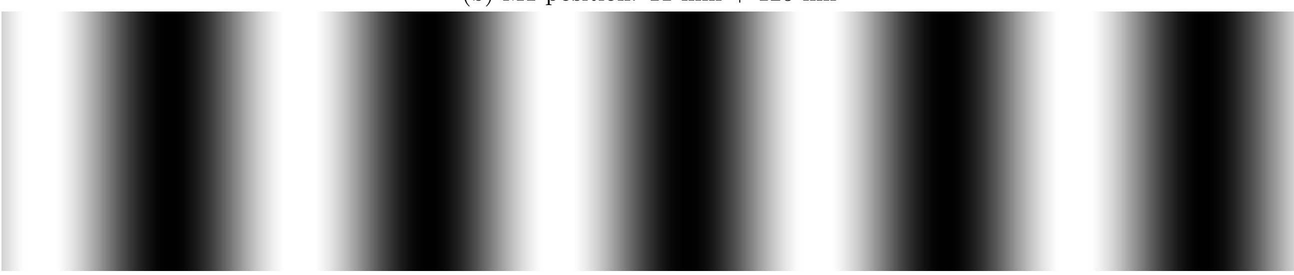
Let the wavelength be 500 nm. The figure 3 illustrates how the interference pattern changes depending on the mirror $M1$ movement.

Also fringes of equal inclination were modeled (fig. 4). They also depend on position of the mirror.



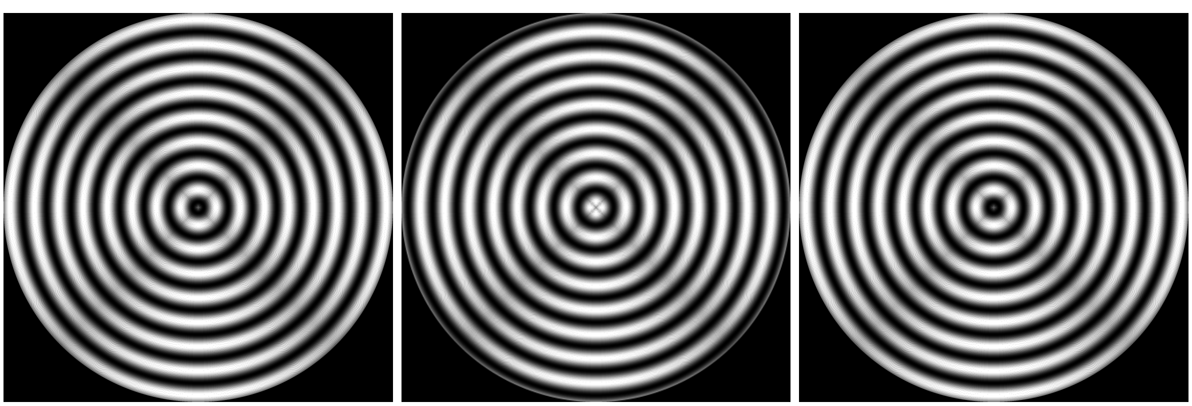
(a) M1 position: 11 mm

(b) M1 position: 11 mm + 125 nm



(c) M1 position: 11 mm + 250 nm

Figure 3: Changes in the interference pattern (wavelength is 500 nm)



(a) M1 position: 11 mm (b) M1 position: 11 mm + 125 nm (c) M1 position: 11 mm + 250 nm

Figure 4: Fringes of equal inclination (wavelength is 500 nm)

6. CONCLUSION

This article describes the main foundations of interferometry, the main features of the Michelson interferometer and its advantages in measurement of linear displacements (including vibrations). The mathematical model of the interference pattern is designed for investigation of dependence of interference pattern on the changes in lengths of the interferometer arms. This mathematical model in MATLAB environment will help in arrangement of the Michelson interferometer scheme for hight precision measurement of vibrations parameters.

ACKNOWLEDGMENTS

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